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CONDUCTION OF HEAT OF SYNCHROTRON RADIATION THROUGH THE WRAPPED BEAM TUBE WALL

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Abstract

The energy of the synchrotron radiation is deposited initially in a very narrow stripe on the "away from center" side of the inner surface the beam tube. The energy is transported through the beam tube wall and the exterior wrapping to the liquid helium. The temperature distribution on the inner wall of the beam tube and the distribution of energy flux into the helium are calculated for a stainless steel beam tube with its wrapping, with and without a 2 mil copper plating on the inner surface. With the copper plating the maximum temperature difference is less than 0.1 K and the heat flux is distributed over a wide angular range (26 mm, FWHM, or equivalently $\pm 41°$) as it reaches the helium. In the absence of the copper layer, the temperature difference is as much as 0.6 K and the emergent heat flux is confined to an angular range of about $\pm 9°$ or 5.6 mm, FWHM. In neither circumstance is there cause for concern about the vacuum or the helium cooling, except that spacers or other objects impeding the contact of helium with the wrapped tube should be positioned appropriately.

1. Introduction

At 20 TeV, the 73 mA proton beam in the SSC emits approximately 0.14 W/m of synchrotron radiation. This energy impinges on the inner wall of the beam tube in a narrow stripe in the equatorial plane of the machine. The width of the stripe is roughly 1 mm, calculated as follows. The rms angle of emission of synchrotron radiation relative to the direction of particle motion is $\delta\theta \approx \gamma^{-1}$, where γ is the usual relativistic factor. [1] The photons move tangentially to the beam until they strike the inner surface of the beam tube about $d = 18$ m away from the point of emission ($d \simeq \sqrt{2Rb}$, where $b = 1.63$ cm and $R \simeq 10$ km). In that distance, the angular spread translates into a transverse vertical dimension of $\delta y \simeq d\delta\theta = 1.8 \times 10^4\ \gamma^{-1}$ mm. At 20 TeV, $\gamma \simeq 2.1\times10^4$ and so $\delta y \simeq 0.85$ mm. Since the photons are emitted all around the ring, they "paint" a stripe on the outer side of the inside of the beam tube.

For the present purposes, the photons are assumed to be absorbed and converted to heat energy upon first impact. Undoubtedly some scattering occurs, distributing the energy more uniformly around the beam tube wall, but our assumption is conservative. Actual temperature changes will be less and energy flow into the helium more broadly distributed than computed here.

Once the energy has been dumped at the inner surface, it will flow through the beam tube and its associated wrapping of kapton, correction coil windings, kevlar cord, epoxy, etc. to the constant temperture helium bath. For a given steady heat flux distribution postulated on the inner surface, there is a steady-state temperature distribution within the wall and its wrapping and an "output" heat flux distribution into the helium.

The present beam tube has an inner radius of approximately 1.63 cm, a wall thickness of 1 mm, a heterogeneous wrapping of about 0.9 mm total thickness. To lower the resistive wall impedance presented to the beam,

[1] Actually, the angular distribution of synchrotron radiation, integrated over all frequencies, has an rms angular deviation of $\sqrt{22/9}\ \gamma^{-1} = 1.56\gamma^{-1}$. (See equation (14.84) of Jackson's book.)

it is planned to plate a 2 mil copper layer on the inside surface. Because there are large non-isotropic Lorentz forces produced during a quench by eddy currents in the copper, it has been suggested to omit the copper in the equatorial region where these forces are largest. Since this is the region where the synchrotron heat is deposited, there is interest in the thermal response with and without a copper layer.

2. Model

The combined wall and wrapping thickness of 1.9 mm is roughly one tenth of the radius of curvature of the beam tube. Since the spread of the synchrotron radiation is of order 1 mm, the curvature can be neglected and the problem analyzed locally in terms of flat slabs of copper, steel and wrapping. The incident synchrotron energy input is modeled (because of betatron oscillations of the beam, etc.) by a Gaussian distribution with a standard deviation σ:

$$S_x(0,y) = \frac{dP}{dz} \cdot \frac{1}{\sqrt{2\pi}\,\sigma} \exp(-y^2/2\sigma^2) \tag{1}$$

where the coordinates are chosen with the origin at the inner surface of the copper layer (if present) in the equational plane, the x direction into the beam tube, the y direction tangential to the beam tube surface, and z parallel to the tube axis.

The two-dimensional heat conduction problem has the temperature T(x,y) satisfying Laplace's equation, the temperature constant as a function of y at the helium bath, defined by $x = t_1 + t_2 + t_3$, where t_1, t_2 and t_3 are the thicknesses of the copper, steel, and wrapping layers, respectively. The boundary condition at $x = 0$ is that the heat flux,

$$S_x(0^+, y) = -\mu_1[\partial T(x,y)/\partial x]_{x=0^+}$$

is given by Eq. (1). The parameter μ_1 is the thermal conductivity of the copper, with μ_2 and μ_3 for the steel and wrapping. The wrapping, through heterogeneous in reality, is modeled as a uniform medium. Most of the volume is insulator. The imbedded correction coils cover about two thirds

of the surface. Within them, the 8 mil diameter wires are spaced about 13 mils apart. Thus the wires cover in projected area only about 40% of the surface on average (62% near the equator), and iN a disconnected fashion from the point of view of heat conduction. The thermal conductivities μ_i are taken as constants, independent of temperature. This is justified a posteriori by the smallness of the temperature differences found.

The two-dimensional Laplace's equation has exponential solutions in one direction and oscillatory solutions in the other. We choose to employ exponentials (actually $\sinh kx$ and $\cosh kx$) in the x direction and $\exp(iky)$ in the y-direction. The temperature in one of the slabs is written as

$$T(x,y) = \int dk\ e^{iky}(a(k)\cosh ky + b(k)\sinh ky)$$

The matching of solutions between slabs requires equating the temperatures and the x components of the heat flux in adjacent faces for all values of y. The solutions for two quantities of interest, the temperature $T(0,y)$ along the inner face of the beam tube and the heat flux $S_x(t_1+t_2+t_3,y)$ into the helium bath, are

$$T(0,y) = \frac{1}{\pi\mu_3}\frac{dP}{dz}\int_0^\infty dk\frac{\cos\ ky}{k}e^{-\sigma^2k^2/2}\frac{N}{D} \tag{2}$$

$$S_x(t_1+t_2+t_3,y) = \frac{1}{\pi}\frac{dP}{dz}\int_0^\infty dk\ \cos ky\ e^{-\sigma^2k^2/2}/D \tag{3}$$

where

$$N = c_1c_2s_3 + \frac{\mu_2}{\mu_1}s_1s_2s_3 + \frac{\mu_3}{\mu_2}c_1s_2c_3 + \frac{\mu_3}{\mu_1}s_1c_2c_3 \tag{4}$$

$$D = c_1c_2c_3 + \frac{\mu_1}{\mu_2}s_1s_2c_3 + \frac{\mu_2}{\mu_3}c_1s_2s_3 + \frac{\mu_1}{\mu_3}s_1c_2s_3 \tag{5}$$

with

$$s_i \equiv \sinh kt_i \text{ and } \mathrm{c_i} \equiv \cosh \mathrm{kt_i}.$$

3. Numerical Results

Expressions (2) and (3) are evaluated for beam tubes copper plated and not copper plated in the neighborhood of the impact region for synchrotron radiation. The numerical input values of thermal conductivity are

Copper	$\mu_1 = 2.4$ W/cm·K
Steel	$\mu_2 = 10^{-3}\ \mu_1$
Teflon	$\mu_3 = 4.5 \times 10^{-4}$ W/cm·K

Note that the wrapping is modeled as Teflon for its thermal properties. There is some uncertainity introduced by this assumption, but in the absence of measurements on the actual wrappings it is probably reasonable. The values were read from graphs in the BNL Cryogenics Handbook and/or R. B. Scott, Cryogenic Engineering, Van Nostrand, Princeton (1959), p. 344-5. They can be taken as representative, at least.

The thicknesses used in the computations are $t_1 = 5 \times 10^{-2}$ mm, $t_2 = 0.95$ mm, $t_3 = 0.9$ mm. The latter value (for the wrapping) is inferred from a detailed description of the various layers on the outside of the beam tube supplied by Per Dahl (private communication). For the calculations without copper, the parameters were changed to $t_1 + t_2 = 1.0$ mm, $\mu_1 = \mu_2 = 2.4 \times 10^{-3}$ W/cm · K. Most calculations have $\sigma = 1$ mm in Eq. (1).

The results are contained in Figs. 1 and 2. Figure 1 displays the temperature on the inner surface for $dP/dz = 1.42 \times 10^{-3}$ W/cm. Without copper plating, the peak temperature relative to the helium is roughly 0.6 K with an approximately Gaussian shape and an rms deviation of 1.2 mm. With 2 mils of copper (actually 50 μm), the maximum temperature is less than 0.1 K and the rms deviation is of the order of 5 mm (although the curve is only roughly Gaussian). The dashed curves show that narrowing the incident energy flux to $\sigma = 0.5$ mm has little impact on the distributions.

Figure 2 shows the normalized energy flux from the outside of the wrapping into the helium. The presence of the copper causes the heat to be

Figure 2 shows the normalized energy flux from the outside of the wrapping into the helium. The presence of the copper causes the heat to be distributed over a much larger area than without it. The full widths at half maximum are 5.6 mm and 26 mm, respectively. Taking the outer radius of the assembly to be 18.2 mm, these correspond to angular ranges in azimuth of $\pm 8.8°$ and $\pm 41°$, without and with copper plating in this angular region.

3. Comments

With beam currents of the order of 75 mA at 20 TeV, the synchrotron radiation causes a temperature rise of the inner wall of the beam tube in the equatorial plane of no more than 0.6 K if there is no copper plating, and less than 0.1 K with 2 mils of copper plating. The heat is deposited into the helium bath over an angular range of the order of $\pm 10°$ (no copper) or $\pm 45°$ (2 mils of copper plating).

Neither of these situations seems to pose difficulties for the vacuum or the helium cooling system. One point is worth mentioning. The present configuration of the beam tube inside the coil has G-10 spacers located near the equator (see Fig. 5.2-8, p. 287, CDR). If these could be located elsewhere (e.g., at 45°), the removal of the heat by the helium would be incrementally improved. Apart from such details, heat conduction of the synchrotron radiation energy is not an issue in consideration of copper plating in the equatorial region of the beam tube.

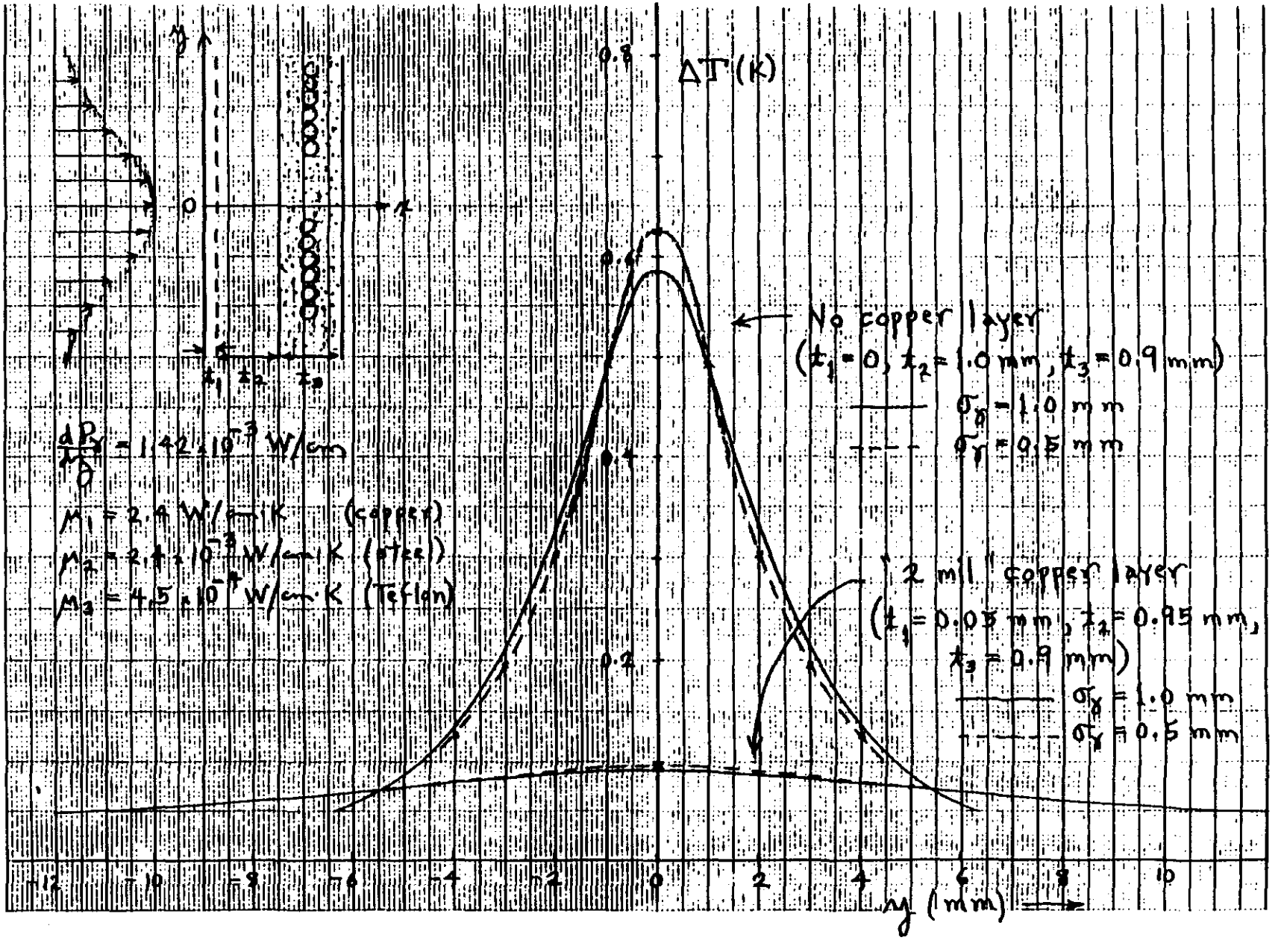

Figure 1. Temperature profile near the equator on the inner beam tube wall. Solid (dashed) curve is for synchrotron radiation rms spread of 1.0 (0.5) mm. Upper (lower) curves are for the beam tube without (with) 50 μm layer of copper on its inner surface.

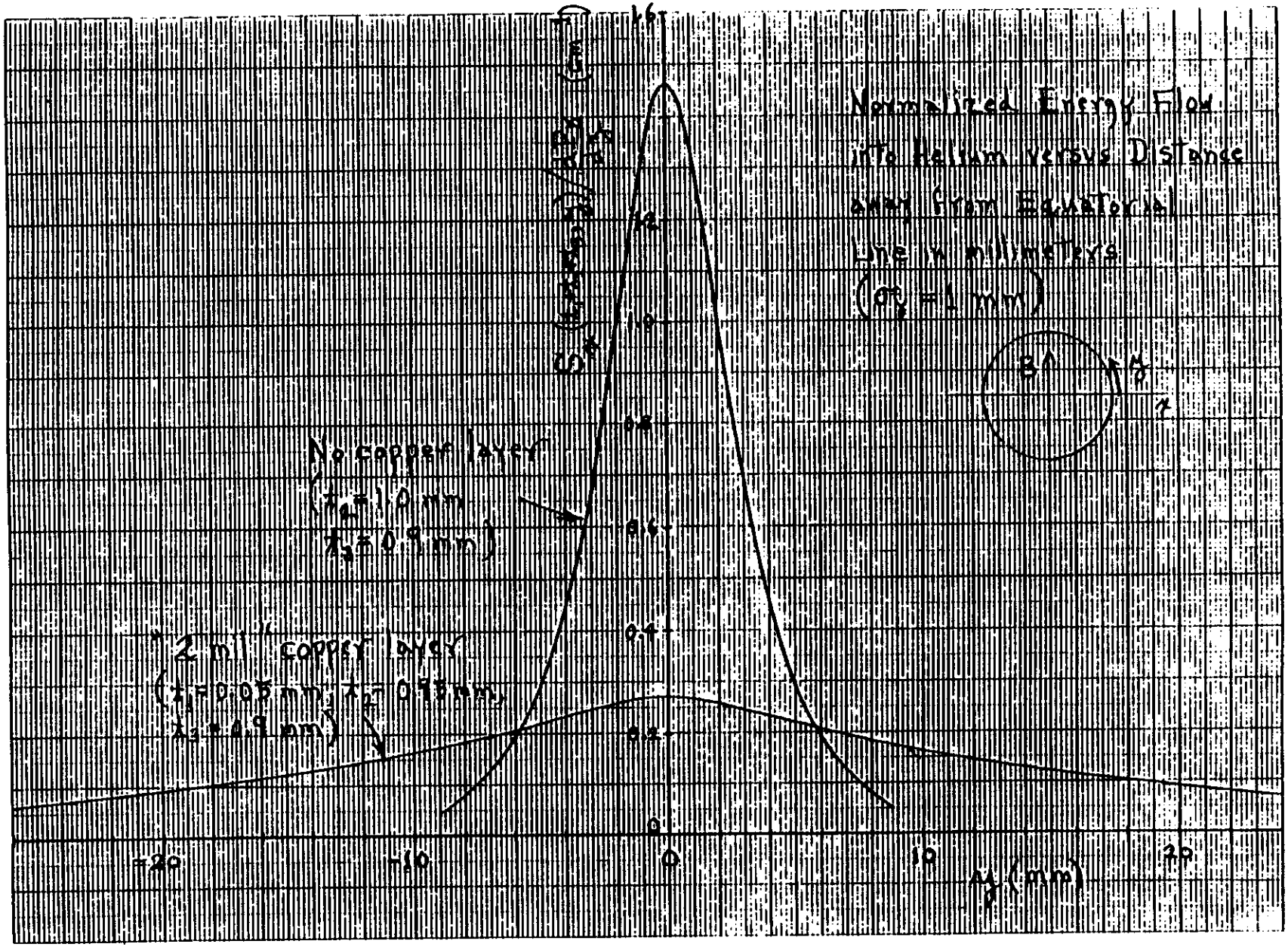

Figure 2. Normalized heat flow profile into the helium from the outside of the wrapping as a function of distance away from the equator. Upper (lower) curve is for the beam tube without (with) 50 μm layer of copper on its inner surface.