USING COIL SPACER WEDGES AS CURRENT-CARRYING FIELD-TRIMMING ELEMENTS

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Introduction

Present SSC dipole magnet designs contain passive, wedge-shaped, copper spacer elements as part of the optimum coil design, as shown in Fig. 1. The suggestion of this note is that these parts be made instead from superconducting material (filmented to avoid gratuitous magnetization) and used as the elements of multipole trim circuits. Such circuits are needed, for example, to compensate for field non-uniformity caused by persistent currents. The currents required to give a "pure sextupole" compensation of strength equal to one "unit" are indicated in Fig. 2. A prescription for determining this (or other) compensation multipoles will be described and the quality of the compensation investigated.

In the current SSC design this compensation is performed using "bore tube compensation coils". These can be designed as near-ideal pure multipole coils and, in that regard, are superior to the coils suggested in this note. On the other hand, bore tube coils have certain disadvantages. They reduce the aperture (with resultant reduction in peak operating current); compared to the main coils they are fragile and their presence complicates the manufacturing process; and they are also inflexible, being specific to a particular multipole. Choice between bore tube coils and the scheme suggested here will depend on assessment of their relative merits in these various areas and perhaps on considerations of ease of powering the coils.

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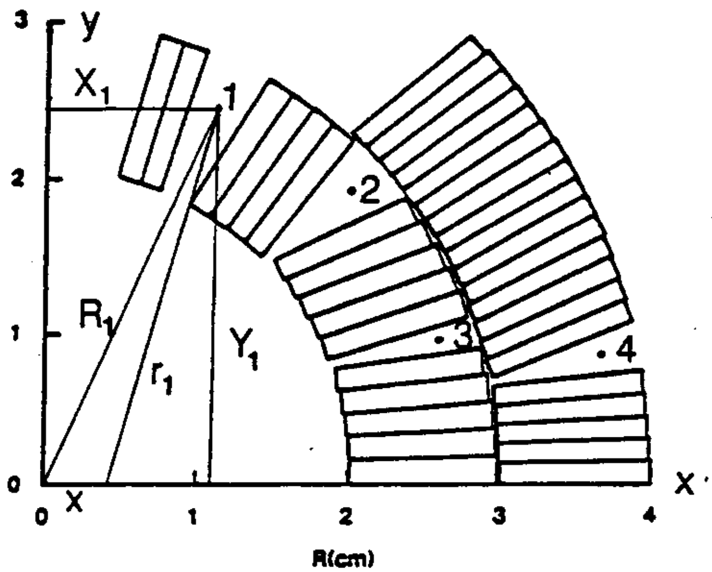

Figure 1. Dipole coil design showing wedge-shaped regions between superconducting turns and defining geometric quantities.

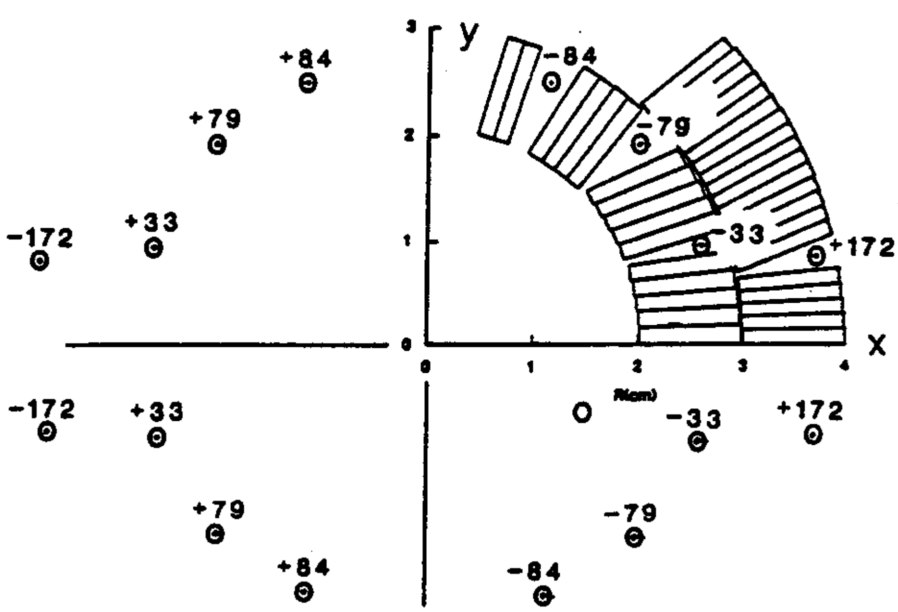

Figure 2. Currents needed for "pure sextupole" compensation of "unit" strength.

Determination of the Trim Currents

The vertical magnetic field component at position x on the horizontal axis due to current I_i in the i'th element is given by

$$B_i^{(y)}(x,0) = 10^2 \frac{\mu_0}{2\pi} \frac{I_i}{r_i} \frac{X_i - x}{r_i} \tag{1}$$

where the geometric quantities are defined in Fig. 2. M.K.S. units are in use except that a factor 10^2 has been included explicitly so that distances can be measured in centimeters. (Relevant distances are of order 1 in these units and it has become conventional to express field non-uniformity at a displacement of 1 centimeter.)

It is assumed that design requirements of the basic magnet fix the wedge locations, for example as shown in Table 1. The idea is to adjust the

Table 1.
Parameters for "Pure Sextupole" Compensation

Quantity	Unit	Wedge Number 1	2	3	4
X_i	cm	1.10	2.0	2.55	3.60
Y_i	cm	2.45	1.9	0.93	0.90
C_i		1.329	-0.1025	-1.5304	-1.7647
I_i	amp	-83.76	-79.06	-33.10	172.16
area	mm^2	27	22.5	10	18
resistance (if copper)	μohms/m	5.9	7.1	16.0	8.9
power	watts/m	0.0416x4	0.0444x4	0.0175x4	0.264x4

currents I_i; i=1, 2, 3, 4 to give the desired trim field. It would not be obligatory for these elements to be powered with the same symmetry as the main dipole coil but, since the main purpose of these coils is to compensate for persistent currents having this same symmetry, we will abide by such a constraint. In that case the field can be expanded as

$$B^{(y)}(x,0) = (b_0 + b_2 x^2 + b_4 x^4 + b_6 x^6)\ 10^{-4} B_0 \tag{2}$$

where the series has been truncated after four terms, which is all that can be fit with four independent current elements. Also a quantity 10^{-4} times the nominal dipole field B_0, has been factored out so that the multipole coefficients b_{2r} are measured in the standard "units". The value of B_0 will be taken to be 6.6 Tesla.

Summing (1) over the four assumed wedges and the other 12 symmetrically placed elements yields for the field

$$B^{(y)}(x,0) = 10^2 \frac{\mu_0}{2\pi} \sum_{i=1}^{4} 2I_i \left[\frac{X_i - x}{Y_i^2 + (X_i - x)^2} + \frac{X_i + x}{Y_i^2 + (X_i + x)^2} \right]$$

$$= \sum_{i=1}^{4} a_i \frac{1 - (x/R_i)^2}{1 + C_i (x/R_i)^2 + (x/R_i)^4}\ 10^{-4}\ B_0 \tag{3}$$

where

$$a_i = \frac{10^4}{B_0}\ 10^2 \frac{\mu_0}{2\pi} \frac{4X_i}{R_i^2} I_i \ ,$$

$$C_i = \frac{2(Y_i^2 - X_i^2)}{R_i^2} \ . \tag{4}$$

Expansion of (3) in powers of x^2 yields the result

$$B^{(y)}(x,0) \cong \sum_{r=0}^{3} \left(\sum_{i=1}^{4} d_{ri} a_i \right) x^{2r}\ 10^{-4} B_0 \tag{5}$$

where the coefficients d_{ri} form a matrix $D = (d_{ri})$ with

$$
\begin{aligned}
d_{0i} &= 1, \\
d_{1i} &= -\frac{1 + C_i}{R_i^2}, \\
d_{2i} &= \frac{-1 + C_i + C_i^2}{R_i^4}, \\
d_{3i} &= \frac{1 + 2C_i - C_i^2 - C_i^3}{R_i^6} .
\end{aligned}
\tag{6}
$$

The currents I_i, or equivalently the vector of coefficients $A = (a_1, a_2, a_3, a_4)^T$ is to be adjusted to yield some desired multipole correction, expressible also as a vector $B = (b_0, b_2, b_4, b_6)^T$. Equating (2) and (5) yields a matrix equation

$$B = D\,A \tag{7}$$

which can be solved for A

$$A = D^{-1}B. \tag{8}$$

With the wedge locations given in Table 1 the matrices D and D^{-1} are given by

$$
D = \begin{pmatrix} 1.0 & 1.0 & 1.0 & 1.0 \\ -0.32290 & -0.11794 & 0.07200 & 0.05553 \\ 0.04027 & -0.01886 & -0.00347 & 0.00184 \\ -0.00129 & 0.00180 & -0.00205 & -0.000057 \end{pmatrix},
$$

$$
D^{-1} = \begin{pmatrix} 0.066241 & -1.54846 & 9.54662 & -38.2316 \\ 0.185401 & -2.51852 & -23.4626 & 41.6992 \\ 0.103169 & -1.38868 & -27.7677 & -439.250 \\ 0.645188 & 5.45566 & 41.6838 & 435.782 \end{pmatrix}. \tag{9}
$$

The currents shown in Fig. 2. and Table 1 are given by (8) with

$$B = B^{(S)} = (0,\ 1,\ 0,\ 0)^T \tag{10}$$

which defines a "pure" sextupole of unit strength; more explicitly

$$I_i^{(S)} = 10^{-2}\ \frac{2\pi}{\mu_0}\ \frac{R_i^2}{4X_i}\ 10^{-4}\ B_0(D^{-1})_{i2}. \tag{11}$$

The quality of this nominally-sextupole field can be inferred by evaluating the exact expression (3); the result is shown in Table 2. It can be seen that for values of x less than a centimeter the field accuracy is much better than one percent and at 2 centimeters the discrepancy in only 2 percent. Since the "needed" aperture is only 5 mm this would seem like an adequate precision.

Table 2
Field Quality of "Pure Sextupole" Trim Configuration

Displacement	Perfect Sextupole	Actual Field	Fractional Error
0.0 cm	0.00	0.00000	0.0
0.1	0.01	0.01000	4.0×10^{-5}
0.2	0.04	0.04000	5.0×10^{-5}
0.5	0.25	0.25001	4.0×10^{-5}
1.0	1.00	1.00029	3.0×10^{-4}
1.5	2.25	2.2578	3.5×10^{-3}
2.0	4.00	4.087	2.1×10^{-2}

The currents can be adjusted to give other multipoles. For example they could be used to given a horizontal steering correction via b_0 or a decapole via b_4 or, for that matter, any linear combination of the lowest four multipoles.

This scheme would be especially attractive if it were unnecessary for the wedges to be superconducting. Assuming they are copper the power dissipation in the "unit sextupole" configuration has been worked out and recorded in

Table 1. Since the power is much greater than the nominal synchrotron radiation power of 0.14 watts/m this scheme appears to be ruled out, which is why, for this note, superconducting elements have been assumed.

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