SSC-N-271

GEOMETRY OF 60° AND 90° LATTICES

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ABSTRACT

The geometry of the CDR 60° cell lattice with 5 dipoles per half cell, and of three proposed substitutes[1-2-3] 90° cell lattice with 6 dipoles per half cell, is described.

1 Geometry of the CDR 60° Lattice

Some approximations have to be made:

-- uniform bending within each cell is assumed

-- the calculations below ignore vertical bending

The whole ring has 384 cells of 192 m each. This total includes the dispersion suppressors cells, which are identical in length and curvature to normal cells. Hence:

$$\theta_{cell} = \theta_c = \frac{360°}{384} = .9375° = 16.362462 \text{ mrad}$$

The average radius of the arcs and dispersion suppressors is:

$$R_0 = \frac{L_{cell}}{\theta_c} = \boxed{11{,}734.17564 \text{ m} = R_0}$$

1.1 Description of the Arcs

Each (upper or lower) arc has 166 cells with a total length of 31,872 m.

$$\theta_{arc} = \theta_a = 166\theta_c = 155.625°$$

$$\frac{\theta_a}{2} = 77.8125°$$

1.2 Description of a Cluster Module

Each (near or far) cluster contains four cluster modules. A module, moving clockwise, contains: 3 cells (dispersion suppressor: d.s.), straight section (6 cells equivalent length), and 3.5 cells (d.s.).

To avoid asymmetry, it is possible to describe a module as below:

	3.25 Cells	Straight Section	3.25 Cells
Length	624 m	$\boxed{L_s = 1152 \text{ m}}$	624 m
Angle	$\boxed{\theta_D = 3.046875°}$ = 53.178 mrad	0° = 0.0 mrad	θ_D = 3.046875° = 53.178 mrad
Radius	R_0	∞	R_0

The total length of a cluster module is 2400 m.

1.3 Description of the Ring

The complete ring consists of eight straight sections of length L_s separated by arcs of various lengths, with radius of curvature R_0. Its total length is $\boxed{82{,}944\ \text{m}}$.

The geometry is defined by the dipoles, and the ring has reflection symmetry about the major and minor axes. These axes intersect the ring in the center of five-dipole strings of a half-cell.

The top right quadrant of the ring (Fig. 1a, page 5) consists, starting from a, of an arc with angle θ_D, a straight section of length L_s, an arc of angle $2\theta_D$, another straight section of length L_s, another arc of angle θ_D, and a final big arc of angle $\frac{\theta_a}{2}$.

The other three quadrants are obtained by reflection about the major and minor axes. To obtain the complete figure we first calculate the coordinates of the centers of curvature of the arcs. These points define a polygon with eight sides of lengths L_s at the angles to each other enumerated above.

Within the top right quadrant, the coordinates P, Q, T of the polygon are as follows:

1.3.1 Coordinates of the Central Polygon

$$P \begin{cases} x_P = L_s(\sin\theta_D + \sin 3\theta_D) = 244.2368\ \text{m} \\ y_P = 0 \end{cases}$$

$$Q \begin{cases} x_Q = L_s \sin 3\theta_D = 183.0046\ \text{m} \\ y_Q = L_s \cos\theta_D = 1150.3715\ \text{m} \end{cases}$$

$$T \begin{cases} x_T = 0 \\ y_T = L_s(\cos\theta_D + \cos 3\theta_D) = 2287.7427\ \text{m} \end{cases}$$

1.3.2 Coordinates of Critical Points of the Ring

From the arc centers on the polygon, we find the coordinates of the critical points of the ring a to g:

a: geometric cluster center

$$\begin{cases} x_a = x_p + R_0 = \boxed{11{,}978.412 \text{ m} = 1/2 \text{ minor axis}} \\ y_a = 0 \end{cases}$$

$$b \begin{cases} x_b = x_p + R_0 \cos \theta_D = 11{,}961.825 \text{ m} \\ y_b = R_0 \sin \theta_D = 623.706 \text{ m} \end{cases}$$

$$c \begin{cases} x_c = x_Q + R_0 \cos \theta_D = 11{,}900.593 \text{ m} \\ y_c = y_Q + R_0 \sin \theta_D = 1774.077 \text{ m} \end{cases}$$

$$d \begin{cases} x_d = x_Q + R_0 \cos 3\theta_D = 11{,}768.173 \text{ m} \\ y_d = y_Q + R_0 \sin 3\theta_D = 3014.441 \text{ m} \end{cases}$$

$$e \begin{cases} x_e = R_0 \cos 3\theta_D = 11{,}585.168 \text{ m} \\ y_e = y_T + R_0 \sin 3\theta_D = 4151.812 \text{ m} \end{cases}$$

$$f \begin{cases} x_f = R_0 \cos 4\theta_D = 11{,}469.710 \text{ m} \\ y_f = y_T + R_0 \sin 4\theta_D = 4764.963 \text{ m} \end{cases}$$

g: geometric upper arc center

$$\begin{cases} x_g = 0 \\ y_g = y_T + R_0 = \boxed{14{,}021.918 \text{ m} = ½ \text{ major axis}} \end{cases}$$

1.3.3 Coordinates of Arc and Cluster Centers and Boundaries

Due to the quarter cell counter-clockwise shift made to symmetrize the cluster module, the geometric symmetry points a and g do not fall at quadrupole centers.

For distribution systems, etc., it is necessary to list the coordinates of the quadrupole centers a', g' which are, respectively, 1/4 cell clockwise less 2.59 m (see note down this page, and Fig. 1b, page 5.)

$$a' \begin{cases} x_{a'} = x_P + R_0 \cos\left(\frac{\theta_c}{4} - \frac{2.59 \times 360}{2\pi R_0}\right) = 11978.325 \text{ m} \\ y_{a'} = -R_0 \sin\left(\frac{\theta_c}{4} - \frac{2.59 \times 360}{2\pi R_0}\right) = -45.410 \text{ m} \end{cases}$$

$$g' \begin{cases} x_{g'} = R_0 \sin\left(\frac{\theta_c}{4} - \frac{2.59 \times 360}{2\pi R_0}\right) = 45.410 \text{ m} \\ y_{g'} = y_T + R_0 \cos\left(\frac{\theta_c}{4} - \frac{2.59 \times 360}{2\pi R_0}\right) = 14021.830 \text{ m} \end{cases}$$

In addition, we list the true boundary f' between the arc and the cluster, which is the beginning of the dispersion suppressor:

$$f' \begin{cases} x_{f'} = R_0 \cos\left[4\theta_D - \left(\frac{\theta_c}{4} - \frac{2.59 \times 360}{2\pi R_0}\right)\right] = 11479.211 \text{ m} \\ x_{f'} = y_T + R_0 \sin\left[4\theta_D - \left(\frac{\theta_c}{4} - \frac{2.59 \times 360}{2\pi R_0}\right)\right] = 4720.558 \text{ m} \end{cases}$$

Notes: - Fig. 1a has reflection symmetry about OP and OT (see above) but also has local symmetry about the middle of bc, cd, and de.

- Points a,f, and g are the center of dipole strings between two quads. The mid-half cell points, defined by quads, are 2.59 m (see Fig. 1b, page 5) counterclockwise from points a,f, and g. This is because the correction coil packages displace the dipole strings from a centered arrangement between the quadrupoles.

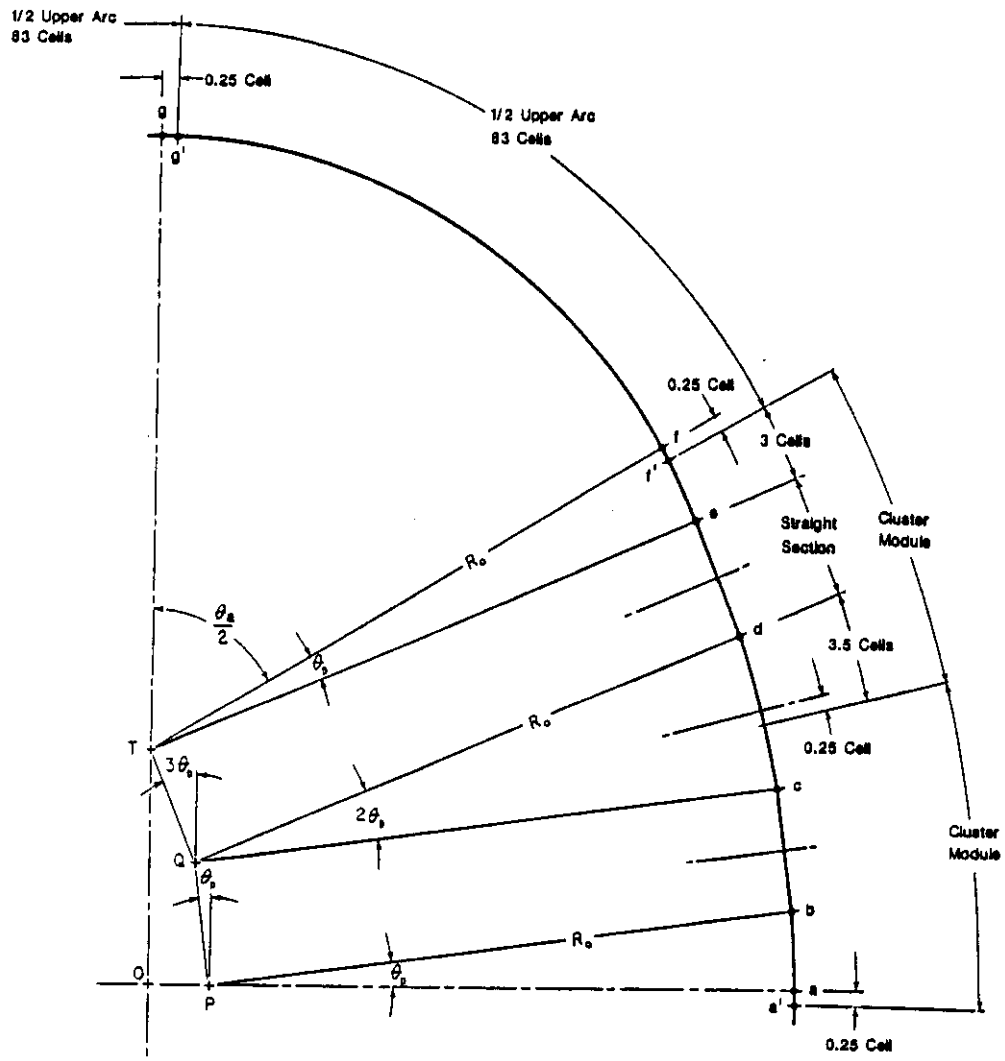

Fig. 1a : Geometry of CDR Lattice

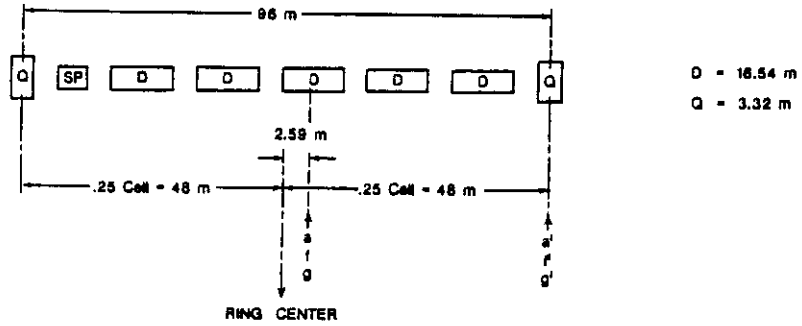

Fig.1b : CDR Lattice : Half Cell

SSC CENTRAL DESIGN GROUP	
DRAWING NO. 84A188	TITLE CDR LATTICE GEOMETRY
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2 Geometry of a 90° Lattice (Ref. 1: SSC-SR-1024 App. B)

2.1 Description of the Arcs

Each (upper or lower) arc has 143 cells, of length 228 m, containing 12 x 16.54 m (magnetic effective length) normal dipoles for a total length of 32,604 m.

2.2 Description of a Cluster Module

Again each (near or far) cluster contains four modules. A module, moving clockwise, contains: .5 normal cell, dispersion suppressor (d.s.), straight section (s.s.), d.s., 1 normal cell.

Again we symmetrize:

.75 cell d.s. s.s. d.s. .75 cell

The dispersion suppressor has two cells, each .75 the length of a normal cell so its total length is 342 m. Each of its two cells has eight reduced length dipoles (16.485441 m). The straight section has a length equivalent to 5.5 cells, L_s = 1254 m. Thus the module is:

	.75 cell	d.s.	s.s.	d.s.	.75 cell
Length	171 m	342 m	L_s = 1254 m	342 m	171 m
Bends	9x16.54 m	16x16.485441 m	0	16x16.485441 m	9x16.54 m

Hence, each module has 18 normal 16.54 m dipoles, and 32 reduced-length 16.485441 m (magnetic effective length) dipoles for a total length of 2280 m.

2.3 Description of the Ring

The ring contains:

$$2\,[(143 \times 12) + (18 \times 4)] = 3576 \text{ normal dipoles}$$

$$2\,(32 \times 4) = 256 \text{ short dipoles}$$

The magnetic radius of the ring is:

$$2\pi\rho = (3576 \times 16.54) + (256 \times 16.485441)$$

$$\rho = 10085.22108 \text{ m}$$

The bend angle of a normal cell is:

$$\theta_c = 12 \times \frac{L_{dip}}{\rho} = 0.019680283 \text{ rad} = 1.127597127°$$

The radius of curvature of the arcs is:

$$R_0 = \frac{L_{cell}}{\theta_c} = \boxed{11{,}585.19955 \text{ m} = R_0}$$

The bend angle of a 3/4 cell is:

$$\theta_0 = .75\theta_c = 14.760212 \text{ mrad} = \boxed{.845697845° = \theta_0}$$

The bend angle of a dispersion suppressor is:

$$\theta_1 = 16\,\frac{L_{short\ dip}}{\rho} = 0.026153820 \text{ rad} = \boxed{1.498503499° = \theta_1}$$

The radius of curvature of the dispersion suppressors is:

$$R_1 = \frac{L_{d.s.}}{\theta_1} = \boxed{13{,}076.48371 \text{ m} = R_1}$$

The total length of the ring is $\boxed{83448 \text{ m}}$.

We proceed as with the CDR lattice to first establish the polygon of the arc centers (points A to G), then the critical points of the ring (a to k) and finally the arc and cluster real centers and boundaries a', j', k'.

However, the central polygon is more complex than in the CDR case, due to the different radii of curvature for the normal cells and the dispersion suppressors. As before, the ring has reflection symmetry about two perpendicular axes so it suffices to trace out one quadrant (Fig. 2a, page 11).

2.3.1 Coordinates of the Different Centers

$$A \begin{cases} X_A = 0 \\ Y_A = 0 \end{cases}$$

$$B \begin{cases} X_B = -(R_1 - R_0) \cos \theta_0 = -1491.121705 \text{ m} \\ Y_B = -(R_1 - R_0) \sin \theta_0 = -22.01087077 \text{ m} \end{cases}$$

$$C \begin{cases} X_C = X_B - L_S \sin (\theta_0 + \theta_1) = -1542.413583 \text{ m} \\ Y_C = Y_B + L_S \cos (\theta_0 + \theta_1) = 1230.939704 \text{ m} \end{cases}$$

$$D \begin{cases} X_D = X_C + (R_1 - R_0) \cos (\theta_0 + 2\theta_1) = -54.482146 \text{ m} \\ Y_D = Y_C + (R_1 - R_0) \sin (\theta_0 + 2\theta_1) = 1330.881964 \text{ m} \end{cases}$$

$$E \begin{cases} X_E = X_D - (R_1 - R_0) \cos (3\theta_0 + 2\theta_1) = -1538.815388 \text{ m} \\ Y_E = Y_D - (R_1 - R_0) \sin (3\theta_0 + 2\theta_1) = 1187.065261 \text{ m} \end{cases}$$

$$F \begin{cases} X_F = X_E - L_S \sin (3\theta_0 + 3\theta_1) = -1692.347787 \text{ m} \\ Y_F = Y_E + L_S \cos (3\theta_0 + 3\theta_1) = 2431.630969 \text{ m} \end{cases}$$

$$G \begin{cases} X_G = X_F + (R_1 - R_0) \cos (3\theta_0 + 4\theta_1) = -217.5640 \text{ m} \\ Y_G = Y_F + (R_1 - R_0) \sin (3\theta_0 + 4\theta_1) = 2652.8575 \text{ m} \end{cases}$$

$$O \begin{cases} X_O = 0 \\ Y_O = 0 \end{cases} \quad A \begin{cases} X_A = 217.5640 \text{ m} \\ Y_A = 0 \end{cases} \quad B \begin{cases} X_B = -1273.5577 \text{ m} \\ Y_B = -22.0109 \text{ m} \end{cases} \quad C \begin{cases} X_C = -1324.8496 \text{ m} \\ Y_C = 1230.9397 \text{ m} \end{cases}$$

$$D \begin{cases} X_D = 163.0819 \text{ m} \\ Y_D = 1330.8820 \text{ m} \end{cases} \quad E \begin{cases} X_E = -1321.2514 \text{ m} \\ Y_E = 1187.0653 \text{ m} \end{cases} \quad F \begin{cases} X_F = -1474.7838 \text{ m} \\ Y_F = 2431.6310 \text{ m} \end{cases} \quad G \begin{cases} X_G = 0 \\ Y_G = 2652.8575 \text{ m} \end{cases}$$

2.3.2 Coordinates of Critical Points of the Ring

$$a \begin{cases} x_a = x_A + R_0 = \boxed{11802.764 \text{ m} = \tfrac{1}{2} \text{ minor axis}} \\ y_a = 0 \end{cases}$$

$$b \begin{cases} x_b = x_A + R_0 \cos \theta_0 = 11801.502 \text{ m} \\ y_b = y_A + R_0 \sin \theta_0 = 170.994 \text{ m} \end{cases}$$

$$c \begin{cases} x_c = x_B + R_1 \cos (\theta_0 + \theta_1) = 11791.983 \text{ m} \\ y_c = y_B + R_1 \sin (\theta_0 + \theta_1) = 512.852 \text{ m} \end{cases}$$

$$d \begin{cases} x_d = x_C + R_1 \cos (\theta_0 + \theta_1) = 11740.691 \text{ m} \\ y_d = y_C + R_1 \sin (\theta_0 + \theta_1) = 1765.802 \text{ m} \end{cases}$$

$$e \begin{cases} x_e = x_C + R_1 \cos (\theta_0 + 2\theta_1) = 11722.236 \text{ m} \\ y_e = y_C + R_1 \sin (\theta_0 + 2\theta_1) = 2107.294 \text{ m} \end{cases}$$

$$f \begin{cases} x_f = x_D + R_0 \cos (3\theta_0 + 2\theta_1) = 11694.283 \text{ m} \\ y_f = y_D + R_0 \sin (3\theta_0 + 2\theta_1) = 2448.138 \text{ m} \end{cases}$$

$$g \begin{cases} x_g = x_E + R_1 \cos (3\theta_0 + 3\theta_1) = 11656.853 \text{ m} \\ y_g = y_E + R_1 \sin (3\theta_0 + 3\theta_1) = 2788.073 \text{ m} \end{cases}$$

$$h \begin{cases} x_h = x_F + R_1 \cos (3\theta_0 + 3\theta_1) = 11503.321 \text{ m} \\ y_h = y_F + R_1 \sin (3\theta_0 + 3\theta_1) = 4032.639 \text{ m} \end{cases}$$

$$i \begin{cases} x_i = x_F + R_1 \cos (3\theta_0 + 4\theta_1) = 11457.015 \text{ m} \\ y_i = y_F + R_1 \sin (3\theta_0 + 4\theta_1) = 4371.480 \text{ m} \end{cases}$$

$$j \begin{cases} x_j = x_G + R_0 \cos (4\theta_0 + 4\theta_1) = 11430.400 \text{ m} \\ y_j = y_G + R_0 \sin (4\theta_0 + 4\theta_1) = 4540.394 \text{ m} \end{cases}$$

$$k \begin{cases} x_k = 0 \\ y_k = y_G + R_0 = \boxed{14238.057 \text{ m} = \tfrac{1}{2} \text{ major axis}} \end{cases}$$

2.3.3 <u>Coordinates of Arc and Cluster Centers and Boundaries</u>

$$a' \begin{cases} x_{a'} = x_A + R_0 \cos\left(\frac{\theta_0}{3} - \frac{2.60 \times 360}{2\pi R_0}\right) = 11802.636 \text{ m} \\ y_{a'} = -R_0 \sin\left(\frac{\theta_0}{3} - \frac{2.60 \times 360}{2\pi R_0}\right) = -54.400 \text{ m} \end{cases}$$

$$k' \begin{cases} x_{k'} = R_0 \sin\left(\frac{\theta_0}{3} - \frac{2.60 \times 360}{2\pi R_0}\right) = 54.400 \text{ m} \\ y_{k'} = y_G + R_0 \cos\left(\frac{\theta_0}{3} - \frac{2.60 \times 360}{2\pi R_0}\right) = 14237.929 \text{ m} \end{cases}$$

$$j' \begin{cases} x_{j'} = R_0 \cos\left(\frac{11\theta_0}{3} + 4\theta_1 + \frac{2.6 \times 360}{2\pi R_0}\right) = 11439.137 \text{ m} \\ y_{j'} = y_G + R_0 \sin\left(\frac{11\theta_0}{3} + 4\theta_1 + \frac{2.6 \times 360}{2\pi R_0}\right) = 4486.701 \text{ m} \end{cases}$$

<u>Notes:</u> - Fig. 2a has reflection symmetry about OA and OG, but also has local symmetry about the middle of cd, ef and gh.

- Points a,j, and k are the center of dipole strings between two quads. The mid half-cell points, defined by quads, are 2.60 m (see Fig. 2b, page 11) counterclockwise from points a, j and k.

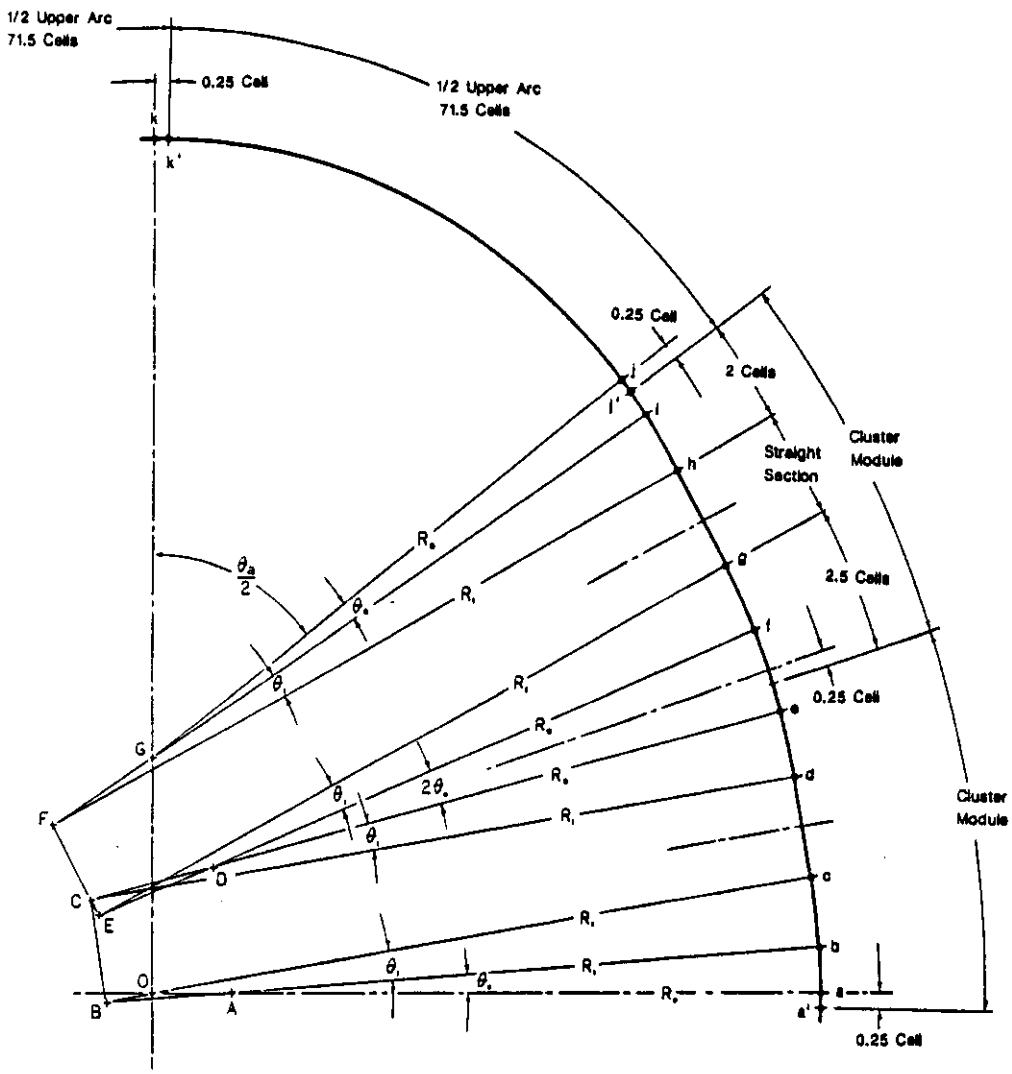

Fig. 2a : Geometry of 90°/Cell Lattice, Option #1 (SSC SR-1024, App. B)

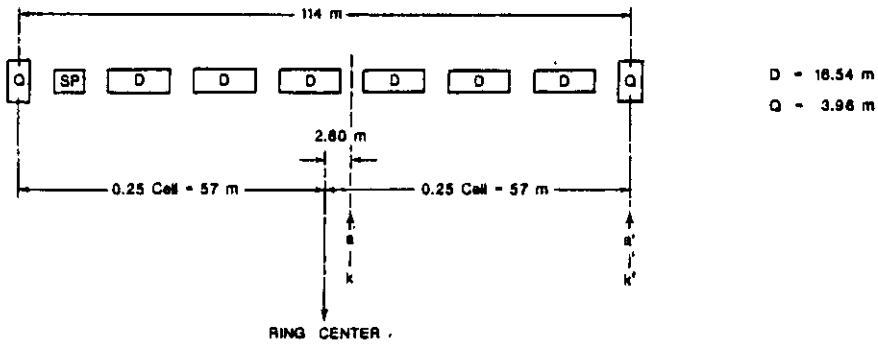

Fig. 2b : 90°/Cell Lattice, Option #1 - Half Normal Cell (SSC SR-1024, App. B)

SSC CENTRAL DESIGN GROUP	
DRAWING NO. 84A189	TITLE OPTIMIZED LATTICE GEOMETRY
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3. Geometry of a Second 90° Lattice (Ref. 2: Synchr run L90 - SSC 90° Cell Lattice - 29Nov86, A.A. Garren, D. Johnson)

3.1 Description of the Arcs

Each (upper or lower) arc has 143 cells, of length 228.5 m, containing 12 x 16.54 m (magnetic effective length) dipoles for a total length of 32,675.5 m.

3.2 Description of a Cluster Module

Again each (near or far) cluster contains four modules. A module, moving clockwise, contains: 0.5 cell, dispersion suppressor (d.s.), straight section (s.s.), d.s., 1 cell.

Again we symmetrize:

.75 cell d.s. s.s. d.s. .75 cell

The dispersion suppressor has two cells, each .75 the length of a normal cell so its total length is 342.75 m. Each of its two cells has eight 16.54 m dipoles. The straight section has a length equivalent to 5.5 cells, L_s = 1256.75 m. Thus the module is:

	.75 cell	d.s.	s.s.	d.s.	.75 cell
Length	171.375 m	342.75 m	L_s = 1256.75 m	342.75 m	171.375 m
Bends	9x16.54 m	16x16.54 m	0	16x16.54 m	9x16.54 m

Hence, each module has 50 dipoles for a total length of 2285 m.

3.3 Description of the Ring

The ring contains:

$$2\,[(143 \times 12) + (50 \times 4)] = 3832 \text{ dipoles}$$

The magnetic radius of the ring is:

$$2\pi\rho = 3832 \times 16.54$$

$$\rho = 10087.44401 \text{ m}$$

The bend angle of a normal cell is:

$$\theta_c = 12 \times \frac{L_{dip}}{\rho} = 0.019675946 \text{ rad} = 1.127348643°$$

The radius of curvature of the arcs is:

$$R_0 = \frac{L_{cell}}{\theta_c} = \boxed{11{,}613.16483 \text{ m} = R_0}$$

The bend angle of a 3/4 cell is:

$$\theta_0 = .75\theta_c = 14.756959 \text{ mrad} = \boxed{.845511482° = \theta_0}$$

The bend angle of a dispersion suppressor is:

$$\theta_1 = 16\,\frac{L_{dip}}{\rho} = 0.026234594 = \boxed{1.503131524 = \theta_1}$$

The radius of curvature of the dispersion suppressors is:

$$R_1 = \frac{L_{d.s.}}{\theta_1} = \boxed{13{,}064.81044 \text{ m} = R_1}$$

The total length of the ring is $\boxed{83631 \text{ m}}$.

We proceed as with the first 90° lattice to first establish the polygon of the arc centers (points A to G), then the critical points of the ring (a to k) and finally the arc and cluster real centers and boundaries a', k', j'.

As before, the ring has reflection symmetry about two perpendicular axes so it suffices to trace out one quadrant (Fig. 3a, page 17).

3.3.1 Coordinates of the Different Centers

$$A \begin{cases} X_A = 0 \\ Y_A = 0 \end{cases}$$

$$B \begin{cases} X_B = -(R_1 - R_0) \cos \theta_0 = -1451.487552 \text{ m} \\ Y_B = -(R_1 - R_0) \sin \theta_0 = -21.42109758 \text{ m} \end{cases}$$

$$C \begin{cases} X_C = X_B - L_S \sin (\theta_0 + \theta_1) = -1502.989261 \text{ m} \\ Y_C = Y_B + L_S \cos (\theta_0 + \theta_1) = 1234.273187 \text{ m} \end{cases}$$

$$D \begin{cases} X_D = X_C + (R_1 - R_0) \cos (\theta_0 + 2\theta_1) = -54.6226670 \text{ m} \\ Y_D = Y_C + (R_1 - R_0) \sin (\theta_0 + 2\theta_1) = 1331.788239 \text{ m} \end{cases}$$

$$E \begin{cases} X_E = X_D - (R_1 - R_0) \cos (3\theta_0 + 2\theta_1) = -1499.480858 \text{ m} \\ Y_E = Y_D - (R_1 - R_0) \sin (3\theta_0 + 2\theta_1) = 1191.574887 \text{ m} \end{cases}$$

$$F \begin{cases} X_F = X_E - L_S \sin (3\theta_0 + 3\theta_1) = -1653.640024 \text{ m} \\ Y_F = Y_E + L_S \cos (3\theta_0 + 3\theta_1) = 2438.834088 \text{ m} \end{cases}$$

$$G \begin{cases} X_G = X_F + (R_1 - R_0) \cos (3\theta_0 + 4\theta_1) = -218.1237410 \text{ m} \\ Y_G = Y_F + (R_1 - R_0) \sin (3\theta_0 + 4\theta_1) = 2654.630236 \text{ m} \end{cases}$$

$$O \begin{cases} X_O = 0 \\ Y_O = 0 \end{cases} \quad A \begin{cases} X_A = 218.1237410 \text{ m} \\ Y_A = 0 \end{cases} \quad B \begin{cases} X_B = -1233.363811 \text{ m} \\ Y_B = -21.42109758 \text{ m} \end{cases} \quad C \begin{cases} X_C = -1284.865520 \text{ m} \\ Y_C = 1234.273187 \text{ m} \end{cases}$$

$$D \begin{cases} X_D = 163.5010740 \text{ m} \\ Y_D = 1331.788239 \text{ m} \end{cases} \quad E \begin{cases} X_E = -1281.357117 \text{ m} \\ Y_E = 1191.574887 \text{ m} \end{cases} \quad F \begin{cases} X_F = -1435.516283 \text{m} \\ Y_F = 2438.834088 \text{m} \end{cases} \quad G \begin{cases} X_G = 0 \\ Y_G = 2654.630236 \text{m} \end{cases}$$

3.3.2 Coordinates of Critical Points of the Ring

$$a \begin{cases} x_a = x_A + R_0 = \boxed{11831.289 \text{ m} = \tfrac{1}{2} \text{ minor axis}} \\ y_a = 0 \end{cases}$$

$$b \begin{cases} x_b = x_A + R_0 \cos \theta_0 = 11830.024 \text{ m} \\ y_b = y_A + R_0 \sin \theta_0 = 171.369 \text{ m} \end{cases}$$

$$c \begin{cases} x_c = x_B + R_1 \cos (\theta_0 + \theta_1) = 11820.472 \text{ m} \\ y_c = y_B + R_1 \sin (\theta_0 + \theta_1) = 513.976 \text{ m} \end{cases}$$

$$d \begin{cases} x_d = x_C + R_1 \cos (\theta_0 + \theta_1) = 11768.970 \text{ m} \\ y_d = y_C + R_1 \sin (\theta_0 + \theta_1) = 1769.670 \text{ m} \end{cases}$$

$$e \begin{cases} x_e = x_C + R_1 \cos (\theta_0 + 2\theta_1) = 11750.434 \text{ m} \\ y_e = y_C + R_1 \sin (\theta_0 + 2\theta_1) = 2111.905 \text{ m} \end{cases}$$

$$f \begin{cases} x_f = x_D + R_0 \cos (3\theta_0 + 2\theta_1) = 11722.367 \text{ m} \\ y_f = y_D + R_0 \sin (3\theta_0 + 2\theta_1) = 2453.495 \text{ m} \end{cases}$$

$$g \begin{cases} x_g = x_E + R_1 \cos (3\theta_0 + 3\theta_1) = 11684.790 \text{ m} \\ y_g = y_E + R_1 \sin (3\theta_0 + 3\theta_1) = 2794.169 \text{ m} \end{cases}$$

$$h \begin{cases} x_h = x_F + R_1 \cos (3\theta_0 + 3\theta_1) = 11530.631 \text{ m} \\ y_h = y_F + R_1 \sin (3\theta_0 + 3\theta_1) = 4041.428 \text{ m} \end{cases}$$

$$i \begin{cases} x_i = x_F + R_1 \cos (3\theta_0 + 4\theta_1) = 11484.130 \text{ m} \\ y_i = y_F + R_1 \sin (3\theta_0 + 4\theta_1) = 4380.999 \text{ m} \end{cases}$$

$$j \begin{cases} x_j = x_G + R_0 \cos (4\theta_0 + 4\theta_1) = 11457.405 \text{ m} \\ y_j = y_G + R_0 \sin (4\theta_0 + 4\theta_1) = 4550.276 \text{ m} \end{cases}$$

$$k \begin{cases} x_k = 0 \\ y_k = y_G + R_0 = \boxed{14267.795 \text{ m} = \tfrac{1}{2} \text{ major axis}} \end{cases}$$

3.3.3 Coordinates of Arc and Cluster Centers and Boundaries

$$a' \begin{cases} x_{a'} = x_A + R_0 \cos\left(\frac{\theta_0}{3} - \frac{2.885 \times 360}{2\pi R_0}\right) = 11831.162 \text{ m} \\ y_{a'} = -R_0 \sin\left(\frac{\theta_0}{3} - \frac{2.885 \times 360}{2\pi R_0}\right) = -54.240 \text{ m} \end{cases}$$

$$k' \begin{cases} x_{k'} = R_0 \sin\left(\frac{\theta_0}{3} - \frac{2.885 \times 360}{2\pi R_0}\right) = 54.240 \text{ m} \\ y_{k'} = y_G + R_0\left(\cos \frac{\theta_0}{3} + \frac{2.885 \times 360}{2\pi R_0}\right) = 14267.668 \text{ m} \end{cases}$$

$$j' \begin{cases} x_{j'} = R_0 \cos\left(\frac{11\theta_0}{3} + 4\theta_1 + \frac{2.885 \times 360}{2\pi R_0}\right) = 11466.134 \text{ m} \\ y_{j'} = y_G + R_0 \sin\left(\frac{11\theta_0}{3} + 4\theta_1 + \frac{2.885 \times 360}{2\pi R_0}\right) = 4496.701 \text{ m} \end{cases}$$

Notes: - Fig. 2a has reflection symmetry about OA and OG, but also has local symmetry about the middle of cd, ef and gh.

- Points a,j, and k are the center of dipole strings between two quads. The mid half-cell points, defined by quads, are 2.885 m (see Fig. 3b, page 17) counterclockwise from points a, j and k.

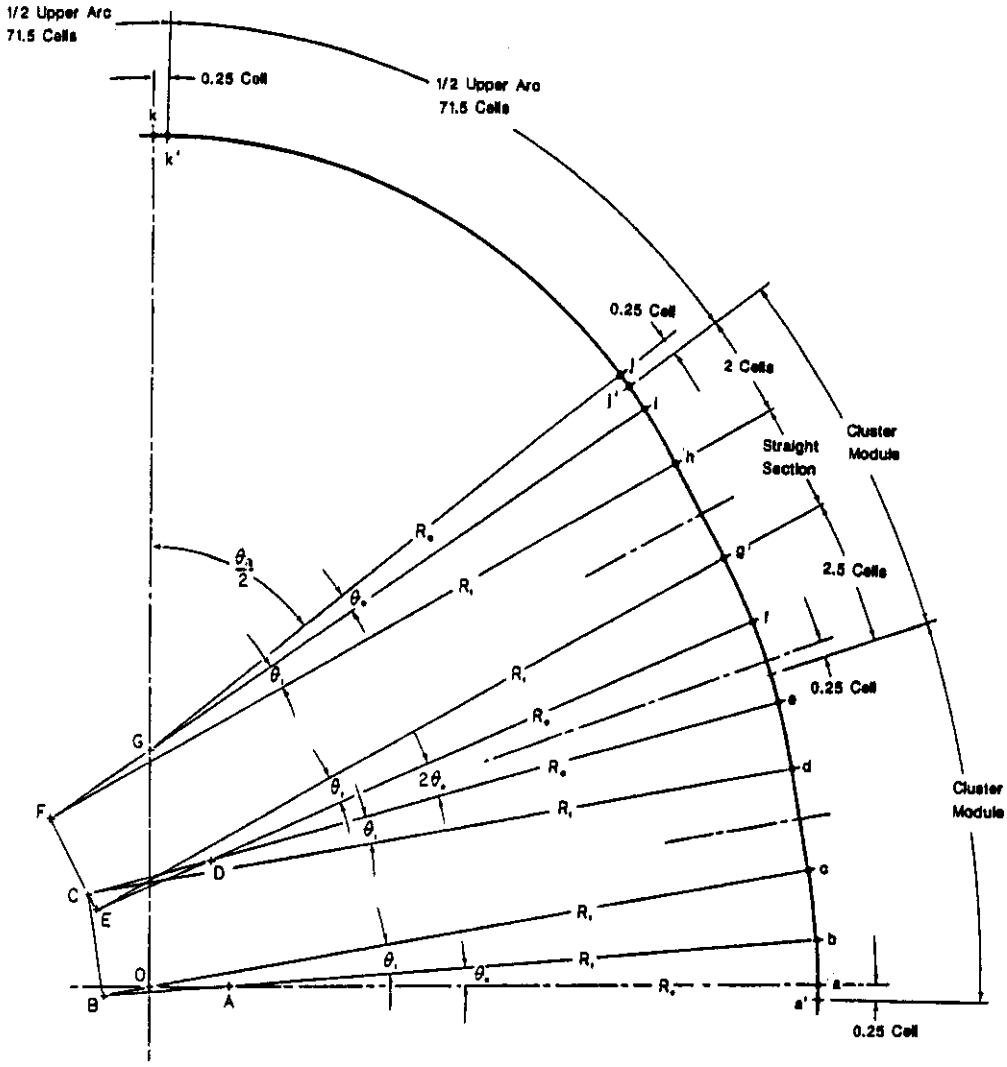

Fig. 3a : Geometry of 90°/Cell Lattice, Option #2 (See Reference 2, Page 23)

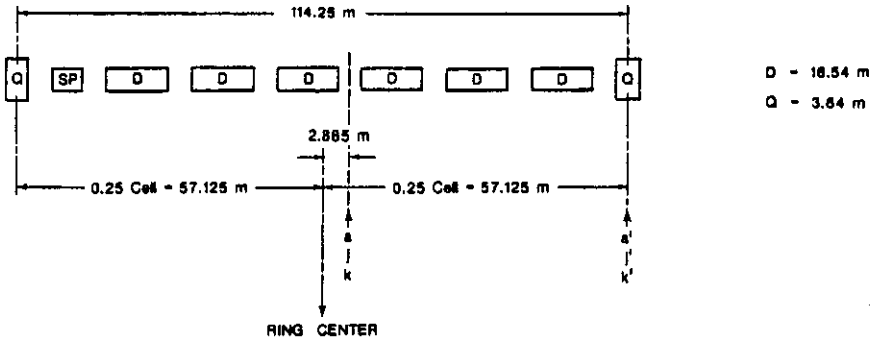

Fig. 3b : 90°/Cell Lattice, Option #2 - Half Normal Cell (See Reference 2, Page 23)

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4. Geometry of a Third 90° Lattice - (Ref 3: SSC Coordinating Committee: Meeting of December 31)

This third version of the 90° lattice is designed to be able to accommodate a by-pass[4] for the clustered regions.

For this purpose the last two cells of each arc are only half-filled with six dipoles and two additional cells also filled with 6 dipoles are inserted between the arc and the cluster.

These four cells form the transition zone.

4.1 Description of the Arcs

Each (upper and lower) arc has 139 cells of length 229.14 m containing 12 x 16.54 m (magnetic effective length) dipoles for a total length of 31,850.46 m.

4.2 Description of the Transition Zones

Each of the four transition zones has four cells of length 229.14 m containing 6 x 16.54 m dipoles for a total length of 916.56 m.

4.3 Description of a Cluster Module

Each cluster module has the same composition as in 3.2:

	0.75 Cell	ds	ss	ds	0.75 cell
Length	171.855 m	343.71 m	L_s = 1260.27 m	343.71 m	171.855 m

Hence each module has 50 dipoles for a total length of 2291.4 m

4.4 Description of the Ring

The ring contains the same number of dipoles as in 3.3:

$$2[(139 \times 12) + (2 \times 4 \times 6) + (50 \times 4)] = 3832 \text{ dipoles}$$

$$\theta_c = 1.127348643°$$

$$R_0 = \frac{L_{cell}}{\theta_c} \quad \boxed{11{,}645.69186 \text{ m} = R_0}$$

$$\boxed{\theta_0 = .845511482°}$$

$$\boxed{\theta_1 = 1.503131524°}$$

$$R_1 = \frac{L_{ds.}}{\theta_1} = \boxed{13{,}101.40334 \text{ m} = R_1}$$

The bend angle of a half-full cell of the transition zone is $\theta_c/2$

The bend angle of the transition zone is $2\theta_c = \boxed{2.254697286° = \theta_2}$

The radius of curvature of this transition zone is $R_2 = 2R_0$

$$\boxed{R_2 = 23{,}291.38372 \text{ m}}$$

The total length of the ring is $\boxed{85{,}698.36 \text{ m}}$.

We proceed as with the other lattices to first establish the polygon of the arc centers (A to I), then the critical points of the ring (a to ℓ) and finally the arc and cluster real centers and boundaries a', ℓ', j', k'.

As before, the ring has reflection symmetry about two perpendicular axes, so it suffices to trace one quadrant (Fig. 4a, page 24).

2.3.1 <u>Coordinates of the Different Centers</u>

→ $A \begin{cases} X_A = 0 \\ Y_A = 0 \end{cases}$ →

$$B \begin{cases} X_B = -(R_1 - R_0) \cos \theta_0 = -1455.552979 \text{ m} \\ Y_B = -(R_1 - R_0) \sin \theta_0 = -21.48109528 \text{ m} \end{cases}$$

$$C \begin{cases} X_C = X_B - L_S \sin (\theta_0 + \theta_1) = -1507.198938 \text{ m} \\ Y_C = Y_B + L_S \cos (\theta_0 + \theta_1) = 1237.730233 \text{ m} \end{cases}$$

$$D \begin{cases} X_D = X_C + (R_1 - R_0) \cos (\theta_0 + 2\theta_1) = -54.77565800 \text{ m} \\ Y_D = Y_C + (R_1 - R_0) \sin (\theta_0 + 2\theta_1) = 1335.518412 \text{ m} \end{cases}$$

$$E \begin{cases} X_E = X_D - (R_1 - R_0) \cos (3\theta_0 + 2\theta_1) = -1503.680709 \text{ m} \\ Y_E = Y_D - (R_1 - R_0) \sin (3\theta_0 + 2\theta_1) = 1194.912341 \text{ m} \end{cases}$$

$$F\begin{cases} X_F = X_E - L_S \sin(3\theta_0 + 3\theta_1) = -1658.271655 \text{ m} \\ Y_F = Y_E + L_S \cos(3\theta_0 + 3\theta_1) = 2445.664959 \text{ m} \end{cases}$$

$$G\begin{cases} X_G = X_F + (R_1 - R_0) \cos(3\theta 0 + 3\theta 1) = -218.7346780 \text{ m} \\ Y_G = Y_F + (R_1 - R_0) \sin(3\theta_0 + 4\theta_1) = 2662.065524 \text{ m} \end{cases}$$

$$H\begin{cases} X_H = X_G - R_0 \cos(4\theta_0 + 4\theta_1) = -11708.23021 \text{ m} \\ Y_H = Y_G - R_0 \sin(4\theta_0 + 4\theta_1) = 761.1101550 \text{ m} \end{cases}$$

$$I\begin{cases} X_I = X_H + R_0 \cos(4\theta_0 + 4\theta_1 + \theta_2) = -302.4165600 \text{ m} \\ Y_I = Y_H + R_0 \sin(4\theta_0 + 4\theta_1 + \theta_2) = 3112.610525 \text{ m.} \end{cases}$$

$$\vec{O}\begin{cases} X_0 = 0 \\ Y_0 = 0 \end{cases} \quad A\begin{cases} X_A = 302.41656 \text{ m} \\ Y_A = 0 \end{cases} \quad B\begin{cases} X_B = -1153.136419 \text{ m} \\ Y_B = -21.48109528 \text{ m} \end{cases} \quad C\begin{cases} X_C = -1204.782378 \text{ m} \\ Y_C = 1237.730233 \text{ m} \end{cases}$$

$$D\begin{cases} X_D = 247.640902\text{m} \\ Y_D = 1335.518412\text{m} \end{cases} \quad E\begin{cases} X_E = -1201.264149\text{m} \\ Y_E = 1194.912341\text{m} \end{cases} \quad F\begin{cases} X_F = -1355.855095\text{m} \\ Y_F = 2445.664959\text{m} \end{cases} \quad F\begin{cases} X_G = 83.681882\text{m} \\ Y_G = 2662.065524\text{m} \end{cases}$$

$$H\begin{cases} X_H = -11405.81365 \\ Y_H = 761.110155\text{m} \end{cases} \quad I\begin{cases} X_I = 0 \\ Y_I = 3112.610525 \end{cases}$$

4.3.2 Coordinates of Critical Points of the Ring

$$a \begin{cases} x_a = x_A + R_0 = \boxed{11948.108 \text{ m} = \tfrac{1}{2} \text{ minor axis}} \\ y_a = 0 \end{cases}$$

$$b \begin{cases} x_b = x_A + R_0 \cos \theta_0 = 11946.840 \text{ m} \\ y_b = y_A + R_0 \sin \theta_0 = 171.849 \text{ m} \end{cases}$$

$$c \begin{cases} x_c = x_B + R_1 \cos (\theta_0 + \theta_1) = 11937.261 \text{ m} \\ y_c = y_B + R_1 \sin (\theta_0 + \theta_1) = 515.415 \text{ m} \end{cases}$$

$$d \begin{cases} x_d = x_C + R_1 \cos (\theta_0 + \theta_1) = 11885.615 \text{ m} \\ y_d = y_C + R_1 \sin (\theta_0 + \theta_1) = 1774.627 \text{ m} \end{cases}$$

$$e \begin{cases} x_e = x_C + R_1 \cos (\theta_0 + 2\theta_1) = 11867.027 \text{ m} \\ y_e = y_C + R_1 \sin (\theta_0 + 2\theta_1) = 2117.824 \text{ m} \end{cases}$$

$$f \begin{cases} x_f = x_D + R_0 \cos (3\theta_0 + 2\theta_1) = 11838.881 \text{ m} \\ y_f = y_D + R_0 \sin (3\theta_0 + 2\theta_1) = 2450.367 \text{ m} \end{cases}$$

$$g \begin{cases} x_g = x_E + R_1 \cos (3\theta_0 + 3\theta_1) = 11801.199 \text{ m} \\ y_g = y_E + R_1 \sin (3\theta_0 + 3\theta_1) = 2801.995 \text{ m} \end{cases}$$

$$h \begin{cases} x_h = x_F + R_1 \cos (3\theta_0 + 3\theta_1) = 11646.608 \text{ m} \\ y_h = y_F + R_1 \sin (3\theta_0 + 3\theta_1) = 4052.748 \text{ m} \end{cases}$$

$$i \begin{cases} x_i = x_F + R_1 \cos (3\theta_0 + 4\theta_1) = 11599.978 \text{ m} \\ y_i = y_F + R_1 \sin (3\theta_0 + 4\theta_1) = 4393.270 \text{ m} \end{cases}$$

$$j \begin{cases} x_j = x_G + R_0 \cos(4\theta_0 + 4\theta_1) = 11573.177 \text{ m} \\ y_j = y_G + R_0 \sin(4\theta_0 + 4\theta_1) = 4563.029 \text{ m} \end{cases}$$

$$k \begin{cases} X_K = X_I + R_0 \cos(4\theta_0 + 4\theta_1 + \theta_2) = 11405.814 \text{ m} \\ Y_K = Y_I + R_0 \sin(4\theta_0 + 4\theta_1 + \theta_2) = 5464.111 \text{ m} \end{cases}$$

$$\ell \begin{cases} x_\ell = 0 \\ y_\ell = y_I + R_0 = \boxed{14758{,}302 \text{ m} = ½ \text{ major axis}} \end{cases}$$

2.3.3 <u>Coordinates of Arc and Cluster Centers and Boundaries</u>

$$a' \begin{cases} x_{a'} = x_A + R_0 \cos\left(\frac{\theta_0}{3} - \frac{2.885 \times 360}{2\pi R_0}\right) = 11947.981 \text{ m} \\ y_{a'} = -R_0 \sin\left(\frac{\theta_0}{3} - \frac{2.885 \times 360}{2\pi R_0}\right) = -54.400 \text{ m} \end{cases}$$

$$\ell' \begin{cases} x_{\ell'} = R_0 \sin\left(\frac{\theta_0}{3} - \frac{2.885 \times 360}{2\pi R_0}\right) = 54.400 \text{ m} \\ y_{\ell'} = y_I + R_0 \cos\left(\frac{\theta_0}{3} - \frac{2.885 \times 360}{2\pi R_0}\right) = 14758.175 \text{ m} \end{cases}$$

$$j' \begin{cases} x_{j'} = X_H + 2R_0 \cos\left(4\theta_0 + 4\theta_1 - \frac{54.4 \times 360}{4\pi R_0}\right) = 11581.995 \text{ m} \\ y_{j'} = y_H + 2R_0 \sin\left(4\theta_0 + 4\theta_1 - \frac{54.4 \times 360}{4\pi R_0}\right) = 4509.340 \text{ m} \end{cases}$$

$$k' \begin{cases} x_{k'} = R_0 \cos\left(\frac{11\theta_0}{3} + 4\theta_1 + \theta_2 + \frac{2.885 \times 360}{2\pi R_0}\right) = 11416.674 \text{ m} \\ y_{k'} = y_I + R_0 \sin\left(\frac{11\theta_0}{3} + 4\theta_1 + \theta_2 + \frac{2.885 \times 360}{2\pi R_0}\right) \; 5410.806 \text{ m} \end{cases}$$

<u>Notes:</u> - Fig. 2a has reflection symmetry about OA and OI, but also has local symmetry about the middle of cd, ef and gh.

- Points a,j', k and ℓ are the center of dipole strings between two quads. The mid half-cell points, defined by quads, are 2 m (see Fig. 4b, page 24) counterclockwise from points a, j, k and ℓ.

References

[1] SSC-SR-1024, Optimization of the Cell Lattice Parameters for the SSC, Appendix B.

[2] Internal Document: "Synchr. run L90-SSC 90 degree Cell Lattice - 29 Nov. 86. A.A. Garren and D. Johnson.

[3] SSC Coordinating Committee: meeting of December 31.

[4] SSC-N-186, A Possible By-Pass for the SSC Clustered IR Regions, D. Johnson.

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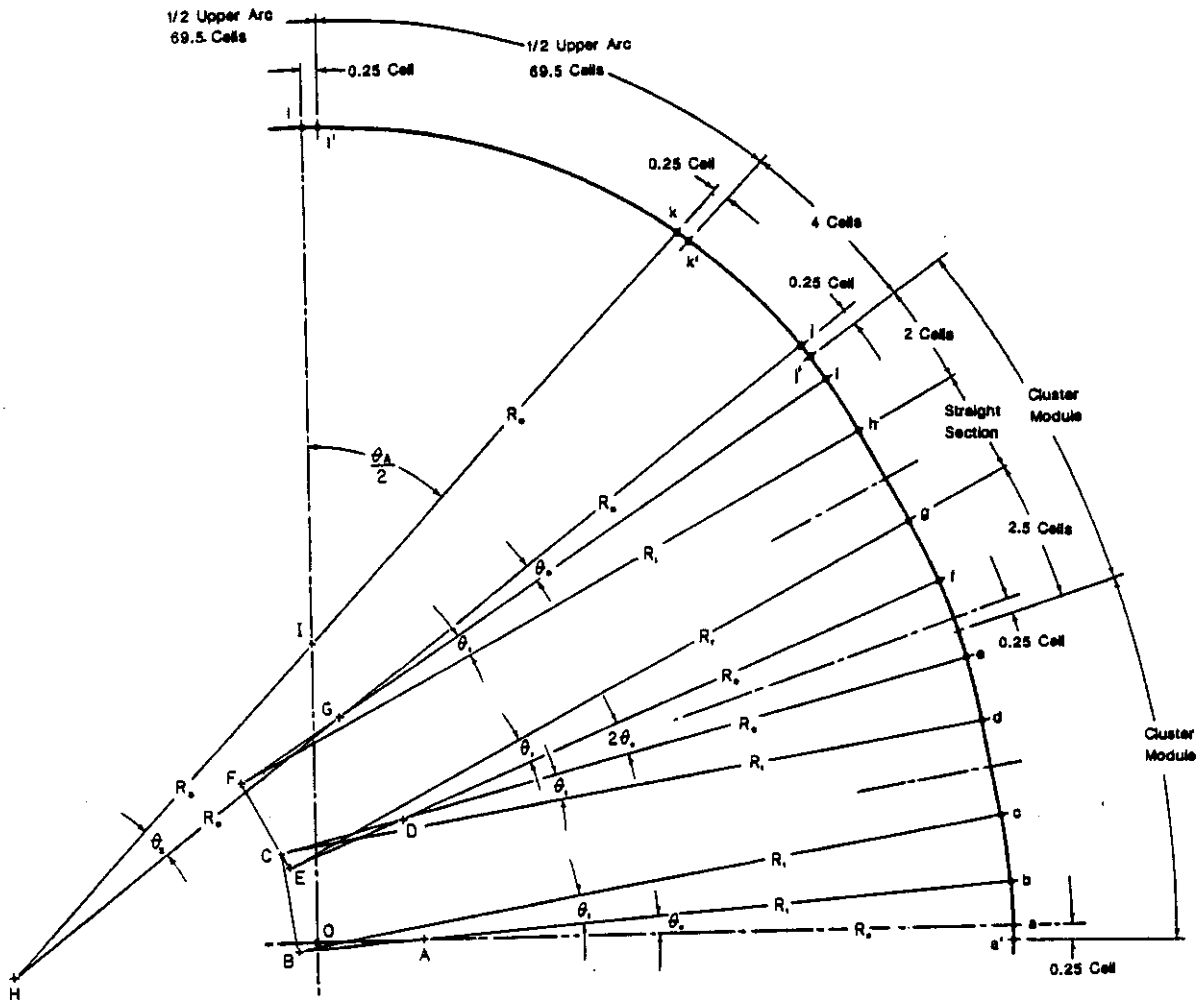

Fig. 4a : Geometry of 90°/Cell Lattice, Option #3 (See Reference 3, Page 23)

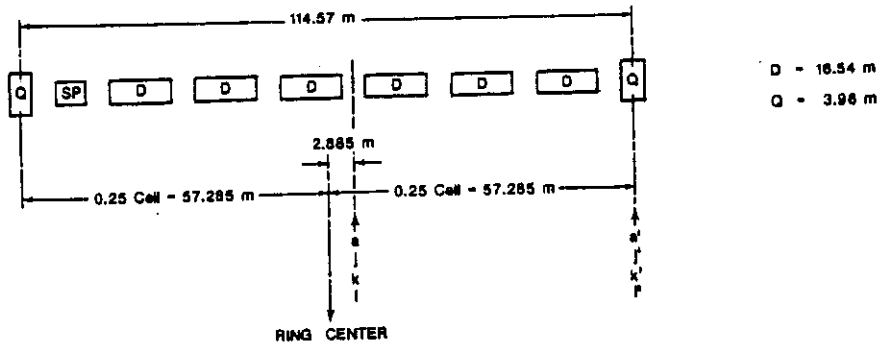

Fig. 4b : 90°/Cell Lattice, Option #3 – Half Normal Cell (See Reference 3, Page 23)

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