SSC-N-215

PRELIMINARY ASSESSMENT OF MAGNET ACCELERATED LIFE TESTS

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Introduction

Accelerated life tests will be necessary to assure that the SSC magnets can be expected with specified confidence levels to perform at their specified reliability and availability levels over their specified lifetimes. Our purpose here is to examine rudimentary analyses of accelerated life tests for the SSC magnets as a preliminary in order to perceive what kinds of more meaningful analyses can and should be addressed. The ultimate goal of our analysis program will be to structure the accelerated life test protocol and parameter ranges that will yield the most critical information affecting the magnet program.

We begin by reviewing the operational lifetime requirements as they have been specified in the SSC Conceptual Design Report and the accelerated life test protocol that has been suggested in collaboration of SSC/CDG and Brookhaven magnet development department. We then describe the assumptions and method of analysis that, although it is much over-simplified in our present treatment, suggests a structure by which the accumulated information of successive developments may be fed back into the analysis so as to maximize its value for subsequent analysis and testing decisions. We compute estimates of confidence in various specified magnet lifetimes as functions of testing time within the context of different prior assumptions. The results are shown to include certain relations that are useful for test analysis and are independent of certain prior assumptions and/or test parameters.

Representative of the conclusions that can be obtained is, e.g., that 99% confidence that the mean magnet life is ≥20 yr. can be achieved with the order of 100-150 magnet-weeks of accelerated life testing, depending on the number of failures sampled. More detailed relations among observations and test parameters and assumptions are given.

Specifications

Operational life time requirements that individual magnets must be designed to meet have been specified with the following equivalents:[1,2]

Machine lifetime	~20 years
Magnetic cycles	~10^4 years
Thermal cycles	~20 cycles
Quench cycles	~50 cycles.

The accelerated life test protocol that has been considered in collaboration by SSC/CDG and Brookhaven magnet development department[3] consists of the following 25 steps:

1 step of	10 full-strength magnetic-ramp cycles,
	5 quench and recovery cylces;
plus,	
24 steps of	250 full-strength magnetic-ramp cycles,
	1 full 300-4 degree thermal cycle,
	250 full-strength magnetic-ramp cycles,
	2 quench and recovery cycles.

Each test step after the first preliminary step is about the equivalent of 1/20 of the specified lifetime.

The time-budgets for the various components of a test step have been estimated as:[3]

1 full 300-4-degree thermal cycle = 120 hr./90	= 5.6 day,
500 full magnet-ramp cycles = 500(100 sec.)/.90	= 0.6 day,
2 quench and recovery cycles = 2(3hr.)/.90	= <u>0.3</u> day,
Total test-step	6.5 day.

A 90% processing efficiency has been figured into these estimates.[3] So after the first step, we can expect to test the equivalent of 1/20 of the design-specification operational lifetime in each test step, requiring about 6.5 days.

We need to point out that the magnet ramp rate assumed for the above estimates[3] is about 20 times faster than the operating rate of 1000 seconds per ramp specified in the SSC Conceptual Design Report (p. 206). If this slower ramp rate were used for the test the 0.6 day per step for ramping would become 12 days, and would make the whole test step require 17.9 days instead of 6.5 days--a three-fold increase.

At the same time the 5.6 days budgeted in the above estimate for the thermal cycle is about twice the 3 days estimated in SSC CDR (p. 361), based on the ability to concentrate the large pumping capacity in the SSC for quick replacement of a magnet in the ring. This pumping capacity is not expected to be available for early testing at the system test facility. It could be available with the establishment of a string testing program involving early-constructured parts of the operational SSC.[4]

Analysis

The objective of this analysis is to quantify our confidence R for a given value of the mean operational lifetime τ expected on the basis of an accelerated life test period y of a sample of N magnets. The test period y can be denominated in units of equivalent years of actual operation, which, by the above discussion, each requires 6.5 days of testing.

The method by which we attempt this estimate is based on several assumptions, none of which are particularly essential, and all are most conveniently described as they occur within a description of the method.

To begin we assume a constant failure rate $1/\tau_i$ for a single representative magnet such that its probability of not failing in a period of y equivalent operational years is the exponential e^{-y/τ_i}. This so-called exponential-failure-function hypothesis presumes the individual magnets fail from a complex variety of causes. Its use is based mostly on the simplicity of computation that it allows for obtaining rough estimates.[s] With this assumption the probability of $\leq n$ failures in a sample of N magnets tested for a period y at failure rate $1/\tau_i$ is:

$$P_i = \sum_{\ell=0}^{n} \binom{N}{\ell}(1-e^{-y/\tau_i})^{\ell}(e^{-y/\tau_i})^{N-\ell}$$

$$= e^{-yN/\tau_i} + N(1-e^{-y/\tau_i})e^{-(N-1)y/\tau_i} \qquad (1)$$

$$+ \frac{N(N-1)}{2!}(1-e^{-y/\tau_i})^2 e^{-(N-2)y/\tau_i} + \ldots$$

The above is based on a particular value of $1/\tau_i$ for the rate. We do not know this rate. If we knew it we would know the expected mean life. What we can ask is: What is the probability R that the true mean life τ of the magnets represented by the N magnet sample is $\geq\tau_{i'}$; given $\leq n$ failures occur in a test period y? We use

$$R(\tau\geq\tau_{i'}, n, N, y) = \frac{\sum_{i>i'} p_i P_i}{\sum_i p_i P_i} \quad . \qquad (2)$$

In this expression P_i = the probability of the rate being $1/\tau_i$ that we compute with. The P_i are conditioned by prior information: results of previous tests, intuition, or parameterized for computational determination as e.g. by Lagrangian entropy maximization. So here enters out next major assumption: choosing the P_i. We have considered time bins 5 units wide with $\tau_i = 5i + 2$ in $5i < \tau_i \leq 5(i+1)$ and 2 distributions of P_i:

$$\text{A)}\ P_i = c,\ i = (1,2,3,4,5,6)$$
$$P_i = 0,\ i \neq (1,2,3,4,5,6)\ ; \qquad (3)$$

$$\text{B)}\ P_i = c,\ i = (1,2,3,\ldots 7,8,9)$$
$$P_i = 0,\ i \neq (1,2,3,\ldots 7,8,9)\ .$$

The early-time bins i = 0,1 contribute almost nothing in the probabilities R that we compute. Also, the distributions cut off above some maximum τ_{ic}; ic = 6 for set A and ic = 9 for set B.

The effects of the upper cut-off of the prior distribution may be seen as follows: First we re-express R as

$$R = \frac{\sigma}{\sigma+\delta} = \frac{1}{1+\frac{\delta}{\sigma}} \simeq 1 - \frac{\delta}{\sigma} \qquad (5)$$

$$1-R \simeq \frac{\delta}{\sigma}\ ,$$

where $\sigma = \sum_{i \geq i'} p_i P_i$ and $\delta \cong p_{i'-1} P_{i'-1}$. Because p_i falls rapidly with i below i = i', the $P_{i'-1}$ contribution in δ is dominant. So,

$$1 - R(\tau \geq \tau_{i'}, n, N, y) \approx \frac{p_{i'-1} P_{i'-1}}{\sum_{i \geq i'} p_i P_i} \quad . \tag{6}$$

It is easily seen from this expression that extending the prior distribution to high cut-off in τ_i eventually leads to a behavior something like

$$1 - R(\tau \geq \tau_{i'}, n, N, y) \underset{ic>>i'}{\sim} \frac{P_{i'-1}}{(p_{ic} - p_{t'-1}) \left(\frac{ic - i'}{2}\right)} , \tag{7}$$

where the cutoff value is ic>>i'. It is also easily seen in this expression why

$$1 - R(\tau \geq \tau_{i'}, n, N, y) \sim e^{-yN\alpha} , \tag{8}$$

where α is independent of yN for n=0 and nearly for n=1,2; for this failure law. This behavior is evident in the plots of our results in Figs. 1-5.

We have computed $R(\tau \geq \tau_{i'}, n, N, y)$ with the prior distribution A for

i' = 3, 4, 5, i.e., $\tau \geq$ 15, 20, 25,
n = 0, 1,
N = 5, 10;

and with distribution B for

i' = 3, 4, 5, i.e., $\tau \geq$ 15, 20, 25,
n = 0, 1, 2,
N = 5 .

These results are plotted in Figures 1-5.

The figures show some interesting relations that are more or less independent of the priors for the failure law assumed here. All the results can be well approximated as

$$1 - R(\tau \geq \tau_{i'}, n, N, y) \sim e^{-\alpha Ny} , \tag{10}$$

where the effective rate

$$\alpha = \alpha(\tau \geq \tau_{1'}, n, ic, N) \tag{11}$$

depends on $\tau_{1'}$, n and the prior distribution cutoff ic (= 6 for set A, and = 9 for set B). But the ratios

$$\beta(n_1, n_2, N) \equiv \frac{\alpha(\tau \geq \tau_{1'}, n_1, ic, N)}{\alpha(\tau \geq \tau_{1'}, n_2, ic, N)} \tag{12}$$

are independent of $\tau_{1'}$ and ic,

$$\beta(n_1 = 0, n_2 = 1, N = 5) = 1.24, \text{ set A and B},$$

$$\beta(n_1 = 0, n_2 = 2, N = 5) = \begin{matrix} 1.64 \text{ set A}, \\ 1.68, \text{ set B} \end{matrix} . \tag{13}$$

The β's do depend weakly on N, as

$$\beta(n_1 = 0, n_2 = 1, N = 10) = 1.13, \text{ set A}'. \tag{14}$$

As can be seen in Eqs. (1,2), for n = 0 $R(\tau \geq \tau_{1'}, n = 0, N, y)$ is an entire function of the product Ny, and as $\alpha(\tau \geq \tau_{1'}, n = 0, ic, N)$ is almost independent of y, so also it is almost independent of N. Actually, the α's are slightly increasing functions of y in the region of $y < \tau_{1'} < \tau_{ic}$. This shows as a steepening slope of $\ln(1-R)$ vs y that is more noticable in n = 2 cases. But to the degree that the α's are (nearly) constant a useful practical rule of thumb follows: We accumulate the same level of confidence R that the mean life exceeds some chosen value $\tau \geq \tau_{1'}$ in y units of test time on an N-magnet sample as in ky units of test time on a sample of N/k magnets. This relationship holds as a rule of thumb approximation for n = 1 and n = 2 cases; less than for n = 0 by an amount indicated by the slight N dependence of the β's, e.g.

$$\frac{\beta(n_1 = 0, n_2 = 1, N = 5}{\beta(n_1 = 0, n_2 = 1, N = 10)} = \frac{1.24}{1.13} = 1.10 . \tag{15}$$

The question of estimating the prior probability distribution, i.e., the P_i's, remains. This is a weakness of the present method in that it allows the introduction of subjective judgment in selection of the prior probability distribution.(*) But in our present usage it is also an advantage in that it facilitates the incorporation of new information as it is accumulated in the testing process(es). In particular, in the two trial distributions illustrated here we have been rather conservative by not introducting a preference for the prior probability that any one τ_i was more nor less likely than any of the rest up to some cutoff value $\tau_{i'}$, above which we assign zero prior probability. It may be that our subjective choices of trial cutoff values are far too conservative. In this case if the accelerated life tests approach the trial cutoff i.e. τ_{ic} and we decide that it was chosen too timidly then we can move it higher and recompute our estimated confidence R. This will yield a higher confidence estimate for a given $\tau_{i'}$, n, N. Also, as we gain confidence that the true meanlife exceeds some particular $\tau_{i'}$ we may want to decrease the relative prior probabilities assigned for values of τ_i considerably below that particular $\tau_{i'}$, and again recompute the confidences $R(\tau \geq \tau_{i'})$ for all $\tau_{i'}$ of interest. The result will again be an increase in our estimates R.

The analysis described here has been naive. However, it already offers several worthwhile observations: 1) It suggests some useful model-independent rules of thumb relevant to more detailed analysis of the accelerated life tests. 2) It suggests a modality within which planning, testing and analysis can begin and still accommdate successive upgrading as the program proceeds. 3) It is in consonance with the step-progressive protocol for the accelerated life tests that has been considered previously. As seen in the figures, we can obtain fairly high confidence levels, $1 - R \simeq 0.01$ or 0.001, of lifetimes $\tau \geq 15, 20, 25$ years with testing times of the order of 1/2 to one year, assuming the tests do not reveal serious design failures.[7]

References

(1) SSC Conceptual Design Report, SSC-SR-2020, Sec.5.2.1, p. 267.

(2) Report of Task Force on SSC Magnet System Test Site, SSC-SR-1001, Sec. III, A., p. 9.

(3) V. Karpenko, correspondence with D. Brown, 7/1/86, and D. Brown, correspondence with V. Karpenko, 7/10/86.

(4) Ref. 4., Sec II, p. 8.

(5) We are here considering a first gross overview of a composite system test. The expected lifetime of component subsystems can be analysed separately. We assume that extensive and systematic studies on component parts and subsystems have been separately performed during the previous development of the magnets. As an example, the study and resolution of the problem of collar fatigue failure at the Tevatron is described in Sec. III, A, p. 9-10 of Ref. (2). Results of such subsystem analyses will be integral parts essential to later, more detailed extensions of the overall problem at hand. We are just not considering that level of detail yet.

(6) There are further criticisms of principle that go beyond the susceptibility of errant subjectivity to, inter alia, issues like whether or not a prior probability distribution can even properly be assigned on a physical independent variable. Eq. (2) is of a form known as Bayes' theorem, about which controversy has attended a large fraction of all attempts to exploit it ever since its discovery in the 18th century. We acknowledge this controversy and forge on.

(7) Arguements can be made on the grounds of elementary reliability and availability theory that the expected mean lifetime of individual magnets might be required to be much larger than the requirements aimed for here. We will discuss this elsewhere.

Figure Captions

Fig. 1. $1 - R(\tau \geq \tau_{i'}, n, N, y)$ *versus* y for $\tau_{i'}$ = 15, 20, 25; n = 0, 1; N = 5; where $R(\tau \geq \tau_{i'}, n, N, y)$ is the probability that the mean life τ exceeds $\tau_{i'}$, given that a test sample of N = 5 magnets survives y equivalent operating years of accelerated life test with $\leq$ n failures, and assuming the prior distribution P_i = c, i = 1-6, all other P_i = 0 (set A).

Fig. 2. $1 - R(\tau \geq \tau_{i'}, n, N, y)$ *versus* y for $\tau_{i'}$ = 15, 20, 15; n = 0, 1; N = 10. Same as Fig, 1 except that in this case the number of magnets in the test sample in N = 10.

Fig. 3. $1 - R(\tau \geq \tau_{i'}, n, N, y)$ *versus* y for $\tau_{i'}$ = 25; n = 0, 1, 2; N = 5; where $R(\tau \geq \tau_{i'}, n, N, y)$ is the probability that the mean life τ exceeds $\tau_{i'}$ = 25, given that a test sample of N = 5 magnets survives y equivalent operating years of accelerated life of accelerated life test with $\leq$ n failures, and assuming the prior distribution P_i = c, i = 1-9, all other P_i = 0 (set B).

Fig. 4. $1 - R(\tau \geq \tau_{i'}, n, N, y)$ *versus* y for $\tau_{i'}$ = 20; n = 0, 1, 2; N = 5. Same as Fig. 3 except that in this case R is the probability that the mean lifetime τ exceeds $\tau_{i'}$ = 20 years of standard operation.

Fig. 5. $1 - R(\tau \geq \tau_{i'}, n, N, y)$ *versus* y for $\tau_{i'}$ = 15; n = 0, 1, 2; N = 5. Same as Fig. 3 and 4 except that in this case R is the probability that the mean lifetime τ exceeds $\tau_{i'}$ = 15 years of standard operation.

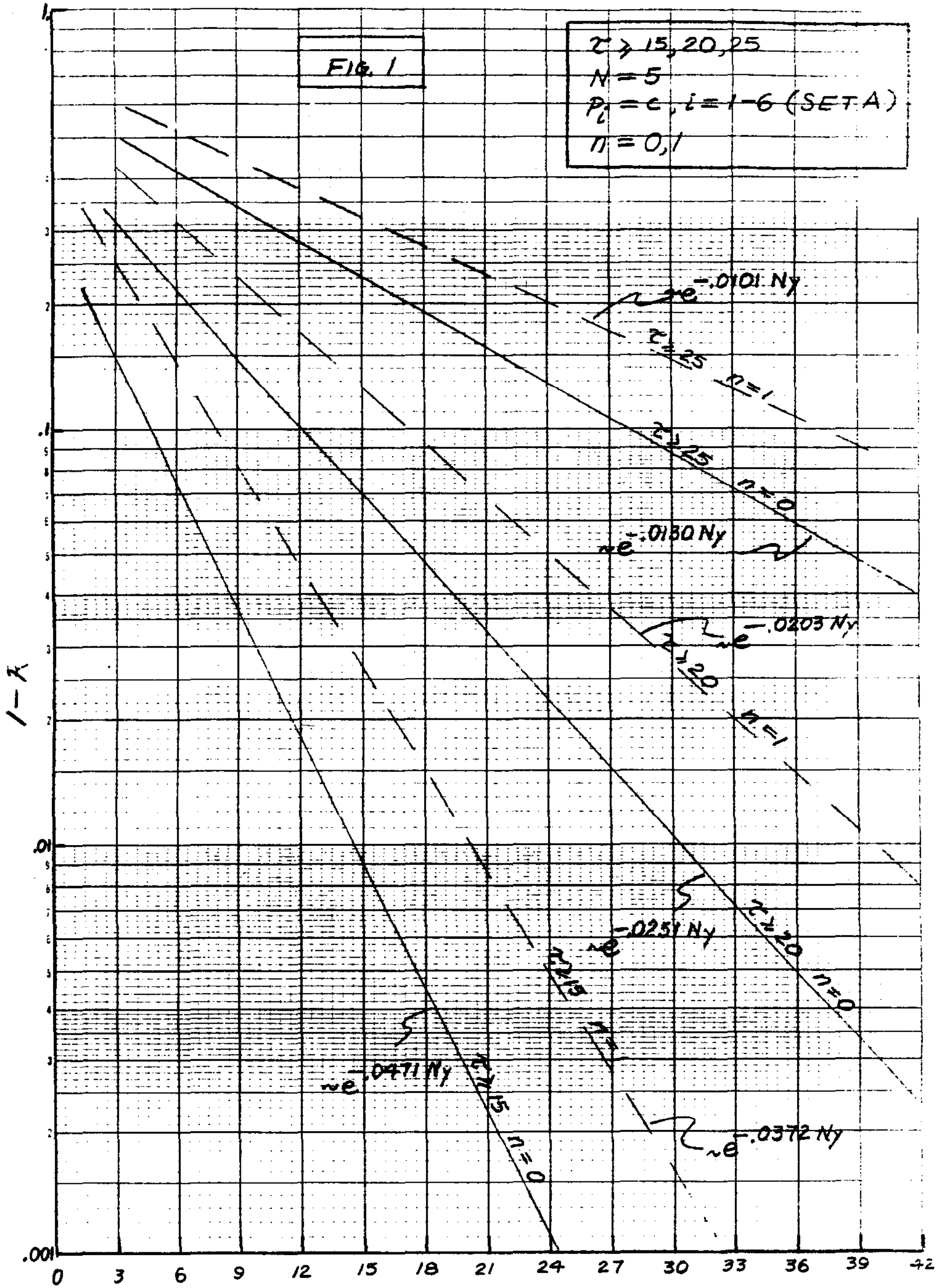
FIG. 1
τ ≥ 15, 20, 25
N = 5
P_i = c, i = 1–6 (SET A)
n = 0, 1
1 − λ
~e^{−.0101 Ny}
τ ≥ 25 n=1
τ ≥ 25 n=0
~e^{−.0180 Ny}
~e^{−.0203 Ny}
τ ≥ 20 n=1
~e^{−.0251 Ny}
τ ≥ 20 n=0
τ ≥ 15 n=1
~e^{−.0471 Ny}
τ ≥ 15 n=0
~e^{−.0372 Ny}
1.
.1
.01
.001
0 3 6 9 12 15 18 21 24 27 30 33 36 39 42

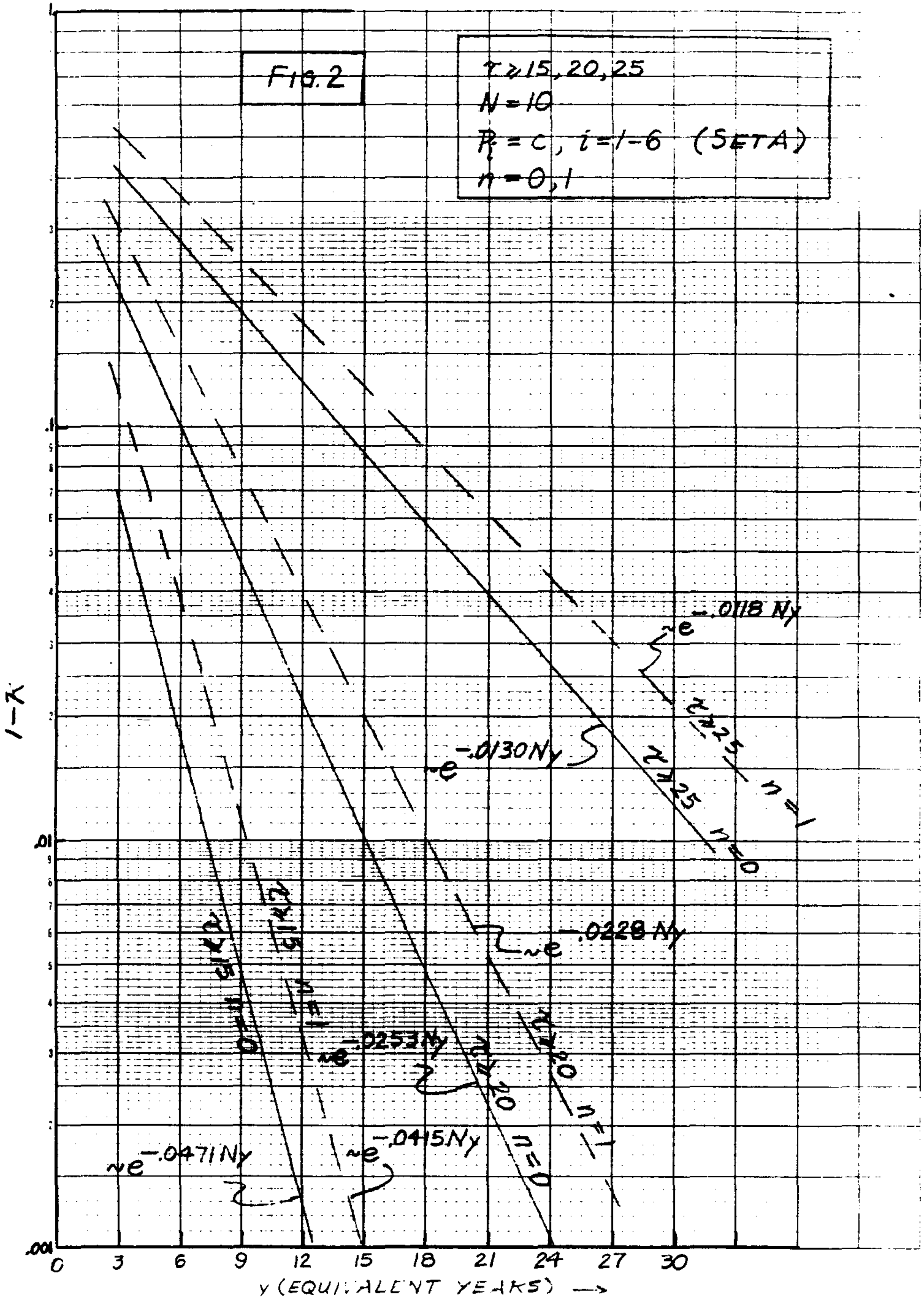

FIG. 2
τ ≥ 15, 20, 25
N = 10
P_i = C, i = 1-6 (SET A)
n = 0, 1
~e^{-.0118 Ny}
~e^{-.0130 Ny}
τ ≥ 25 n = 1
τ ≥ 25 n = 0
~e^{-.0228 Ny}
~e^{-.0253 Ny}
τ ≥ 20 n = 1
τ ≥ 20 n = 0
τ ≥ 15 n = 0
τ ≥ 15 n = 1
~e^{-.0471 Ny}
~e^{-.0415 Ny}
1 − K
y (EQUIVALENT YEARS) →
0 3 6 9 12 15 18 21 24 27 30
.001 .01 .1 1

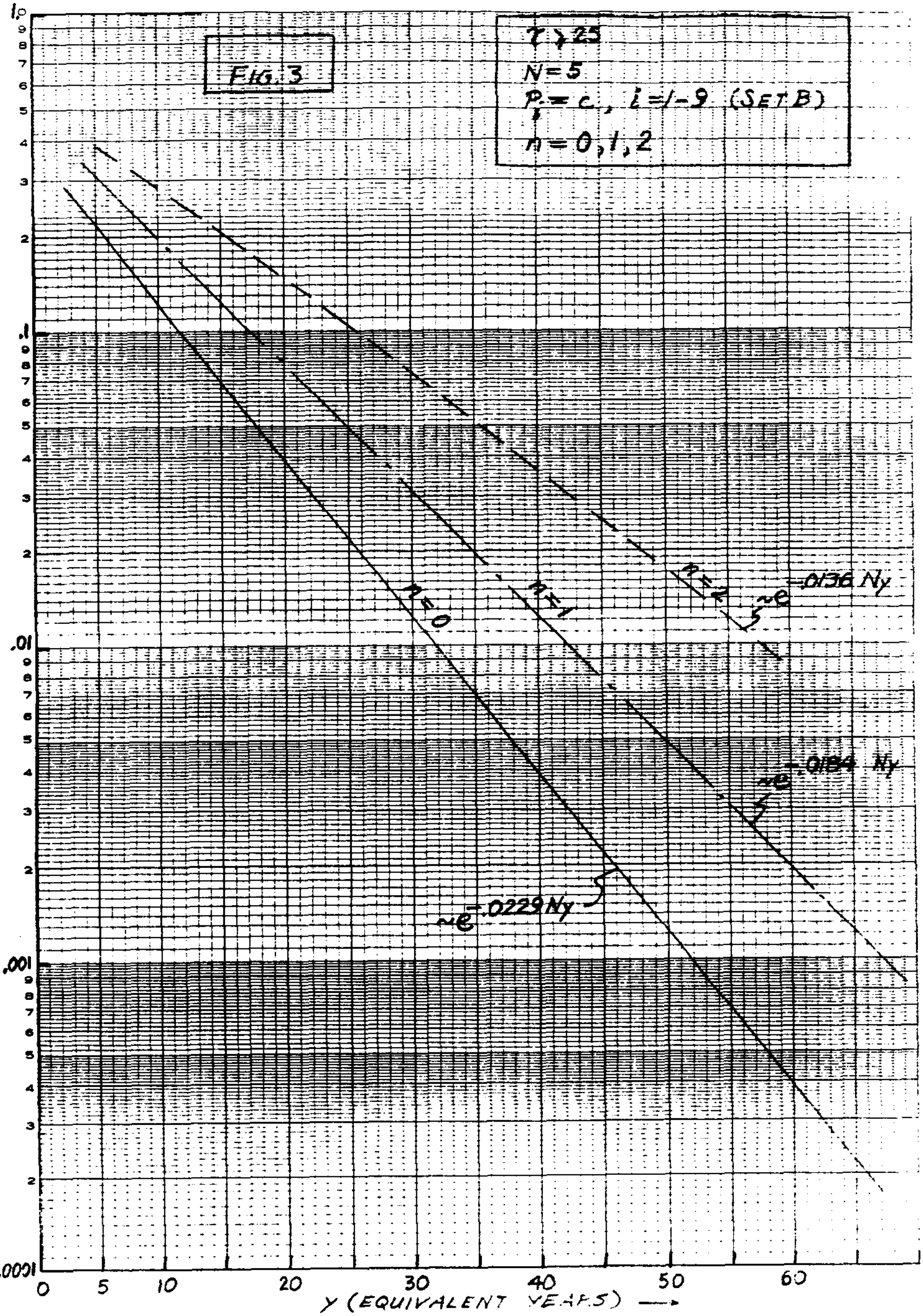

FIG. 3
τ > 25
N=5
P_i = c, i = 1-9 (SET B)
n = 0, 1, 2
n=0
n=1
n=2
~$e^{-.0136 Ny}$
~$e^{-.0184 Ny}$
~$e^{-.0229 Ny}$
Y (EQUIVALENT YEARS) ⟶

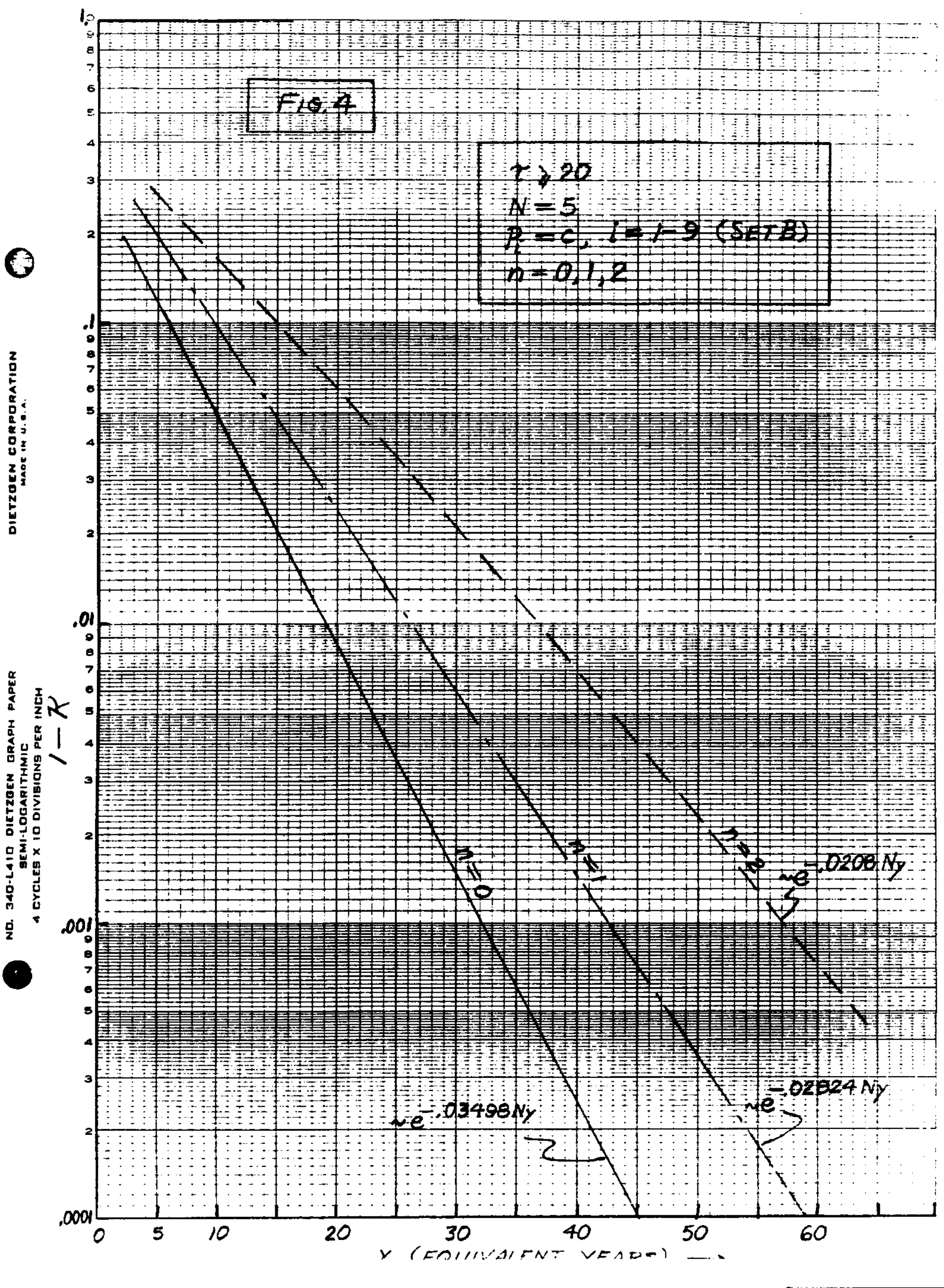

FIG. 4
τ ≥ 20
N = 5
R = c, l = 1-9 (SET B)
n = 0, 1, 2
n = 0
n = 1
n = 2
~e^-.0208 Ny
~e^-.02824 Ny
~e^-.03498 Ny
1 − R
Y (EQUIVALENT YEARS) →
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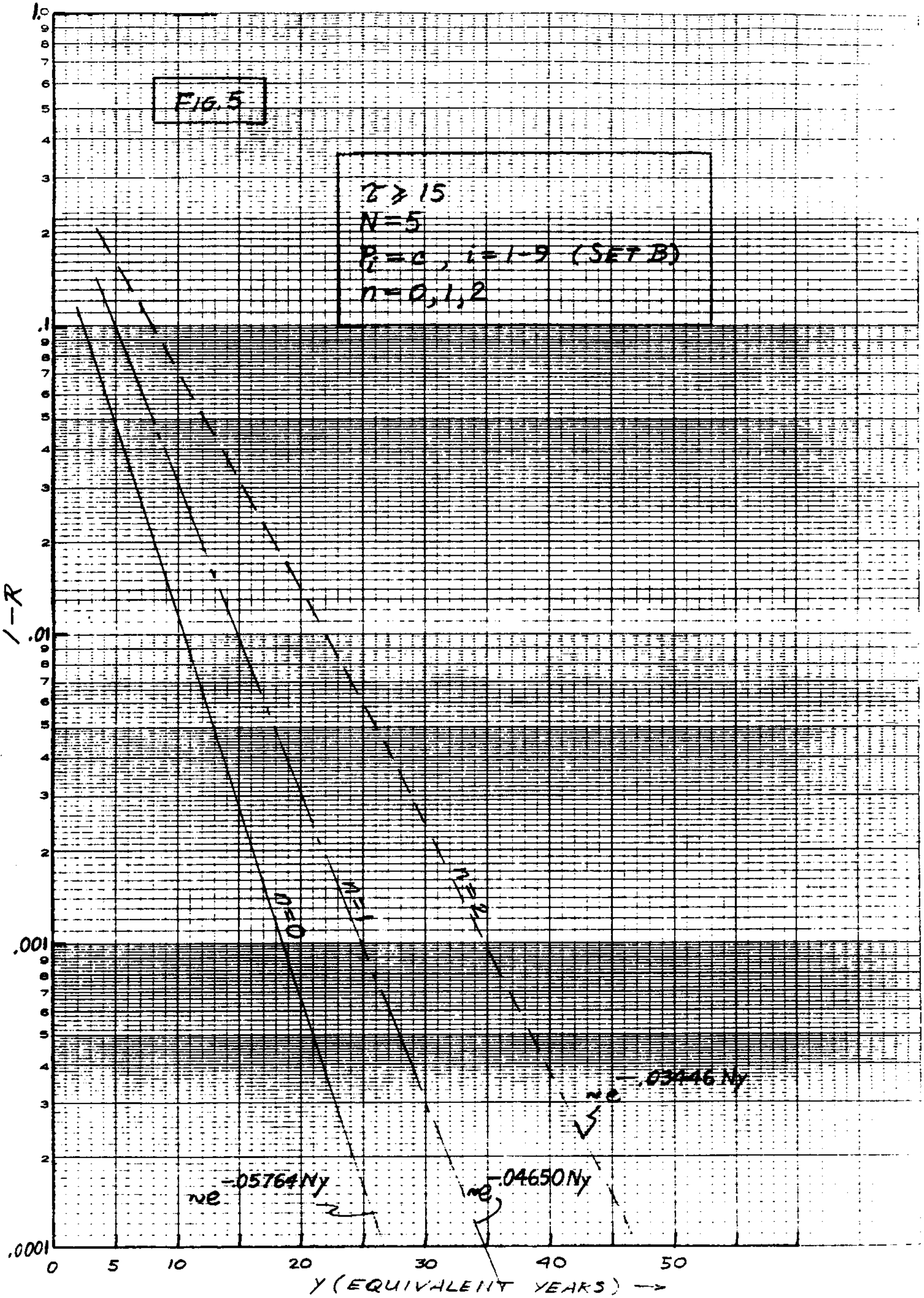
FIG. 5
τ ≥ 15
N=5
P_i = c, i=1-9 (SET B)
n=0,1,2
n=0
n=1
n=2
ne^-.03446 Ny
ne^-.05764 Ny
ne^-.04650 Ny
1-R
1.0
.1
.01
.001
.0001
0
5
10
20
30
40
50
Y (EQUIVALENT YEARS) →