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FIELD VARIATIONS DUE TO ELLIPTICAL DEFORMATION OF A CIRCULAR COIL

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We have proposed in the past that by squeezing the collared dipole coil on the midplane, the random sextupole (b_2) can be corrected, leaving at the same time the higher multipoles virtually unchanged (Ref. 1). To avoid a complicated stress analysis, we have assumed a) the perturbed coil shape is elliptical, and b) the perturbed coil spacing is determined by either of the following two cases:

Case I - The coil moves radially, preserving the azimuthal angle of each turn (azimuthal compression permitted).

Case II - The coil moves both radially and azimuthally, preserving the coil spacing (azimuthal compression not permitted).

It is the purpose of this note to summarize results obtained for both cases.

Case I - Radial Movement Only

If a point on a circular arc R_0 at θ is perturbed radially and placed on an ellipse at r and θ (Fig. 1), we can express its new location as:

$$r^2 = \frac{b^2}{1 - k^2 \sin^2 \theta} \qquad (1)$$

$$k = 1 - \frac{b^2}{a^2} = \frac{4\epsilon}{(1 + \epsilon)^2}$$

$$a = R_o(1 + \epsilon)$$

$$b = R_o(1 - \epsilon)$$

$$\epsilon = \frac{\delta}{R_o}$$

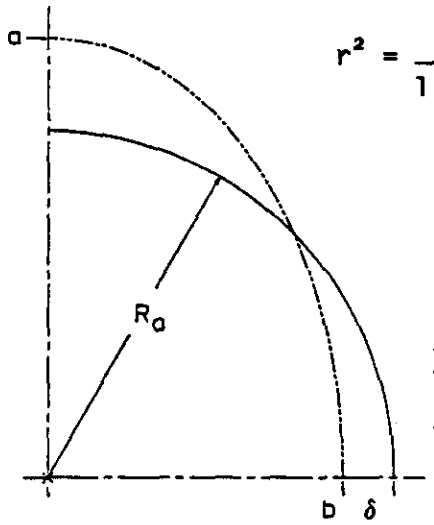

Fig. 1

A perturbation of δ = 0.001", used to reposition the coordinates of the current lines shown in Fig. 2, results in the following multipole harmonic changes (R_{fe} = 5.588 cm, $\mu = \infty$) (Ref. 1):

$\Delta b_o = 2.5$

$\Delta b_2 = 1.35$

$\Delta b_3 = -0.03$

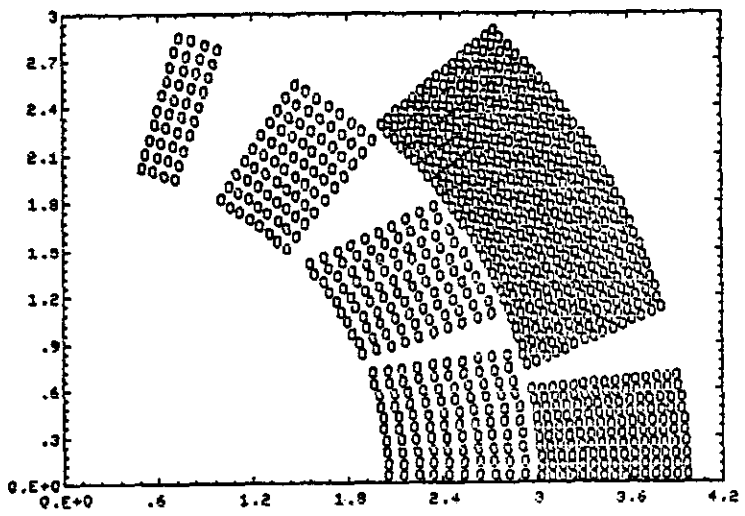

Fig. 2

For $\epsilon \ll 0$, it can be shown that Eq. 1 reduces to

$$r = R_o(1 - \epsilon \cos 2\theta) \qquad (2)$$

Using Eq. (2), L. J. Laslett has shown (Ref. 2) that, based on such a radial perturbation of a thin "cosθ" coil placed at R_o (iron at R_{fe}), the perturbed field is expressed:

$$B^* = B_x - iB_y = \frac{\mu_0 J_0}{2} \left\{ \left[\left[1 + \left(\frac{R_0}{R_{fe}}\right)^2 \right] + \frac{\epsilon}{2} \left[1 - \left(\frac{R_0}{R_{fe}}\right)^2 \right] \right] i \right.$$

$$\left. - \frac{3\epsilon}{2} \left[1 - \left(\frac{R_0}{R_{fe}}\right)^6 \right] \frac{2xy + (y^2 - x^2)\, i}{R_0^{\,2}} \right\} \qquad (3)$$

The perturbed dipole coefficient is:

$$\Delta b_0 = \frac{\frac{\epsilon}{2} \left[1 - \left(\frac{R_0}{R_{fe}}\right)^2 \right]}{\left[1 + \left(\frac{R_0}{R_{fe}}\right)^2 \right]}$$

The perturbed sextupole coefficient is:

$$\Delta b_2 = \frac{\frac{3\epsilon}{2} \left[1 - \left(\frac{R_0}{R_{fe}}\right)^6 \right] \frac{1}{R_0^{\,2}}}{\left[1 + \left(\frac{R_0}{R_{fe}}\right)^2 \right]}$$

If we choose R_0 = 3.0 cm, R_{fe} = 5.588 cm and ϵ corresponding to δ = 0.001", we get

$$\Delta b_0 = 2.34 \text{ units}$$

$$\Delta b_2 = 1.07 \text{ units}$$

<u>Case II - Radial and Azimuthal Movement</u>

If we retain the arc length in the perturbed geometry of a thin "cosθ" coil, the field is expressed (Ref. 2):

$$B^* = B_x - iB_y = \frac{\mu_0 J_0}{2} \left\{ \left[\left[1 + \left(\frac{R_0}{R_{fe}}\right)^2 \right] + \frac{\epsilon}{4} \left[1 - 3\left(\frac{R_0}{R_{fe}}\right)^2 \right] \right] i \right.$$

$$\left. - \frac{3\epsilon}{4} \left[1 - 3\left(\frac{R_0}{R_{fe}}\right)^6 \right] \frac{2xy + (y^2 - x^2)\, i}{R_0^{\,2}} \right\} \qquad (4)$$

The perturbed dipole coefficient is:

$$\Delta b_0 = \frac{\frac{\epsilon}{4}\left[1 - 3\left(\frac{R_0}{R_{fe}}\right)^2\right]}{\left[1 + \left(\frac{R_0}{R_{fe}}\right)^2\right]}$$

The perturbed sextupole coefficient is:

$$\Delta b_2 = \frac{\frac{3\epsilon}{4}\left[1 - 3\left(\frac{R_0}{R_{fe}}\right)^6\right]\frac{1}{R_0^{\,2}}}{\left[1 + \left(\frac{R_0}{R_{fe}}\right)^2\right]}$$

Using the same R_0, R_{fe}, and ϵ as in Case I, we compute

$$\Delta b_0 = 0.22 \text{ units}$$

$$\Delta b_2 = 0.51 \text{ units}$$

By comparison, Δb_2 is virtually a factor of two smaller than in Case I, and Δb_0 is drastically reduced.

G. Morgan (Ref. 3) has used substantially the same assumptions as in Case II, applying them to a <u>thick</u> coil configuration. His results are listed below:

$$\Delta b_0 = 0.45 \text{ units}$$

$$\Delta b_2 = 0.64 \text{ units}$$

$$\Delta b_4 = -0.01 \text{ units}$$

$$\Delta b_6 = 0.03 \text{ units}$$

Morgan's results agree well with our Case II perturbation calculation for a thin "cose" approximation.

Test Results

We have measured Δb_2 by placing a shim of known size between the iron and the collars at the midplane. Δb_2 varies between 0.5 and 1.0 units/mil with a tendancy towards the lower values. This seems to indicate that Case II is an acceptable model for the effect of a midplane perturbation on the sextupole, with Case I being an upper bound.

References

1. Clyde E. Taylor, "Adjustment of Collared Coil in Iron Yoke to Eliminate Both Quadrupole and Sextupole Errors in the SSC Dipole Magnet," LBL-21521, SSC-MAG-79 (April 24, 1986).

2. L. Jackson Laslett, "Documents Concerning:
   a) The Displacement of a Dipole Winding,
   b) The Elliptical Distortion of a Dipole Winding;"
   SSC-MAG-90 (July, 1986), with somewhat different notation than has been adopted here (especially in regard to the sign of ε).

3. G. Morgan, "Elliptical Deformation of a Circular Coil," BNL-194-1, SSC-MD-145 (October, 1986).