SSC-N-254

October 1986

AVAILABILITY ANALYSIS OF ACCELERATED LIFE TESTS FOR SSC MAGNETS

J. E. SHRAUNER

SSC/CDG and WASHINGTON UNIVERSITY, ST. LOUIS

Introduction

Accelerated life tests are necessary to assure that the SSC magnets can be expected with specified confidence levels to perform at their specified reliability, availability and lifetime.[1,2] Analysis of the probabilistic confidence in specified availability levels obtainable from accelerated life tests on the SSC magnets has been outlined before.[3,4] Our objective here is to amplify and elucidate that analysis in order to affirm its general procedure and establish a firm basis for further analysis.

Analysis

The objective of this analysis is to quantify the probabilistic confidence for the specified stationary availability A_∞ of the SSC magnet system over its specifed 20-year lifetime on the basis of accelerated life test results.

The proposed accelerated life test protocol allows for sampling in about one week of testing the equivalent of about one year of ordinary operational cycling anticipated for the SSC.[3,4]

Availability requirements on the SSC magnet system are severe, because it is comprised of about 9500 individual magnets in a serial-fault configuration. This means that the individual component magnets must be extremely reliable. A general relation among the stationary availability A_∞ , the mean time to repair MTTR, and the mean time between failures MTBF is

$$A_\infty = 1/[\ 1+ \mathrm{MTTR}/\mathrm{MTBF}\ ] \quad . \tag{1}$$

For our purposes MTTR is specifed[2] at about 1 week $\cong$ 1/50 year. This determines MTBF through the above relation for a specified value of A_∞ . Because of the serial fault configuration of the component magnets in the system, a mean effective failure rate λ for the individual component magnets (considered same) can be assigned by:

$$\lambda \cong 1/(9500\ \mathrm{MTBF}) = [(1/A_\infty) -1]/[9500\ \mathrm{MTTR}] \quad . \tag{2}$$

The assignment of a constant effective failure rate implies that the individual magnets fail for a complex variety of causes. This so-called exponential-failure-function hypothesis is chosen mostly for the simplicity of computation that it allows for obtaining rough estimates.

The SSC Conceptual Design Report[1] specifies A_∞ for the magnet system as required to be 96 percent. On the other hand, it also specifies the availability for the whole SSC as required to be 80 percent; and it might be thought that in the sense in which we are concerned the magnet system is in some way equivalent to the whole SSC. For this reason we will consider both these values as perhaps two extremes of a range of values for A_{∞} . The value $A_{\infty} = .80$ in Eq.(2) specifies $\lambda \cong 1/(760$ year); $A_{\infty} = .96$ specifies $\lambda \cong 1/(4560$ year).

In general, the probability that a sample of N magnets will survive y equivalent years of operationl cycling (y weeks of testing) with no more than n failures is

$$P(n,N,y|\lambda) = \sum_{\ell=0}^{n} \binom{N}{\ell} [p_{\lambda}(y)]^{N-\ell} [1-p\ (y)]^{\ell} \quad , \tag{3}$$

where $p_{\lambda}(y)$ is the probability that an individual magnet will survive y equivalent operational years of cycling. Under our assumed exponential failure hypothesis $p_{\lambda}(y) \cong \exp(-\lambda y)$. A more realistic model of the failure function could be inserted into Eq.(3). The important point is that Eq.(3) gives the probability of a sample of N (identical) magnets surviving y equivalent operational years with $\leqslant$ n failures under the assumption of a definite given failure law with a definite given effective failure rate[5] λ . But this effective failure rate is what we want to infer from the test results, namely, that a sample of N magnets survives y equivalent operational years of testing with $\leqslant$ n failures.

We quantify our confidence that the actual effective constant failure rate λ is less than λ' may be inferred on evidence of an N-magnet sample surviving y equivalent operational years of test cycles with no more than n failures as

$$R(\lambda \leqslant \lambda' | n,N,y,c) = \frac{\int_0^{\lambda'} P(n,N,y|\lambda) P_c(\lambda) d\lambda}{\int_0^{\infty} P(n,N,y|\lambda) P_c(\lambda) d\lambda} \quad . \tag{4}$$

In this espression we have assumed a continuous distribution of conceivable values of the integration variable λ . The function $P_c(\lambda)$ weighs the probability distribution of the variable on this continuum on the evidence of all prior information, and the label c in the arguements of R indicates its functional dependence on this prior probability distribution. The numerator integrates over conceivable values of λ up to the hypothetical test value λ' the probability of the outcome of our accelerated life test on condition of the prior distribution. The denominator integrates the same function over all conceivable values of λ . The ratio R is the probability that the value to be inferred from our test results lies in the interval $0 \leqslant \lambda \leqslant \lambda'$, subject to the prior distribution $P_c(\lambda)$.

In order to incorporate a tractable, but definite dependence on this prior probability distribution we will take it for now to be of the form

$$P_c(\lambda) = \text{constant} > 0\ , \quad \lambda \geq \lambda_c\ ,$$
$$= 0 \qquad\qquad \lambda < \lambda_c\ . \tag{5}$$

This simple form will be shown in later work to be a surprisingly astute choice.[6]

With the above choice of the prior probability distribution $P_c(\lambda)$ the expression for R can be integrated and rearranged into the form

$$1-R(\lambda \leq \lambda' | n,N,y,c) = C(n,N,y,\lambda',\lambda_c)\exp[-(N-n)(\lambda'-\lambda_c)y]\ , \tag{6}$$

in which

$$C(n,N,y,\lambda',\lambda_c) = D(n,N,y,\lambda')/D(n,N,y,\lambda_c) \qquad , \tag{7}$$

with

$$D(n,N,y,\lambda) = \sum_{\ell=0}^{n} \binom{N}{\ell} e^{-(n-\ell)\lambda y} \sum_{j=0}^{\ell} \binom{\ell}{j} (-e^{-\lambda y})^j \quad . \tag{8}$$

n = 0 Case

The case of n=0 is simple since for all N, y, λ', and λ_c,

$$C(n=0) = D(n=0) = 1 \tag{9}$$

and

$$\ln(1 - R) = -(\lambda' - \lambda_c)Ny \qquad . \tag{10}$$

This relation sets the scale for all n, as n ≠ 0 cases are related to it by studying C of Eqs.(7,8). Some values of interest related by Eq.(10) are given in Table I.

TABLE I

SCALE OF SAMPLE SIZE AND TESTING PERIOD FOR VARIOUS CONFIDENCE LEVELS AT 2 SPECIFIED AVAILABILITIES FOR VARIOUS PRIOR DISTRIBUTIONS RELATED THROUGH EQ.(10) FOR n=0 .

$\lambda' = 1/(760 \text{ year}) \Longleftarrow A_\infty = .80$

R = .99 ,	λ_c/λ' = .4	: Ny = 5833 =	(200 magnets)x.56yr of test
" = .99 ,	" = .1	: " = 3889 =	(200 magnets)x.37yr of test
" = .99 ,	" = 0	: " = 3500 =	(100 magnets)x.67yr of test

$\lambda' = 1/(4560 \text{ year}) \Longleftarrow A_\infty = .96$

R = .90 ,	λ_c/λ' = .1	: Ny =11666 =	(200 magnets)x1.1yr of test
" = .90 ,	" = .05	: " =11050 =	(200 magnets)x1.06yr of test
" = .90 ,	" = 0	: " =10500 =	(200 magnets)x1.0yr of test
" = .99 ,	" = 0	: " =21000 =	(200 magnets)x2.2yr of test

---

n ≠ 0 Case

For the case of $n \neq 0$ Eq.(6) includes a nontrivial C factor. Instead of the simple form of Eq.(10), it now includes an additive logarithmic term:

$$\ln(1 - R) = -(\lambda' - \lambda_c)(N-n)y + \ln C \quad . \tag{11}$$

A specified level of confidence R can be obtained with tests on the same sample with no failures in a test interval y and with no more than n failures in the test period $[y + (\ln C)/(\lambda' - \lambda_c)]/(1 - n/N)$.

The region of effective support for the last term in Eq.(11) is limited by the behavior under which

$$\ln C(n, N, y, \lambda', \lambda_c) \xrightarrow[y \to 0]{} \ln 1 = 0 \quad ,$$

and

$$\xrightarrow[\lambda_c y \gg 1]{} \ln 1 = 0 \quad . \tag{12}$$

At large values of its arguements Eq.(11) has the same form as Eq.(10), but with (N-n) in place of N, i.e., $\ln(1-R)$ vs. $\lambda' y$ is again a straight line from the origin, but with a slope $(1-n/N)$ times that of the $n=0$ line. However, the interval of interest seldom extends to values of $\lambda' y$ large enough for C to approach unity.

The interval of interest can be estimated as

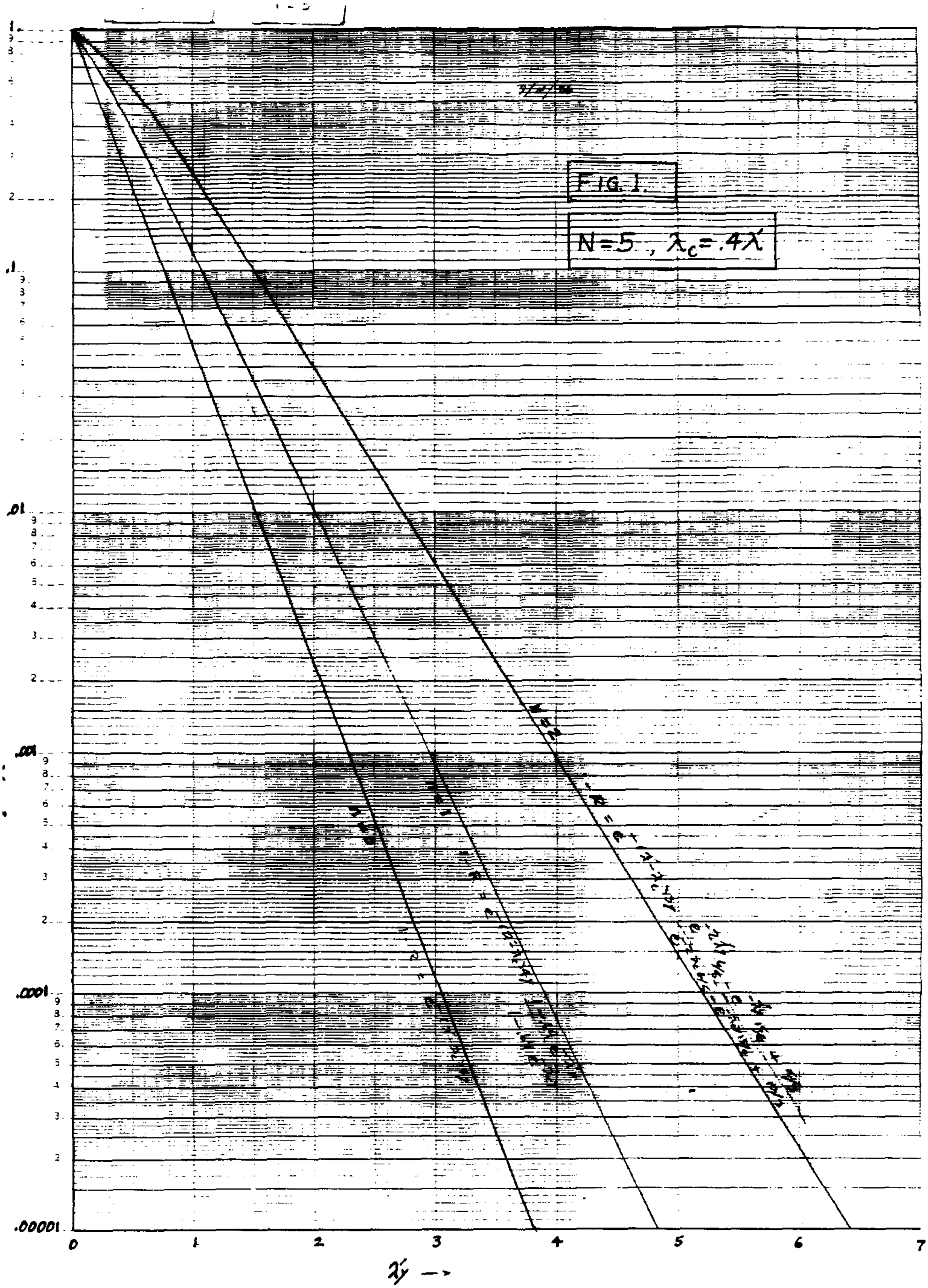
$$\lambda' y \lesssim 2.3d \,/[(N-n)(1 - \lambda_c/\lambda')] \quad , \tag{13}$$

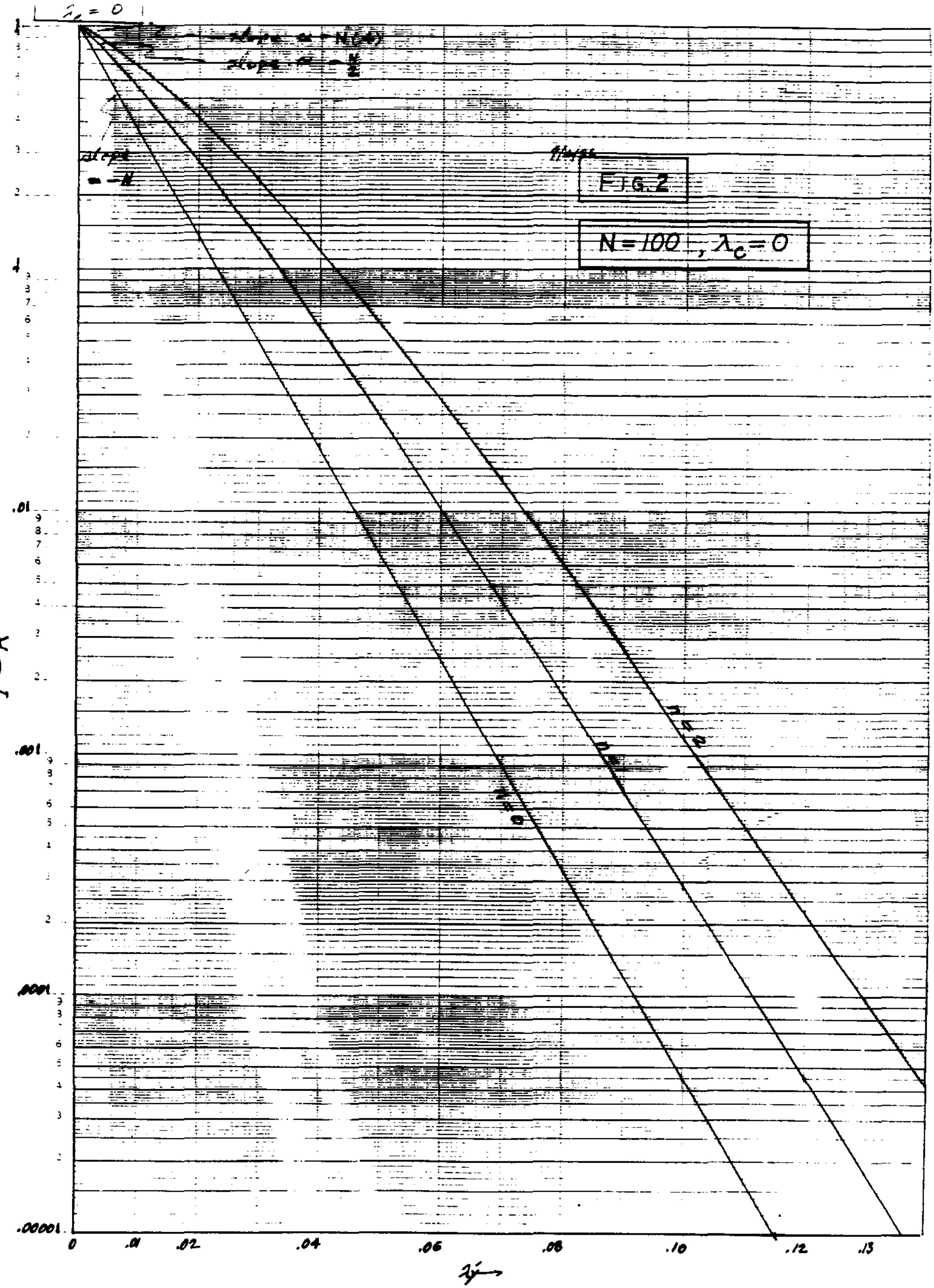
with d specified by $1 - R = 10^{-d}$. Generally, $d \cong 3$-5. The effective support of lnC can be seen from Eqs.(6,7) to extend to about $\lambda' y = 3\,\lambda'/\lambda_c$. So lnC contributes through the upper range of the interval of interest of $\lambda' y$; since for $d \lesssim 5$, $\lambda_c \lesssim .3\lambda'$, and $N-n \lesssim 3$ the $\lambda' y$ of interest is $\lesssim 5 \ll \lambda' y$ of upper extent of support of lnC.

At small values of $(\lambda' - \lambda_c)y$ the factor C modifies the slope of R at the origin such that

$$\ln[1 - R(\lambda' \lesssim \lambda_c | n, N, y, c)] \xrightarrow[y \to 0]{} -(\lambda' - \lambda_c)(N-n)y/(n+1), \tag{14}$$

times a factor like $1/[1+0(n/N)]$ on the right-hand side. Thus, lnC has a broad, slightly-humped plateau extending to above the interval of interest of $\lambda' y$. Effects of lnC show clearly in the graphs of $\ln(1-R)$ vs. $\lambda' y$ in Figures 1 and 2.


1.
.1
.01
.001
.0001
.00001
0
1
2
3
4
5
6
7
FIG. I.
N=5 , $\lambda_c = .4\lambda'$



46 6213

K•E SEMI-LOGARITHMIC 5 CYCLES X 70 DIVISIONS KEUFFEL & ESSER CO. MADE IN U.S.A.

Priors

The prior probability distribution denoted in our analysis here by $P_c(\lambda)$ and the label c in $R(\lambda \leqslant \lambda' | n, N, y, c)$ should represent the totality of all prior information on the distribution of probable values of the variable λ, which is the object of our test program and its analysis. The priors have been taken here as chosen input--mostly for purposes of illustration. If absolutely no prior information relevant to probable distribution of λ were known we should (presumably) take the prior distribution to be uniform over the whole space of conceivable values. Both of these distributions are variants of the so-called microcanonical distribution. As information is accumulated we can incorporate it into our analysis through an up-dateing of the prior distribution used. Preliminary investigations indicate that the extended microcanonical distribution used for illustration here is surprisingly close to that distribution which minimizes the unwarranted bias. Work is continuing on this approach, and will be reported separately.

The prior distribution chosen for illustration here is close to that representing no information when λ_c is taken close to 0. At the other extreme is the case when the prior probability distribution represents almost exact prior knowledge of the value of λ to be expected. For this case the distribution could be any function of narrow compact support: a delta function in the limit, or a narrow microcanonical distribution such as $P_\triangle(\lambda)$ = constant > 0 for $|\lambda - \lambda_\triangle| \leqslant \triangle/2 \ll \lambda_\triangle$, and $= 0$ for λ outside this interval. In these cases $R(\lambda \leqslant \lambda', n, N, y, \triangle)$ becomes a step function rising from zero at $\lambda_\triangle - \triangle/2$ to unity at $\lambda_\triangle + \triangle/2$. This just says that if we know the answer beforehand then the probability of inferring that value from our test will be zero until our test hypothesis includes that value.

FIGURE CAPTIONS

FIGURES 1 and 2 are semi-log plots of $1 - R(\lambda < \lambda' | n,N,y,c)$ versus $\lambda' y$ for $n = 0,1,2$, with the prior distribution Eq.(5) for $N = 5$ in Figure 1, and for $N = 100$ in Figure 2.

References

1.) SSC Conceptual Design Report, SSC-SR-2020, Sec.5.2.1, p.267.

2.) Report of Task Force on SSC Magnet System Test Site, SSC-SR-1001, Sec. III, A., p.9.

3.) V. Karpenko, correspondence with D. Brown, 7/1/86, and D. Brown, correspondence with V. Karpenko, 7/10/86.

4.) E. Shrauner, SSC-N-215, 8/86.

5.) We consider here a broad overview of a composite system test. Reliabilities and availabilities of component subsystems can be analysed separately. We assume that extensive and systematic studies on component parts and subsystems have been separately performed during the previous development of the magnets. As an example, the study and resolution of the problem of collar fatigue failure at the Tevatron is described in Sec. III, A, p. 9-10 of Ref.(2.). Results of such subsystem analyses will be integral parts essential to later, more detailed extensions of the overall problem.

6.) It must be acknowledged that use of formulations such as Eq.(4), which is the essence of our treatment, have been highly controversial. Eq.(4) is a form of what is known as Bayes' theorem. Criticisms focus on the subjectivity in the selected prior probability distribution, and the propriety of attributing any prior probability distribution on a metrical space of conceivable physical variables. The first problem can be mitigated or removed entirely, as we shall show later. The second is philosophical, and subject to evolution, but more persistent. The microcanical type distribution Eq.(5) extended to the open interval $0 \leqslant \lambda < \infty$ is not normalizable on this interval. This does not show up in Eq.(5), which is itself of the form of a normalization condition. If this were to show itself to be problematic it could be made normalizable through the introduction of an upper cutoff on the distribution's domain of support.