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SECOND ORDER PERTURBATION IN SCC

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ABSTRACT

Nonlinear effects, in the SSC including the perturbation of Tune; emittance growth; Hamiltonian resonance strength; generating function resonance strength, fixed points, Chirikov criteria, island width, etc., are investigated and some of our results are discussed and tabulated. Tune diagrams are also included.

I. Introduction

We have studied the nonlinear effects in the SSC using the algorithm ("NONLIN") we have developed [1-3]. In Section II, a brief overview of our theoretical development is given. In Section III, the structure resonaces of the SSC CDR clustered lattice is analyzed and tabulated. Some of these results are also obtained using the alternate program HARMON (given in Reference [4]) and are compared [5].

II. Perturbation Theory

In this section we include a brief discussion of the theoretical development used for investigating the nonlinear effects in the SSC.

A dynamical system such as circular accelerators can be described by the Hamiltonian of the form

$$H = \frac{2\pi}{C} I_x + \frac{2\pi}{C} I_z + V(I_x, I_z, \phi_x, \phi_z, s) \tag{1}$$

where C is the circumference; (I_x, ϕ_x) and (I_z, ϕ_z) are the action angle variables, ν_x and ν_z are the tunes; V includes the nonlinear effects due to sextupoles, octupoles, etc. and is periodic in ϕ_x, ϕ_z and s. To study this Hamiltonian we consider a generating function that can eliminate the nonlinear terms in V, e.g.

$$F(L_x, L_z, \phi_x, \phi_z, s) = L_x\phi_x + L_z\phi_z + \sum_k \frac{f_k(L_x, L_z, s)}{\sin \pi (n_{x_k} \nu_x + n_{z_k})\nu_z} \cos(n_{x_k}\phi_x + n_{z_k}\phi_z + \xi_k) \tag{2}$$

Where $f_k(L_x, L_z, s)$ are the generating function resonant strengths whose magnitude gives the extent to which I_x and I_z varies from the invariants of the motion. The L_x and L_z are the new action variables and ξ_k is the phase, n_{x_k} and n_{z_k} are integers that define a given resonant. Then the transformed Hamiltonian becomes

$$\tilde{H} = H + \frac{\partial}{\partial s} F(L_x, L_z, \phi_x, \phi_z, s) \tag{3}$$

Where we used the generating function F given by eq. (2) with

$$I_x = \frac{\partial F}{\partial \phi_x} \tag{4}$$

$$I_z = \frac{\partial F}{\partial \phi_z} \tag{5}$$

From this Hamiltonian eq. (3), we can obtain the perturbation to betatron tunes [where ν_{o_x} and ν_{o_z} are the unperturbed tunes].

$$\nu_x = \nu_{o_x} + 2\ \alpha_{xx}L_x + 2\ \alpha_{xz}L_z + \ldots. \tag{6}$$

$$\nu_z = \nu_{o_z} + 2\alpha_{xz}L_x + 2\ \alpha_{zz}L_z + \ldots. \tag{7}$$

The $2L_x$ and $2L_z$ are the beam emittance divided by π just before the beam enters the accelerator.

From equations (2, 4 and 5) we obtain the emittance growth, eqs. (8,9). That is the estimates to the upper limit that emittance may grow to as long as the tunes are far from any resonances.

$$E_x \le 2\pi L_x + \sum_k n_{x_k} \frac{\bar{g}_k\ (L_x,\ L_z,\ s)}{\mathrm{Sin}\pi\ (n_{x_k}\nu_x + n_{z_k}\nu_z)} \tag{8}$$

$$E_z \le 2\pi L_z + \sum_k n_{z_k} \frac{g_k\ (L_x,\ L_z,\ s)}{\mathrm{Sin}\pi\ (n_{x_k}\nu_x + n_{z_k}\nu_z)} \tag{9}$$

The above perturbation approach works well when far from resonance. That is when $| n_x\nu_x + n_z\nu_z - P | >> 0$; where n_x, n_z and P are integers. To study the nonlinear behavior when near a resonance we isolate the resonance as we find the fixed points of the system, which is the distance from resonance at which there is no motion in a special reference frame where the Hamiltonian is an invariant.

In that we expand V (in eq. (1)) in a fourier series about ϕ_x, ϕ_z and s. We then find a term in which the argument of the sine and cosine term varies the slowest with s (time variable of the Hamiltonion). We only consider this term since it has the largest effect on the dynamics of the system. Therefore, the Hamiltonian

$$\bar{H} = \frac{2\pi}{C}\, \nu_{o_x} L_x + \frac{2\pi}{C}\, \nu_{o_z} L_z + T\,(L_x, L_z) \tag{10}$$

$$+ \frac{1}{C}\, h\,(L_x,\, L_z)\, \mathrm{Cos}\, (n_x\phi_x + n_z\phi_z - \frac{2\pi}{C}\, ps + \xi)$$

where T (L_x, L_z) is the term causing the perturbation of tune, h (L_x, L_z) is the Hamiltonian resonance strength and ξ is the constant phase. By defining the bandwidth $\delta = n_x\nu_x + n_z\nu_z - p$ that determines how far the tunes (ν_x, ν_z) are from the given resonance (defined by n_x, n_z, p) we can obtain the fixed points [2]. That is, the system will be on a fixed point if

$$\delta = \pm \frac{1}{2\pi h(J_x, J_z)} \left[n_x \frac{\partial h(J_x, J_z)}{\partial J_x} + n_z \frac{\partial h(J_x, J_z)}{\partial J_z} \right] \tag{11}$$

at actions equal to J_x and J_z, where $h(J_x, J_z)$ is the Hamiltonian resonance strengths.

We also obtain a criterion (Chirikov Criterion) that determine whether a nearby resonance is important to the dynamic of the system. That is a resonance can be neglected, if the bandwidth (δ) of a nearby resonance satisfies

$$\delta \gg \frac{4}{C} \sqrt{2\pi \left(n_x \frac{\partial T(L_x, L_z)}{\partial L_x} + n_z \frac{\partial T(L_x, L_z)}{\partial L_z} \right) h(L_x, L_z)} \tag{12}$$

Otherwise, the resonance must be included in order to accurately describe the behavior of the system.

IV. NONLINEAR RESONANCES IN SSC

The operating point of SSC, 0 ($\nu_x = 78.265$, $\nu_z = 78.280$) is near at least 4 resonances of up to the 4th order. $4\nu_z = 313$, $2\nu_x - 2\nu_z = 0$, $2\nu_x + 2\nu_z = 313$, $4\nu_x = 313$, $\nu_x + 3\nu_z = 313$ and $3\nu_x + \nu_z = 313$ (the last two due to the Skew terms). In addition to these, there are higher order resonances that may be important as shown in Figs. (1-4). For example, the eleventh order resonance $5\nu_x + 6\nu_z = 861$ (Fig. 2), the twelfth order resonance $8\nu_x - 4\nu_z = 313$ (Fig. 3), etc.

We have used the SSC clustered lattice with only chromaticity sextupoles [7]. We calculated the resonance strengths, the fixed points, stop bandwidths, island widths and Chirikov Criterion. The perturbation to tune and the emittance growth are also calculated. Some of our results are shown in the following Tables.

The emittance growth and the perturbation to the betatron tune at the operating point of the SSC ($\nu_x = 78.265$, $\nu_z = 78.280$) are given in Table I. These values were calculated for the average beam emittance of $E_0 = 2.49 \times 10^{-7}\pi$ m-rad (corresponding to the expected beam size of 9.1 mm), which is larger than the expected beam emittance) $E_0 = \frac{E_N}{\beta\gamma} = 9.37 \times 10^{-10}$ π m-rad which was obtained from the normalized emittance $E_N = 10^{-4}\pi$ m-rad at injection given in SSC-Design Manual).

This initial emittance was selected because it was close to the estimated value of dynamic aperture as given in SSC-Design Manual. The bandwidth (e) from some of these resonances are calculated and are given in Table III.

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TABLE Ia

CDR LATTICE FOR SSC BET*=6.

Perturbation of tunes

```
vx = vx0 + -716.3168 * Ex + -2445.579 * Ez
vy = vz0 + -2445.579 * Ex + -477.6593 * Ez

where the unperturbed betatron tunes are
vx0 =    78.26496      and  vz0 =    78.27999
with circumference =    82944.00
and     periodicity =        1

Given a beam with emittances
Ex = 2.4939999E-07 and  Ez = 2.4939999E-07
                 (*pi m-rad),
the perturbed tunes become
--------------------------
vx =    78.26417      and  vz =    78.27927

The emittance can grow to
--------------------------
Exmax =  2.7996103E-07 at the 43 th element and
Ezmax =  2.7773686E-07 at the 7 th element.

The resonances are numbered as follows:

No.    Resonance           Strength   Stp bndw
             Fix pts.      Width     Chirikov cr
----------------------------------------------------

 2A   0 vx +4 vz =  313   1.1625E-15 7.4577E-08
             3.9350E-08 1.9500E-10 9.0316E-10

 7B   2 vx -2 vz =    0   8.9925E-11 1.4423E-03
             1.3015E-03 7.7978E-08 3.4943E-07

 9A   2 vx +2 vz =  313   1.6702E-15 5.3576E-08
             5.3576E-08 2.6195E-10 1.9320E-09

13A   4 vx +0 vz =  313   6.8462E-16 4.3921E-08
             3.6246E-08 1.2220E-10 8.4877E-10
```

TABLE Ib

CDR LATTICE FOR SSC BET*=6.
The generating function resonance strength
for the resonances:

Chromaticity sextupoles	Resonance numbers 1	2	3
43	6.84477E-12	1.27363E-12	1.00860E-09
	4	5	6
	2.74030E-09	2.59494E-10	3.84700E-12
	7	8	9
	1.12059E-09	1.36535E-11	7.96476E-13
	10	11	12
	2.24002E-10	9.89518E-11	6.82071E-13
	13	14	15
	3.61567E-13	1.42736E-12	4.54290E-11
Elements	1	2	3
7	6.97217E-12	1.30371E-12	1.00845E-09
	4	5	6
	2.73891E-09	2.59353E-10	3.86933E-12
	7	8	9
	1.12059E-09	1.36619E-11	8.46263E-13
	10	11	12
	2.24590E-10	9.88886E-11	6.77308E-13
	13	14	15
	3.67257E-13	1.42967E-12	4.55659E-11

** The resonance numbers in the above table correspond to the resonance numbers given in the Hamiltonian table (Table Ia).

TABLE IIa

```
CDR LATTICE FOR SSC BET*=.5
-----------------------------------------------------
Perturbation of tunes

vx = vx0 + -2659.335 * Ex + -9080.386 * Ez
vy = vz0 + -9080.386 * Ex + -1773.600 * Ez

where the unperturbed betatron tunes are
vx0 =   78.26497     and  vz0 =   78.27995
with circumference =   82944.00
and periodicity    =         1

Given a beam with emittances
Ex = 2.4939999E-07 and  Ez = 2.4939999E-07
              (*pi m-rad),
the perturbed tunes become
--------------------------
vx =   78.26204     and  vz =   78.27724

The emittance can grow to
--------------------------
Exmax =  3.5259643E-07 at the 331 th element and
Ezmax =  3.4869160E-07 at the 331 th element.

The resonances are numbered as follows:

No.   Resonance          Strength   Stp bndw
            Fix pts.     Width     Chirikov cr
-----------------------------------------------------

 2A  0 vx +4 vz =  313   1.7570E-14 1.1272E-06
            5.9411E-07 3.9343E-10 6.7660E-09

 7B  2 vx -2 vz =    0   3.3390E-10 5.3553E-03
            4.8323E-03 7.7979E-08 1.2975E-06

 9A  2 vx +2 vz =  313   2.6094E-14 8.3701E-07
            8.3701E-07 5.3733E-10 1.4715E-08

13A  4 vx +0 vz =  313   1.1546E-14 7.4075E-07
            6.2142E-07 2.6046E-10 6.7162E-09
```

TABLE IIb

CDR LATTICE FOR SSC BET*=.5
The generating function resonance strength for the resonances:

Elements	Resonance numbers		
	1	2	3
331	2.59284E-11	4.83069E-12	1.94224E-09
	4	5	6
	5.27804E-09	4.99578E-10	1.43731E-11
	7	8	9
	4.16080E-09	5.07310E-11	3.12842E-12
	10	11	12
	8.33345E-10	1.90890E-10	2.51736E-12
	13	14	15
	1.36763E-12	5.30889E-12	1.69158E-10

** The resonance numbers in the above table correspond to the resonance numbers given in the Hamiltonian table (Table IIa).

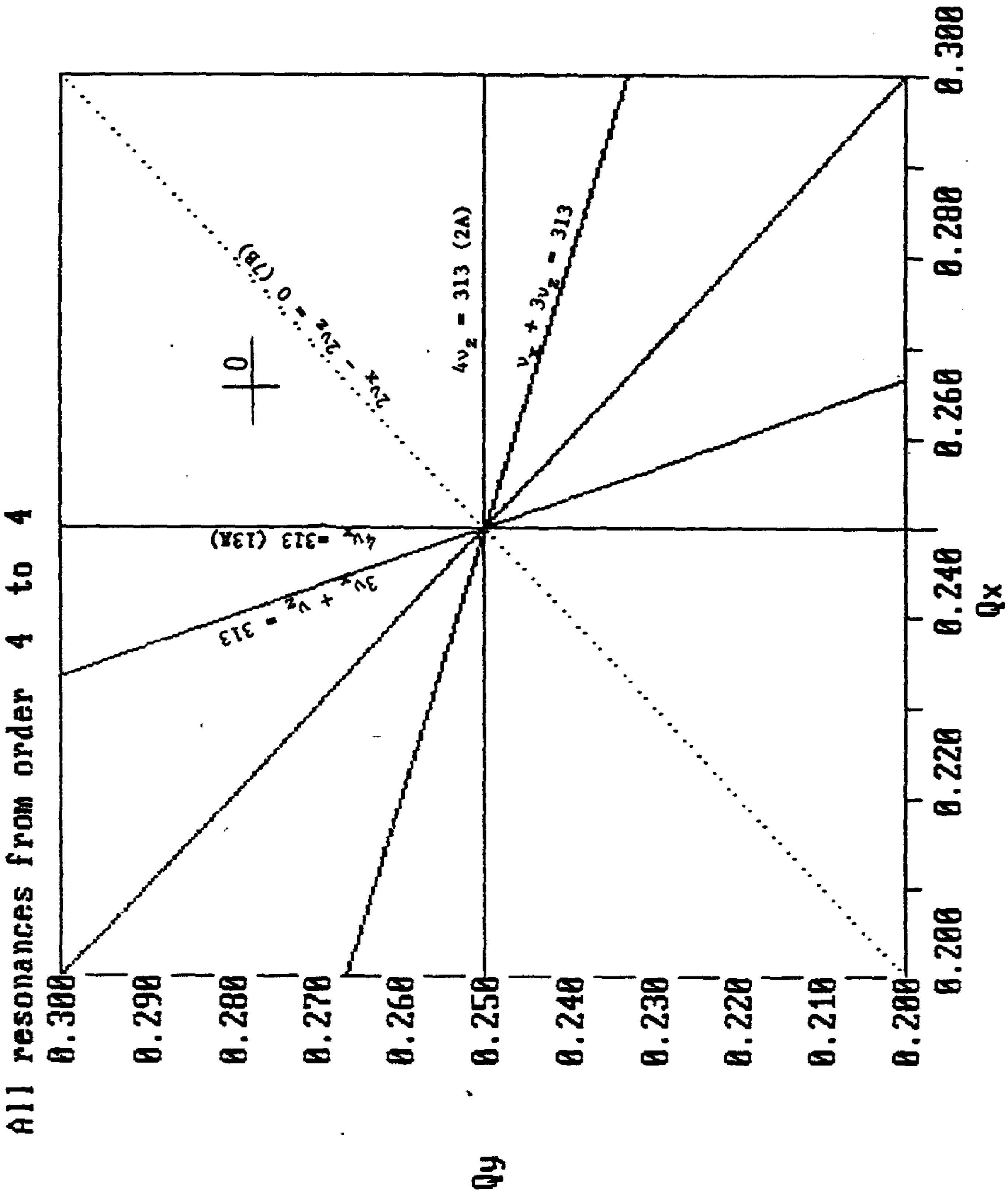


FIG. 1 SSC-CDR LATTICE 0(78.265, 78.280)

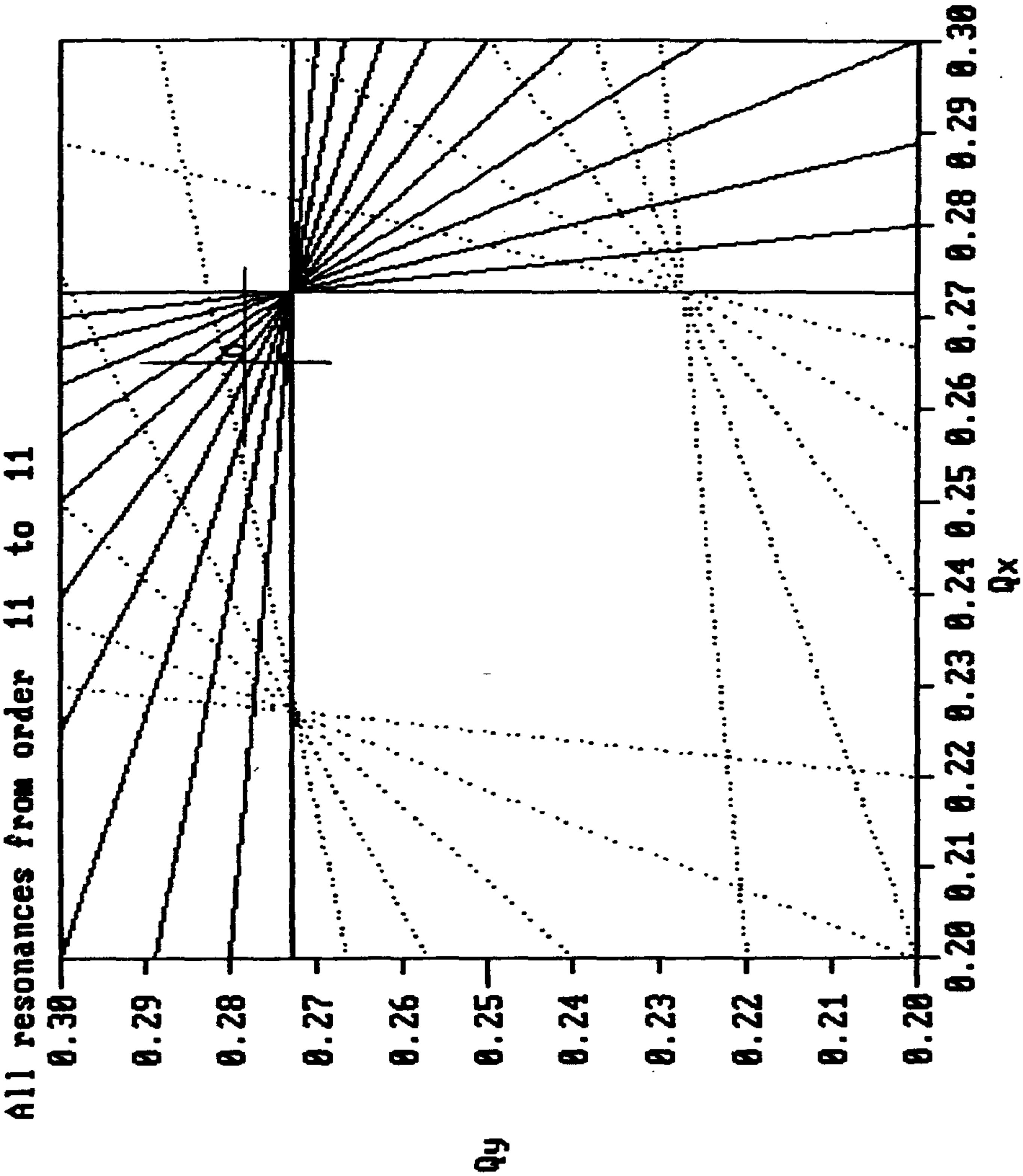
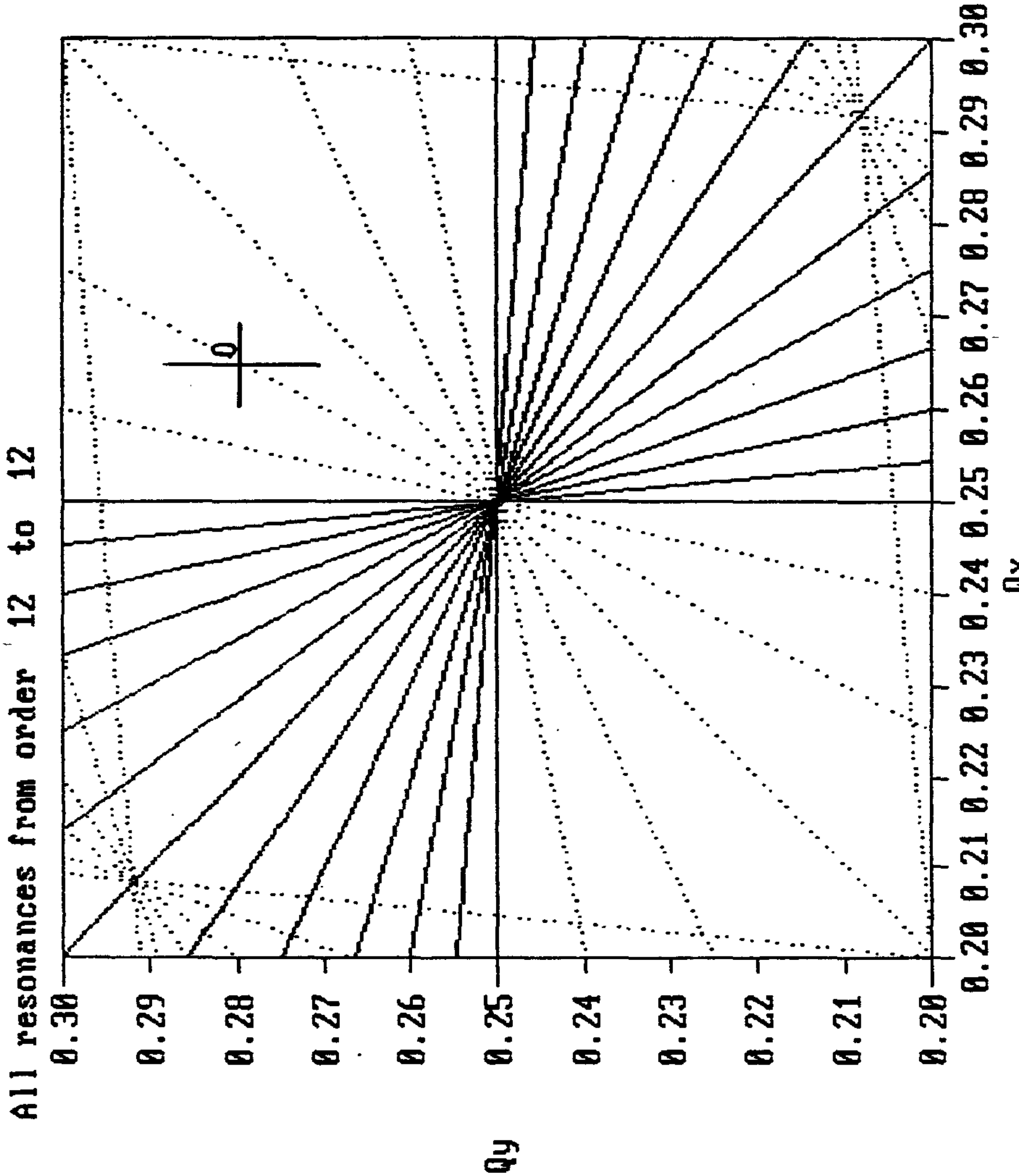


FIG. 2 SSC-CDR CLUSTERED LATTICE O(78.265, 78.280)



<u>FIG. 3</u> SSC-CDR CLUSTERED LATTICE O(78.265, 78.280)

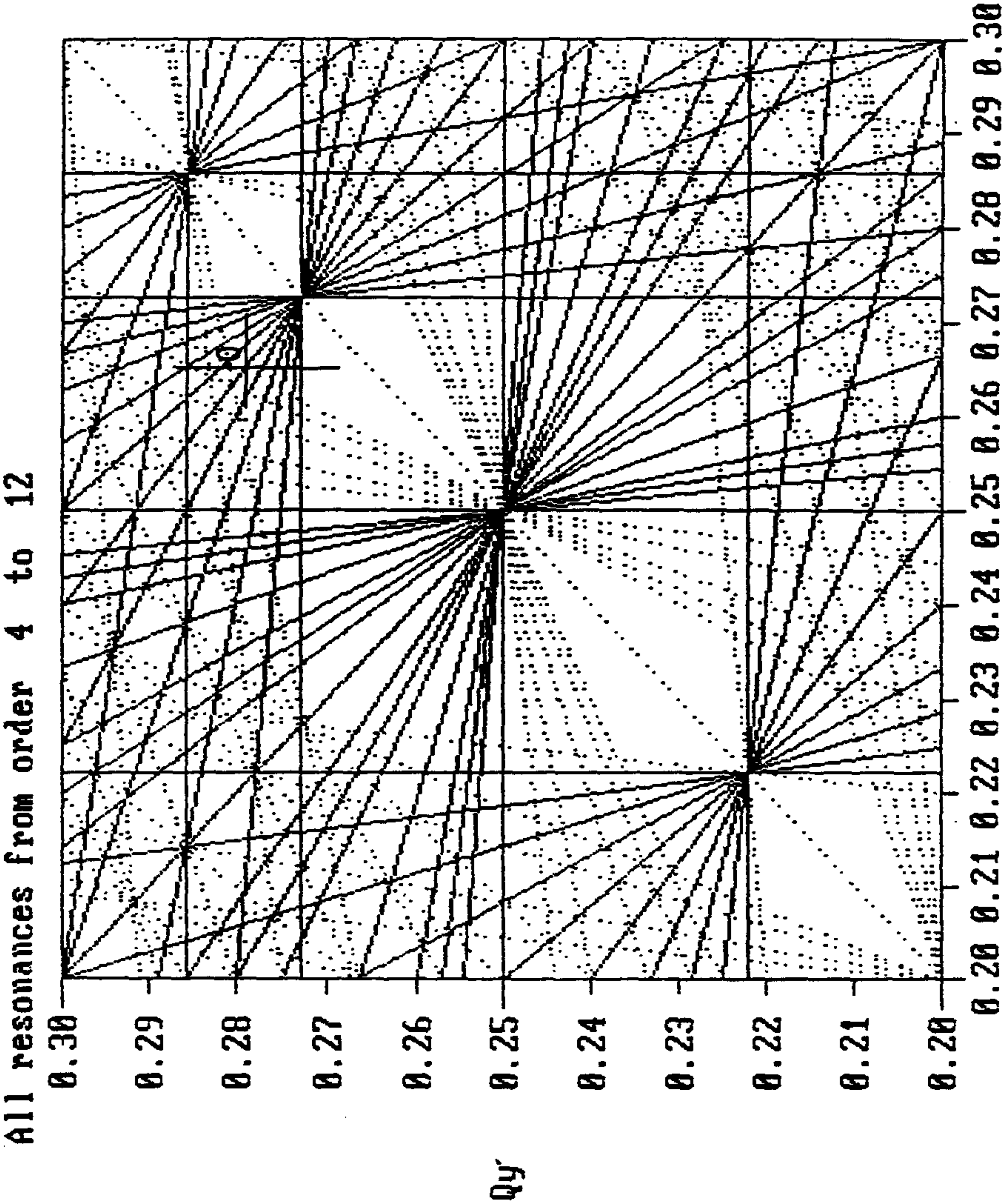


FIG. 4   SSC-CDR CLUSTERED LATTICE O(78.265, 78.280)