BNL SSC-55
CDG-SSC-N-256

LINEAR APERTURE, SMEAR, VARIATION OF PARTICLE ACTION

and

BEAM EMITTANCE IN THE SSC

ZOHREH PARSA

(SSC Physics Note)

October 1986

ACCELERATOR DEVELOPMENT DEPARTMENT

BROOKHAVEN NATIONAL LABORATORY

Associated Universities, INC.
Upton, New York 11973

LINEAR APERTURE, SMEAR, VARIATION OF PARTICLE ACTION AND BEAM EMITTANCE IN THE SSC

Zohreh Parsa

Brookhaven National Laboratory
Accelerator Development Department
Upton, New York 11973

September, 1986

ABSTRACT

We have examined the variation of the particle action, beam emittance, smear and linear aperture for the SSC. The preliminary results obtained for the SSC $\beta^* = 6$ and $\beta^* = 5$ lattices, using the algorithm we have developed are also included.

I. INTRODUCTION

We have investigated and developed an algorithm for calculating the variation of the particle action, beam emittance growth, smear, linear aperture, etc. for accelerators in two dimensions with second order perturbation theory. We describe our theoretical development in section II and present some of our results for the SSC in section III.

II. THEORY

A dynamical system can be described by the Hamiltonian of the form

$$H = \frac{J_x}{\beta_x} + \frac{J_z}{\beta_z} + \sum_{k=3} \sum_{\ell=0}^{k} a_{k\ell}(s)\ \beta_x^{\ell/2}\ \beta_z^{\frac{k-\ell}{2}}\ J_x^{\ell/2}\ J_z^{\frac{k-\ell}{2}}\ \cos^{\ell}\phi_x \cos^{k-\ell}\phi_z$$

$a_{k\ell}(s)$ are the generalized multipole strengths (e.g. for sextupoles $a_{33} = \frac{S}{6}$, $a_{31} = -\frac{S}{2}$, etc; where S = sextupole strength etc.).

Using the generating function G (eq. (4)) and Hamilton's equations (eqs. 2 and 3)) we can find the emittance as a function of the invariants K_x, K_z; angle variables ϕ_x, ϕ_z and s;

$$\frac{\partial H}{\partial \phi_x} = -\frac{dJx}{ds} \tag{2a}$$

$$\frac{\partial H}{\partial J_x} = \frac{d\phi}{ds} \tag{2b}$$

$$\frac{\partial H}{\partial \phi_z} = -\frac{dJ_z}{ds} \tag{3a}$$

$$\frac{\partial H}{\partial J_z} = -\frac{d\phi_z}{ds} \tag{3b}$$

the generating function

$$G = K_x\phi_x + K_z\phi_z + \sum_k \frac{g_k(K_x,K_z,s)}{\sin\pi(n_{x_k}\nu_x+n_{z_k}\nu_z)} \cos(n_{x_k}\phi_x + n_{z_k}\phi_z + \xi_k) \tag{4}$$

Therefore, the emittance become:

$$E_x = 2\pi\left[K_x + \sum_k \frac{n_{x_k}\, g_k(K_x,K_z,s)}{\sin\pi(n_{x_k}\nu_x+n_{z_k}\nu_z)} \left(-\sin(n_{x_k}\phi_x + n_{z_k}\phi_z + \xi_k)\right)\right] \tag{5}$$

$$E_z = 2\pi\left[K_z + \sum_k \frac{n_{z_k}\, g_k(K_x,K_z,s)}{\sin\pi(n_{x_k}\nu_x+n_{z_k}\nu_z)} \left(-\sin(n_{x_k}\phi_x + n_{z_k}\phi_z + \xi_k)\right)\right] \tag{6}$$

where the transformed Hamiltonian can be written as:

$$\bar{H} = \frac{K_x}{\beta_x} + \frac{K_z}{\beta_z} + V(K_x,K_z,\phi_x,\phi_z,s) \tag{7}$$

where V includes the nonlinear terms due to sextupoles, octupoles, etc. for convenience, the emittance can be expressed in the following form

$$E_x = 2\pi\left[K_x + W_{1_{\phi_x}}K_x^{3/2} + W_{2_{\phi_x}}K_x^{1/2}K_z + V_{1_{\phi_x}}K_x^2 + V_{2_{\phi_x}}K_xK_z + V_{3_{\phi_x}}K_z^2 + \ldots\right] \tag{8}$$

and

$$E_z = 2\pi\left[K_z + W_{1_{\phi_z}}K_x^{3/2} + W_{2_{\phi_z}}K_x^{1/2}K_z + V_{1_{\phi_z}}K_x^2 + V_{2_{\phi_z}}K_xK_z + V_{3_{\phi_z}}K_z^2 + \ldots\right] \tag{9}$$

where W_1, W_2, V_1, V_2, V_3 etc. are the terms coming from the coefficients of the generating function given in Reference [1].

From these expressions we can obtain the variation of emittance due to the nonlinear effects in two dimensions [1,2], which includes the coupling effects due to the nonlinear terms to second order in the perturbation. This can be seen from eqs. (8 and 9) where the second order effects due to sextupoles are included in V terms.

In our preliminary runs of NONLIN we found that the second order V terms (which includes octupoles and second order effects of sextupoles) were larger than the first order W terms. Thus, if the contribution from W terms are zero, then the first order perturbation theory (calculations) breaks down and the second order perturbation theory becomes necessary.

SMEAR

We next find the "smear" by considering the variation of the emittance with respect to (s) the time variable of the Hamiltonian. Since, "smear" is defined as the measure of the extent to which the emittance varies from the invariant of the motion. In that we first find the average of the emittance E_x and E_z with respect to s (since ϕ_x and ϕ_z are functions of s);

$$\langle E_x \rangle_s = 2\ K_x$$
$$\langle E_z \rangle_s = 2\ K_z$$

From this we can find $\langle E_x^{1/2} \rangle_s$ and $\langle E_z^{1/2} \rangle_s$ and then the smear [3]

$$\text{Smear} = \left[\frac{\dfrac{\langle E_x \rangle + \langle E_z \rangle}{2}}{\left(\dfrac{\langle E_x^{1/2} \rangle + \langle E_z^{1/2} \rangle}{2}\right)^2} - 1 \right]^{1/2}$$

LINEAR APERTURE

We obtain the linear aperture (a_ℓ) from $a_\ell = \beta_{max} E_O/\pi$, where E_O is the initial beam emittance (that gives a smear = 0.1) and β_{max} is the maximum betatron amplitude.

We incorporated some of our results into an algorithm which was used to obtain results for the SSC $\beta^* = 6$ and $\beta^* = .5$ lattices, given in the following Section III.

CONCLUSION

In this note, for the SSC CDR Clustered Lattices $\beta^* = 6$ and $\beta^* = 0.5$, with only two chromaticity sextupole families [4], we have calculated the variation in action, beam emittance growth, smear and the linear aperture in two dimensions with second order perturbation. We note that the major reason for the difference in the linear aperture obtained for these two lattices is that the chromaticity correcting sextupoles in the $\beta^* = 0.5$ is stronger than those in the $\beta^* = 6$ lattice. Since although the β functions are identical in the arcs of these two lattices the natural chromaticity is much greater in the $\beta^* = 0.5$ lattice than that for $\beta^* = 6$ lattice.

III. SSC CDR Clustered Lattice

Given the unperturbed betatron tunes of the SSC (i.e. operating tunes of $\nu_x^0 = 78.265$ and $\nu_z^0 = 78.280$, periodicity 1) and a beam with initial emittances $E_x^0 = E_z^0 = 2.49\times10^{-7}\ \pi$ m-rad;

<u>for the $\beta^* = 6$;</u>

the emittance can grow to $E_x^{max} = 2.80\times10^{-7}$(at 43th element chromaticity sextupoles) and $E_z^{max} = 2.777\times10^{-7}$ (at the 7th chromaticity sextupoles). The "smear" is found to be 1.586×10^{-2}. Then using the definition for the linear aperture, the beam size at which the "smear" is 0.1, the linear emittance become 1.513×10^{-6} and (with $\beta_{max} = 332$) the corresponding linear aperture is 22.4 mm.

For the $\beta^* = 0.5$;

(At the 331th chromaticity sextupoles) the emittance can grow to $E_x^{max} = 3.526 \times 10^{-7}$ and $E_z^{max} = 3.487 \times 10^{-7}$ π m-rad. The "smear" is found to be 6.136×10^{-2} and the linear emittance when the smear = 0.1 is $4.049 \times 10^{-7} \pi$ m-rad. Then with $\beta_{max} = 332$, the linear aperture is 11.59 mm.

In the following Tables, we find the emittance such that the smear is 0.1 (or close to 0.1). Column 1 is the initial emittance E_o (equal to average emittance) and column 2 is the corresponding "smear" calculated using our algorithm NONLIN.

CONCLUSION

We have calculated the emittance growth, smear and linear aperture for the two SSC CDR clustered lattices (with only the two chromaticity sextupole families) for $\beta^* = 0.5$ and $\beta^* = 6$. In the arcs of these two lattices the β functions are identical but the natural chromaticity is much greater in the $\beta^* = 0.5$ lattice than that for $\beta^* = 6$ lattice. Thus, the chromaticity correcting sextupole in the $\beta^* = 0.5$ (SF = 9.87283E-03, SD = -1.59109E-02) must be stronger than those (SF = 5.12380E-03, SD = -8.25720E-03) in $\beta^* = 6$ lattice. This is the major reason for the difference in the linear aperture obtained for these two lattices.

Figures 1 and 2, show the plots of $E_z^{1/2}$ versus $E_x^{1/2}$ for the two SSC CDR clustered lattices respectively. These types of plots may be used as an alternative to tracking.

REFERENCES

1. Z. Parsa, S. Tepikian and E. D. Courant, Second Order Perturbation Theory for Accelerators (to be published);
   Z. Parsa, S. Tepikian and E. D. Courant, Fourth Order Resonances in the AGS-Booster Lattice, BNL-BST/TN 58, (1986).

2. Z. Parsa, Analytical Method for Treatment of Nonlinear Resonances in Accelerators, Proceedings of the 1986 Summer Study on the Physics of the SSC, Snowmass, Colorado, June 23-July 11, 1986;
   Z. Parsa, Second Order Perturbation in SSC, SSC/TN (1986).

3. Z. Parsa, Analytical Method for Obtaining the Variation of the Beam Emittance, Particle Action and Linear Aperture in Accelerators, Proceedings of the Summer Study on the Physics of the SSC, Snowmass, (1986).

4. SSC SYNCH input file: VAX version, A. Garren, CDG; CDC version, E. D. Courant, BNL.

5. Z. Parsa, Chromatic Perturbation and Resonance Strenghts in the SSC, Proceedings of the Summer Study on the Physics of the SSC, Snowmass, (1986).

6. Z. Parsa, Accelerator Dynamics and Beam Aperture, Proceedings of the 1985 SLAC Particle Accelerator School, (1986).

** We thank Dr. E. D. Courant and S. Tepikian for discussions and assistance.

TABLE IA

CDR LATTICE FOR SSC BET*=6.

	E0= 2Kx=2Kz	SMEAR
a	1.4964000000000000E-06	9.8886398305367912E-02
b	1.5132763787338209E-06	0.1000006207824445
c	1.5132669744559812E-06	9.9999999988740025E-02
d	2.2446000000000001E-06	0.1478648550284760
e	1.5134011346377608E-06	9.9997583074065054E-02
f	1.5134377543299081E-06	0.1000000000669477
g	1.5134377533155871E-06	9.9999999999936624E-02

	ERROR/ITTERATION
a	2.4940000000000001E-07
b	8.4381893669104098E-09
c	4.7021389198322087E-12
d	3.7410000000000002E-07
e	8.5005673188803848E-09
f	1.8309846073659077E-11
g	5.0716052718225910E-16

* abcd = at the position of 43 chromaticity sextupoles
* efg = at the position of 7th chromaticity sextupoles

TABLE IB

```
CDR LATTICE FOR SSC BET*=6.
--------------------------------------------------------

Perturbation of tunes

vx = vx0 + -716.3168 * Ex + -2445.579  * Ez
vy = vz0 + -2445.579 * Ex + -477.6593  * Ez

where the unperturbed betatron tunes are
vx0 =   78.26496     and  vz0 =   78.27999
with circumference =   82944.00
and    periodicity =       1

Given a beam with emittances
Ex = 2.4939999E-07 and Ez = 2.4939999E-07
             (*pi m-rad),
the perturbed tunes become
vx =   78.26417     and  vz =   78.27927

The emittance can grow to
Exmax =  2.7996103E-07 at the 43 th element and
Ezmax =  2.7773686E-07 at the  7 th element.

The smear is found to be
Smear=  1.5864842E-02 at the  43 th element and
Smear=  1.5863875E-02 at the   7 th element

The linear emittance when the smear = .1 is
Elin =  1.5132670E-06 at the  43 th element and
Elin =  1.5134377E-06 at the   7 th element
```

** 43 th element = 43 th chromaticity sextupoles
** 7 th element = 7 th chromaticity sextupoles

TABLE IIA

CDR LATTICE FOR SSC BET*=.5

	E0= 2Kx=2Kz	SMESR
a	3.7410000000000001E-07	9.2359282883863145E-02
b	4.0494992362387551E-07	0.1000030200278785
c	4.0493772455707142E-07	9.9999997762639773E-02
d	4.0493772551578602E-07	9.9999999999954249E-02
e	6.0740658683560714E-07	0.1498154196738448
f	4.0493772551390067E-07	9.9999999999549711E-02

	ERROR/ITTERATION
a	6.2350000000000002E-08
b	1.5424961811937750E-08
c	6.0995334020432968E-12
d	4.7935729934590223E-16
e	1.0123443113926786E-07
f	4.7841462446750210E-16

TABLE IIB

```
CDR LATTICE FOR SSC BET*=.5
--------------------------------------------------

Perturbation of tunes

vx = vx0 + -2659.335 * Ex + -9080.386 * Ez
vz = vz0 + -9080.386 * Ex + -1773.600 * Ez

where the unperturbed betatron tunes are
vx0 =   78.26497     and  vz0 =   78.27995
with circumference =   82944.00
and periodicity =            1

Given a beam with emittances
Ex = 2.4939999E-07 and Ez = 2.4939999E-07
              (*pi m-rad),
the perturbed tunes become
vx =   78.26204     and  vz =   78.27724

The emittance can grow to
Exmax = 3.5259643E-07 at the 331 th element and
Ezmax = 3.4869160E-07 at the 331 th element.

The smear is found to be
Smear= 6.1362222E-02 at the 331 th element and
Smear= 6.1362222E-02 at the 331 th element.

The linear emittance when the smear = .1 is
Elin = 4.0493774E-07 at the 331 th element and
Elin = 4.0493774E-07 at the 331 th element.
```

Figure 1

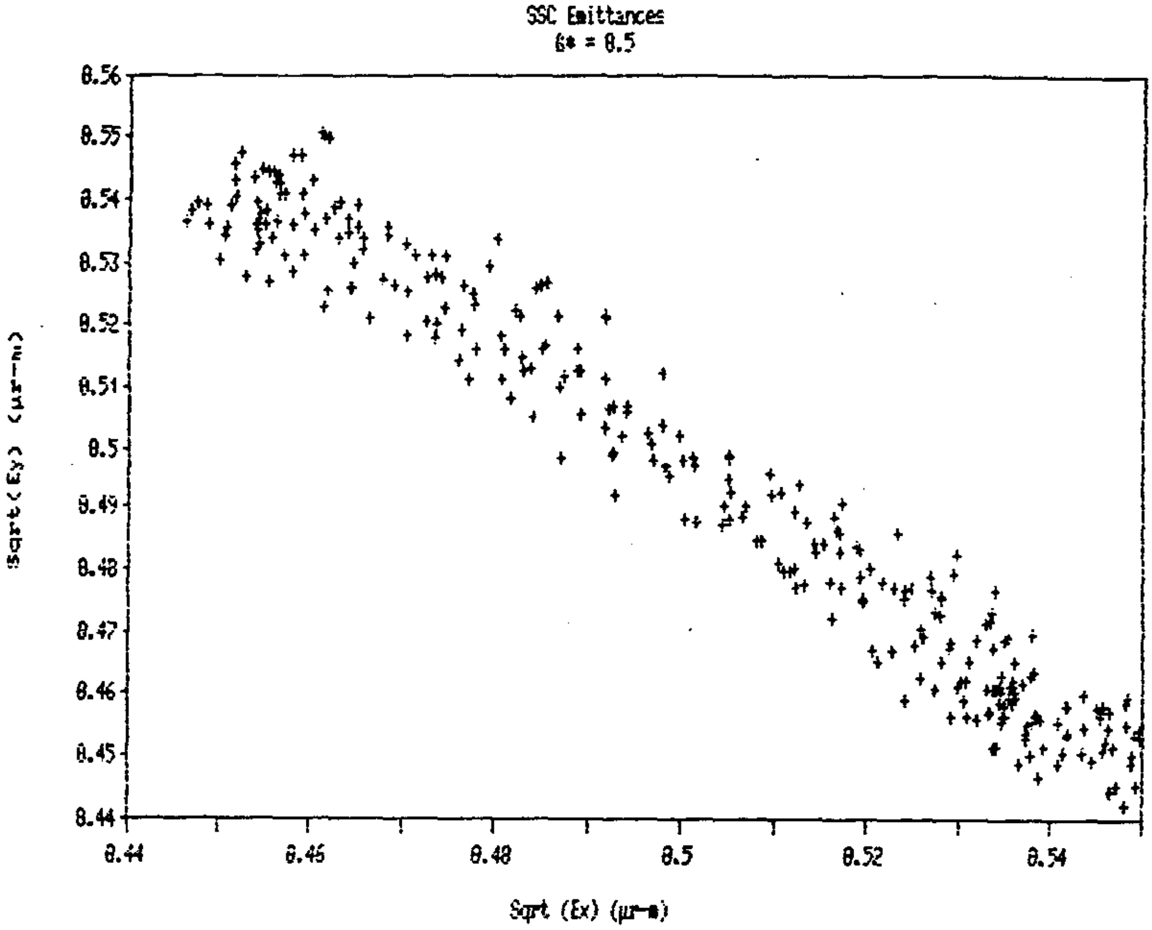

Figure 2