ENERGY ACCEPTANCE OF THE SSC

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I. Introduction

The energy acceptance of the SSC at injection has been taken as 1×10^{-3} and supported by the Aperture Task Force report of April 1985.[1] The energy acceptance is one aspect of the linear aperture requirement, namely, that the change in betatron tune be less than 0.005 both (1) at a betatron amplitude of 7 mm (at β max) with no momentum error and (2) at a betatron amplitude of 5 mm with a momentum deviation of 1×10^{-3}. A lower value, if consistent with the several energy-width requirements, would relax the magnetic tolerances (or ease the required magnetic corrections) and allow a smaller rf system and, thereby, a lower longitudinal impedance. For these reasons, an Auxiliary Working Group on Energy Acceptance convened at Snowmass to review the factors involved.

II. Energy Acceptance Requirements

The energy acceptance of the SSC must accommodate several energy widths or shifts:

(1) Energy spread in the beam. Since the superconducting magnets can be quenched by a small fraction of the incident beam, the energy acceptance of the SSC must include essentially all of the energy spread of the incident beam. This energy width is taken as $\pm\sqrt{6}\sigma_E$, where σ_E is the rms energy width. If the energy distribution were truly Gaussian, this would include 95% of the beam. However, in a more realistic beam, especially if the tails in energy distribution of the incident beam are deliberately trimmed, it can include all of the incident beam. With an rms fractional energy spread of 1.75×10^{-4}, as specified in the CDR, the single-bunch instabilities are well under control, and all of the longitudinal coupled-bunch modes also are stable, except for the lowest (dipole) mode. At 2×10^{-4} Zotter[2] estimates that the dipole mode also will be stable. This latter situation would not eliminate the need for a dipole-feedback correction system, but it would weaken its requirements and thus make it less expensive.

(2) Energy jitter in injected beam. Jitter in the energy match of the injected beam in the SSC can be caused by errors in the match of the magnet currents between the High Energy Booster and the SSC, by collective oscillations in the rf buckets of the booster, and by injection timing errors. The energy jitter observed at injection into the Tevatron is less than 10^{-4} when the system is properly tuned, but typically rises to 1 or 2×10^{-4} due to slow drifts over an hour or so.[3] The current match between the main ring and the Tevatron was designed to be accurate to about one part in 10^4, in good agreement with the observations. At injection from the PS into the SPS at CERN, energy jitter of 1 to 2×10^{-4} is observed, which also is consistent with the accuracy of the current match (again about one part in 10^4 at injection). Because of the experience at the Tevatron, an energy jitter of 1.5×10^{-4} has been assumed for the SSC. However, an energy jitter of 1.5×10^{-4} may be too conservative for a newly designed power supply system. The power supplies involved at the Main Ring at Fermilab and at the PS in CERN were designed without regard to the present energy jitter considerations. Tool, Pfeffer, and Wolff[4] have described two systems, either of which can produce a matching stability of $\pm 2 \times 10^{-5}$.

Energy variation is produced at injection also by timing errors, which produce collective synchrotron oscillations. Raka[5] has estimated that this energy error can be controlled to the 1×10^{-5} level.

(3) Operational Energy Shifts. In operation it is useful to change the average energy of the beam by shifting the frequency of the rf system. Measurements of the corresponding changes in the closed orbit and in the betatron frequency yield the dispersion function and the chromaticity. Another reason for shifting the energy is to compensate for mismatches between the circumference of the collider and the High Energy Booster or between the two rings of the collider. The questions here are the magnitudes of the energy shifts that are required.

(3a) Energy shift for dispersion measurements. The horizontal dispersion $\eta(s)$ at any point is the change in horizontal closed orbit per unit momentum change

$$\eta(s) = \Delta x(s)/(\Delta p/p) \approx \Delta x/(\Delta E/E).$$

If we have a precision of 3×10^{-5}m in the relative displacement $\Delta x(s)$, then an energy shift $\Delta E/E$ of 3×10^{-4} gives a precision in η of 0.10 meter, which is adequate. (The measurement error in $\Delta E/E$ is negligible.)

(3b) Energy shift for chromaticity measurements. The shift in betatron frequency ν can be written as a power series in the fractional energy shift δE

$$\nu = \xi_1\ \delta E + \xi_2\ \delta E^2 + \xi_3\ \delta E^3.$$

The linear, quadratic, and cubic coefficients, ξ_1, ξ_2, and ξ_3, can be corrected by systematic sextupole, octupole, and decapole coils, respectively. To be of operational significance, the fractional tune-shift corresponding to each term must be on the order of 10^{-3}, or more, and thus is easily measurable. Accuracy in fractional tune shift of 10^{-4} is available at the Tevatron and at the SPS and is estimated to be possible also at the SSC.

Tune-shift measurements at four energy shifts ($\pm\ \delta E_m/2$ and $\pm\ \delta E_m$) are adequate to over-determine the three coefficients. To determine the chromaticity to within one unit, a δE shift of only 10^{-4} is adequate if ξ_1 is known to be the dominant term.

However, if the quadratic and cubic terms are of concern, the uncertainty in the determination of the

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chromaticity grows rapidly when one extrapolates outside the region of measurement. This is illustrated in Fig. 1 (due to R. Meller[6]), from which we can conclude that the measurement region must include 0.45 of the energy aperture if an uncertainty in tune shift of 2×10^{-3} at the energy aperture is acceptable and 0.54 of the energy aperture if an uncertainty of only 1×10^{-3} is acceptable.

However, two caveats regarding this measurement of the chromaticity should be considered:

(1) The average value of the tune shift depends on the energy spread σ_E in the beam. At the maximum energy shift for measurement δE_m, the average value of the tune shift is

$$\bar{\nu}(\delta E_m) = \underbrace{\xi_1 \delta E_m + \xi_2 \delta E_m^2 + \xi_3 \delta E_m^3}_{\text{monochromatic shift}} + \underbrace{\sigma_E^2 (\xi_2 + 3\xi_3 \delta E_m)}_{\text{additional shift due to } \sigma_E}$$

The first term in the additional shift due to σ_E is constant, independent of δE_m, and thus is relatively harmless, provided σ_E is not changed during the measurements. The second term in the additional shift is linear in δE_m and can be relatively large and thus can complicate the interpretation of the measurements.

(2) Because of the non-linear chromaticity terms, the spread in betatron tune varies both with σ_E and with δE_m in a complicated way. A rough estimate indicates that this effect can significantly affect the total width of the tune signal and thus degrade the accuracy of the measurement. This decreased accuracy in turn increases the energy shift required for the measurement.

Both the additional tune shift and the increased tune spread caused by the energy spread in the beam can, of course, be avoided by measuring with "monochromatic" beams--i.e., with beams in which the energy spread σ_E is much smaller than the energy shift δE_m required for measurement. This should be possible in the SSC because normally the longitudinal emittance is intentionally increased by a factor of 22 in the High Energy Booster. Thus it should be possible to reduce the energy spread at injection by a factor of 4 (or more) at low beam intensity. However, such "monochromatic" operation may sometimes be operationally inconvenient, so that the ability to measure chromaticity with a beam having the normal energy spread would be a desirable feature.

It is possible that these two energy-spread effects (additional tune shift and increased tune spread) can be measured and analyzed to help determine the chromaticity coefficients. This possibility has not been analyzed.

(3c) If the circumference of the collider-as-built were wrong by 5 cm[7] and the design acceleration frequency f_0 not changed, the average shift in radius from the central orbit would be 8 mm, corresponding to an energy shiftof 2.8×10^{-3}. If, however, the frequency is changed by only 6.0 $\times 10^{-7} f_0$, then there is no radial shift from the central orbit and no energy shift, but there is an effect on the High Energy Booster. If its acceleration frequency is correspondingly changed, the average beam radius in the Booster is changed by about 0.6 mm, corresponding to an energy shift of only 2.1×10^{-4}, which is quite tolerable

Now consider the consequences of slightly different circumferences in the two collider rings. Since they must use a common acceleration frequency and thus a common beam circumference, there must be a relative radius differential with respect to the central orbits and a relative energy shift. Differences in the closed-orbits in the two rings also contribute to this effect. Taking 0.3 mm as a plausible upper limit for the average radial difference between the rings from both construction and closed-orbit effects, then the relative energy differental is only 1.6×10^{-5}, which also is quite tolerable.

III. SSC Energy Acceptance

The energy acceptance of the Supercollider must accommodate all of the energy-width requirements that have been mentioned. But not all of them are required simultaneously. Beam energy spread is always present and so must always be accommodated. However, energy jitter at injection and energy shift for beam analysis are not simultaneous requirements. The energy jitter at injection will be corrected by feedback before an energy shift for beam analysis is undertaken. The energy shifts for compensating circumference errors were shown to be negligible and will not be considered any further.

Thus, we have two combinations of contributions to the energy acceptance (A_e). The first combination (A_{e1}) is the sum of the beam energy spread σ_E (rms) and the energy jitter ΔE_j (rms)

$$A_{e1} = \sqrt{6}\,\sigma_E + \sqrt{6}\,\Delta_{Ej} ,$$

where we have used the 95% area criterion, assuming a slightly truncated Gaussian shape for each distribution. For σ_E of 1.75×10^{-4} (used in the CDR) and an ΔE_j of 2×10^{-5} (from Tool, Wolff, and Pfeffer), $A_{e1} = 4.8 \times 10^{-4}$.

The second combination is the sum of the beam energy spread and the maximum energy shift δE_m needed for beam diagnostics

$$A_{e2} = \sqrt{6}\sigma_E + \delta E_m .$$

Since the energy shift δE_m can be written in terms of the aperture limit, this aperture equation becomes

$$A_{e2} = \sqrt{6}\ \sigma_E + \alpha A_{e2} ,$$

where the α coefficient depends on the tolerance in tune-shift uncertainty at the energy aperture due to tune-measurement errors of 10^{-4}, as discussed in Section II3b. For uncertainties of 1 and 2×10^{-3} and a σ_E of 1.75×10^{-4}, the corresponding A_{e2} values are 9.4 and 7.9×10^{-4}. This combination is the more demanding of the two energy apertures. We note that the energy-spread requirement ($\sqrt{6}\sigma_E$ =) 4.3 $\times 10^{-4}$ is about half of the total. If the energy spread at injection were to be changed to 2×10^{-4}, (as recommended by Zotter for improved beam stability), the two A_e values would be increased to 10.8 and 9.0×10^{-4}, respectively.

It should be noted that the required energy acceptance decreases during the acceleration cycle because the energy spread drops (even though the longitudinal phase space is deliberately enlarged) and the linear aperture requirements are reduced after the injection period. Energy shifts for beam diagnostics will still be needed, however, at high energy.

IV. Consequences of a Reduced Energy Acceptance.

1. Systematic Multipole Tolerances. One motivation for possibly reducing the required energy aperture of the SSC is to relax the tolerances on the systematic multipoles in the dipoles.[8] However, the betatron amplitude at the emittance-energy aperture is an equally important parameter in determining the systematic tolerances. Figure 2 shows these tolerances for the CDR emittance-energy aperture (betatron amplitude of 5 mm at β_{max}, δE of 10^{-3}, in CDR lattice), and for two other possible apertures that illustrate the dependence of the multipole tolerances on betatron amplitude as well as on energy shift--one at half betatron amplitude, full energy shift and the other at full betatron amplitude, half energy shift. Thus, considerations of the betatron amplitude requirements enter the determination of the energy aperture.

2. RF Bucket. The half-height of the rf bucket with 20 MV peak voltage, as used in the CDR, is 7.5 x 10^{-4} at injection. The "linear" region of the bucket is usually taken as the region up to 2/3 of the bucket height, which in this case is 5 x 10^{-4}, which just matches the injection requirement that the quantity A_{e_1} (4.8 x 10^{-4}) be within the linear region. Note that if it were required that the linear region of the bucket should be enlarged to accommodate a larger energy spread or a larger energy jitter at injection, the required rf voltage increases as the square of the bucket height, resulting in a similar increase in the number of accelerating cavities. The consequences of the increased longitudinal emittance on beam stability have not been analyzed.

V. Conclusions

The arguments assembled by the Energy Acceptance Working Group, although incomplete, tend to support the current energy aperture of 10^{-3} as a reasonable value. More analysis or experience is needed before a recommendation to increase or decrease the energy aperture can be made. The loose ends needing more work include:

(1) Width of betatron-tune spread and accuracy in measurement of tune changes. Effects of betatron amplitude, energy spread, non-linear chromaticity, jitter, power supply configuration, lattice parameters.

(2) Amount of emittance smear (both transverse and longitudinal) involved in correcting large-amplitude oscillations. What are the tolerances?

(3) Effects of adding more rf cavities on beam stability.

(4) Betatron-amplitude requirements at the energy aperture.

(5) What beam energy spread at injection should be adopted?

(6) Should energy jitter and timing errors at injection be explored more fully?

References

[1] CDR, p. 137 and SSC-22.
[2] Bruno Zotter, private communication, August 1986.
[3] M. Harrison, private communication, July 1986.
[4] Gerry Tool, Dan Wolff, Howie Pfeffer, Memo to Alex Chao, June 1986.
[5] Eugene Raka, private communication.
[6] R. Meller, SSC-N-253, October 1986.
[7] Upper limit from T. Toohig, private communication, July 1986. Circumference error in the SPS-as-built was 11 cm (J. Gareyte).
[8] CDR, p. 150.

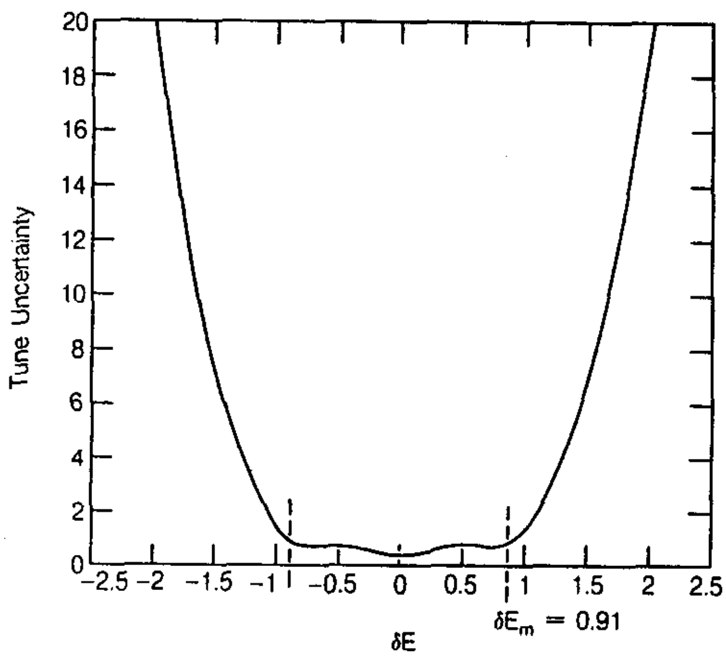

Fig. 1 Tune uncertainity in units of measurement error. Measurements made at 0, ±0.5, and ±0.91 units.

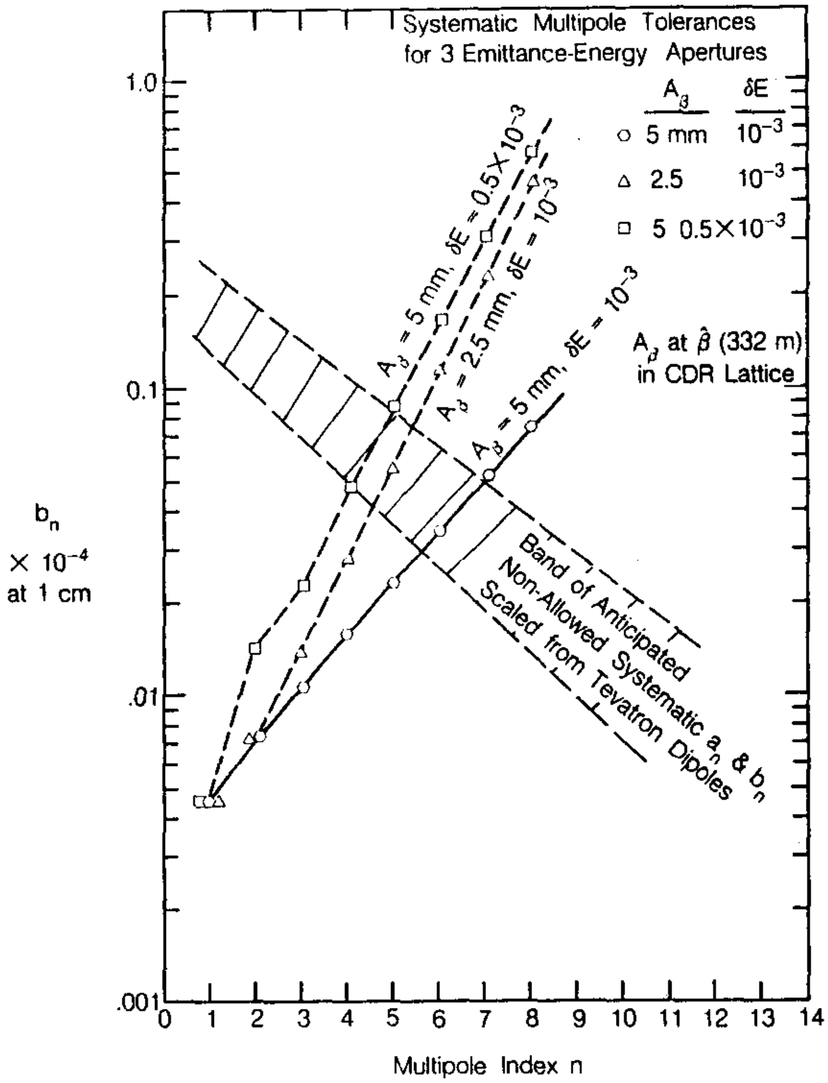

Fig. 2 Systematic multipole tolerances for SSC dipole magnets for three possible emittance-energy aperture criteria.