High Energy Facilities

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EDDY CURRENTS IN PARTIALLY COPPER-PLATED SSC BORE TUBES

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Eddy Currents in Partially Copper-Plated SSC Bore Tubes

Summary

In order possibly to reduce currents induced in SSC dipole bore tube copper plating during a quench, 2 possibilities are calculated for azimuthally discontinuous plating. In one case one would omit the copper at the magnet poles, in the other at the midplanes. It turns out that in the first case a wide region around the poles would have to be free of copper (total $\sim 2/3$ of the tube diameter) in order to reduce the induced current to half its value for full plating. Furthermore, the remaining current would still be in a region where it would have the largest effect on bore tube stresses and deflections. Removing the copper from t[illegible] regions around the midplane would decrease these effects on the bore tube very rapidly; copper removal within $\pm 20°$ around the midplane would reduce stresses and deflections to less than half the values for full plating, and to one quarter for $\pm 30°$. Removal of copper from these regions would require considerations of other matters, concerning beam dynamics and temperature effects, the latter especially regarding impinging synchrotron radiation. Since it follows from the same considerations, a formula applying to eddy currents induced in conductor splices, especially superconducting ones is also given.

When an SSC dipole magnet is quenched, large currents are induced in the copper layer which is plated on the inside of the bore tube through which the particle beam is to pass. As a result large stresses and deflections can be produced in the tube wall, depending on the details of support that can be provided for the tube. These effects were recently reconsidered for the fast quench times predicted for SSC dipoles (SSC Technical Note No. 52).

It has been suggested (P. Limon at SSC CDG) to consider copper plating that is azimuthally discontinuous, thus interrupting the electrically highly conductive copper by low conductivity stainless steel. For now we shall disregard possible effects of discontinuous plating on the beam dynamics, azimuthal temperature distribution due to synchrotron radiation heating, possible difficulty of applying axial copper strips to the inside of a long tube with small diameter, and possible effects on the required high vacuum.

We shall first consider that plating has been omitted at the magnet poles for some width. This can be expected to inhibit the flow of axially induced eddy currents returning from one side of the tube to the other, thus forcing currents to return independently on each side. Currents would thus be reduced, and, in addition, the current density distribution would be affected such as to decrease stresses and deflections.

For simplicity, we will consider a flat sheet only and calculate the effect of an unplated region along the center of the sheet (the "pole") on total currents induced. Calling E_z the electric field induced axially (in direction z), E_x the field induced transversely to z, and B_y the magnetic field acting perpendicularly to the x, z plane, Maxwell's equation, curl $E = -10^{-4}\dot{B}$, becomes here

$$\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} = -10^{-4}\,\dot{B}_y \tag{1}$$

(B in Tesla). Calling j_z and j_x the current densities in the sheet in z and x directions, respectively, we can set

$$E_z = \rho_z j_z$$

$$E_x = \rho_x j_x \tag{2}$$

where ρ_z, ρ_x are the resistivities. We also must satisfy the equation div $E = 0$ for our case.

$$\frac{\partial E_z}{\partial z} + \frac{\partial E_x}{\partial x} = 0 \tag{3}$$

Within each of the two regions, copper plating and stainless steel, $\rho_z = \rho_x$, and therefore from eqs. 1 to 3, one obtains two Poisson equations for the two regions, which, in principle, could be solved with the boundary conditions

$$j_{1x}t_{cu} = j_{2x}t_{ss} \tag{4}$$

along the boundaries between copper (subscript 1 and thickness t_{cu}) and stainless steel (subscript 2 and thickness t_{ss}). Furthermore, at the ends of the tube (here plate) ($z = \ell/2$) $j_{1z} = j_{2z} = 0$. We can also assume axial symmetry around the center of the magnet ($z = 0$), and, of course, transversely around the center line of the stainless steel (polar) region. For the present purpose we are only interested in an approximate result, and therefore we can write

$$\frac{\partial E_{2z}}{\partial x} \approx -\frac{2E_{1zav}}{(\delta + w)} \tag{5}$$

where E_{1zav} is the average magnitude of the induced electric field along each of the two copper strips. These fields are directed in opposite directions, and therefore we have the factor of 2. δ is the width of the region without plating and w the width of each of the two copper strips. Next,

$$E_{1zav} = \frac{I_{1z}\rho_{1z}}{wt_{cu}} (= j_{1zav}\rho_{1z}) \tag{6}$$

where I_{1z} is the total current in the copper. We assume that only a negligibly small axial current j_{2z} is induced in the stainless steel region. From eqs. 3 and 4, it follows that

$$j_{2z}t_{ss} = \frac{E_{2z}t_{ss}}{\rho_{2z}} = -\frac{\partial I_{1z}}{\partial z} \tag{7}$$

Entering eqs. 6 and 7 into eq. 1 gives

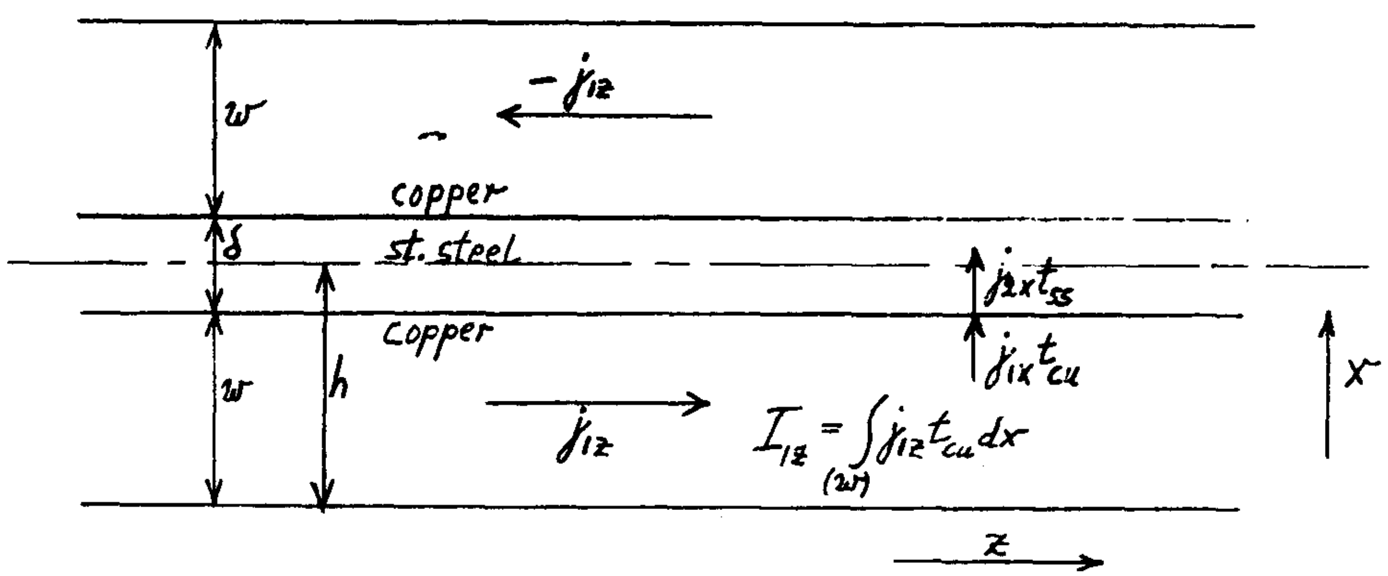
$$\frac{d^2 I_{1z}}{dz^2} = \alpha I_z - \beta \tag{8}$$

where

$$\alpha = \frac{2t_{ss}\rho_{1z}}{(h^2 - \frac{\delta^2}{4})t_{cu}\rho_{2z}} \tag{9}$$

$$\beta = 10^{-4}\frac{\dot{B}_y t_{ss}}{\rho_{2z}}$$

Here we have called $h = w + \frac{\delta}{2}$ the half-width of the sheet. Therefore $w(\delta + w) = (h^2 - \frac{\delta^2}{4})$. The following sketch illustrates various definitions.



The boundary equations for eq. 8 are:

$$z = 0: \quad j_{2z} \sim \frac{dI_{1z}}{dz} = 0 \tag{10}$$

$$z = \frac{\ell}{2}: \quad I_{1z} = 0,$$

where ℓ = magnet length.

While solution of the mentioned two Poisson equations, for the copper and stainless steel regions, would result in currents returning at the tube ends within each of the two copper regions, and in additional currents returning across the stainless steel from one region to the other, the approximation used serves only to calculate the crossover currents. From the results, this will appear justified, since the crossover current does not chang with the stainless steel width δ, except near the ends or when the stainless steel width becomes unreasonably wide. Furthermore, when currents return at each side only, they are smaller than for $\delta \approx 0$ and affect stresses and deflections in the tube much less.

The solution for eq. 8 becomes

$$I_{1z} = \frac{10^{-4}}{2} \frac{\dot{B}_y t_{cu}(h^2 - \frac{\delta^2}{4})}{\rho_{1z}} \left(1 - \frac{\cos h(\alpha^{1/2} z)}{\cos h(\alpha^{1/2}\ell/2)}\right) \tag{11}$$

Since ℓ is quite large, the hyperbolic cosine in the denominator can become very large. Unless z is also large, I_{1z} can remain constant, and equal to the term in front of the second parenthesis for a large range of z. I_{1z} depends then on copper plating gap width δ only through the term $(h^2 - \frac{\delta^2}{4})$. Results are shown in fig. 1, having used the following data:

$\dot{B}_y = 7.5T/0.1sec$

$t_{cu} = 0.01cm$

$t_{ss} = 0.1cm$

$h = 1.75cm (\approx \textit{bore tube radius})$

$\ell = 1700cm$

$\rho_{1z} = 5 \times 10^{-8} Ohm-cm$ (low temperature copper resistivity, including magneto–resistance effect)

$\rho_{2z} = 7 \times 10^{-5} Ohm - cm$ (low temperature stainless steel resistivity).

Figure 1 shows I_{1z} as a function of z, having also varied ρ_{2x} for illustration. Also indicated is I_{1z} at $z = 0$, varying copper gap width δ. We see that, for small δ, I_{1z} remains at its value for $z = 0$ up to large z. This means that the induced current returns only near the ends of the magnet for the given data. If t_{ss} were equal to t_{cu}, which, according to the expression for α, would be equivalent to increasing ρ_{2x} by the ratio $t_{ss}/t_{cu} = 10$, then I_{1z} begins to decrease at smaller z as fig. 1 shows for $\rho_{2x} = 7 \times 10^{-4}$.

These, on first sight, somewhat surprising results can be explained by the fact that, while ρ_{2x} is much larger than ρ_{1x}, δ is also much smaller than ℓ and current can pass in the x direction over a large range in z ($\ell \gg w$). If the magnet were shorter I_{1z} would begin to decrease nearer $z = 0$. For large values of z, $\cosh(\alpha^{1/2} z) \to \exp(\alpha^{1/2} z)$, and therefore the term in the parenthesis becomes

$$\left(1 - e^{-\alpha^{1/2}(\ell/2 - z)}\right)$$

The dependence on δ, given in fig. 1, shows that I_{1z} at $z = 0$ is decreased substantially only when δ approaches h, the half-width of the sheet considered here. When $\delta \approx 2/3$ of the total width, $2h(= 3.5cm)$, of the sheet I_{1z} is reduced to half its value for $\delta = 0$. Not much copper plating is left in this case.

We conclude that discontinuous copper plating, leaving a stainless gap at the poles may not be feasible, since the induced current cannot be reduced sufficiently in this manner. Another possibility is to omit copper plating near the magnet midplane. This would reduce the induced current in the region where the resulting (horizontal) forces have the largest effect on stresses and deflections in the bore tube.

Disregarding end effects, we can set $E_z = 0$, and therefore have, from eq. 1, only

$$\frac{dE_z}{dx} = -10^{-4} \dot{B}_y \tag{12}$$

Therefore

$$j_z = -10^{-4} \frac{\dot{B}_y x}{\rho_z} + C \tag{13}$$

Assume that copper is plated over a strip between $\theta_o \leq \theta \leq \theta$, ($\theta = 0$ in midplane). Then, taking into account that the total induced current (integrated over the width of the strip) must be zero, one obtains

$$| j_z | = 10^{-4} \frac{\dot{B}_y r}{\rho_z} \left(\cos\theta - \frac{\sin\theta_1 - \sin\theta_o}{\theta_1 - \theta_o} \right) \tag{14}$$

which for a strip symmetrical with respect to the pole ($\theta_1 = 180° - \theta_o$) is

$$| j_z | = 10^{-4} \frac{\dot{B}_y r}{\rho_z} \cos\theta \tag{15}$$

r is the average radius of the copper plating. The force distribution $f(\theta)$ on the tube wall is therefore proportional to $\cos\theta (\sim B_y j_z)$.

In CBA Technical Note No. 364 it was shown that stresses and deflections resultir from $f(\theta)$ are proportional to an aggregate of expressions which are themselves proportiona. to

$$S_n = \frac{1}{(n^2-1)^2} \int_o^{2\pi} f(\theta) \left(\cos\theta \cos n\theta + \frac{\sin\theta \ \sin n\theta}{n} \right) d\theta \tag{16}$$

(There are additional terms if the tube is supported.) As seen above, $f(\theta) = 0$ for $-\theta_o < \theta < +\theta_o$ and $180 - \theta_o < \theta < 180 + \theta_o$. One obtains

$$S_2 = \frac{1}{36} \left(3\pi - 6\theta_o - 4\sin 2\theta_o - \frac{1}{2} \sin 4\theta_o \right) \tag{17}$$

S_3, S_5, etc., are zero because of the assumed symmetry about the poles. The next term becomes

$$S_4 = \frac{1}{450} \left(\frac{3}{4} \sin 2\theta_o - \sin 4\theta_o - \frac{1}{4} \sin 6\theta_o \right)$$

The maximum conceivable value for S_4 is 1/225 (higher n-terms are much smaller), and the minimum for S_4 is $\approx$ 1/8. Therefore, for the present calculations we can neglect a" terms higher than S_2.

S_2 is plotted, as a percentage of its value at $\theta_o = 0$, in fig. 2. One sees that S_2 decreases quite rapidly, to 45% at $\theta_o = 20°$, and to 25% at 30°. Stresses and deflections decrease correspondingly from previously calculated values as high as $\delta_{max} = 83 kpsi$ and maximum deflections of 0.022" for unsupported tubes (the actual amount of support that can be provided is uncertain). Thus, if beam dynamics and thermal considerations permit, omitting some of the copper plating near the midplane may result in very manageable stresses during magnet quenches, even without considering possibilities for support. (One must remember that during a quench the bore tube is also exposed to a substantial external helium pressure.)

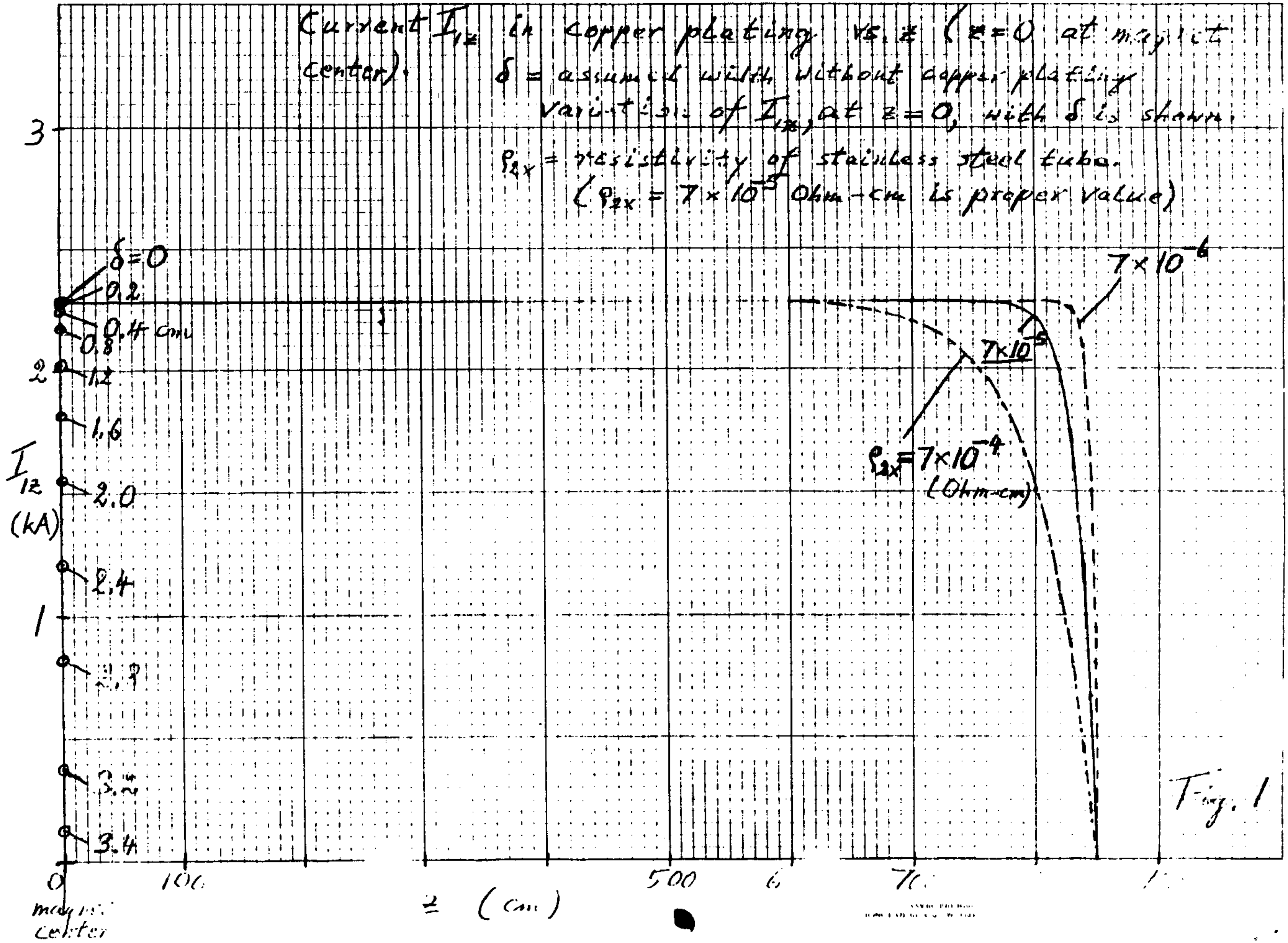
Finally, unrelated to the above, it may be of interest to modify eq. 11 for a "sandwich" of two flat conductors which are pressed or soldered together, for instance for a splice of leads in a superconducting magnet. Calling d the thickness of one of the two conductors and b its width we then obtain for an induced current in a changing field B that is parallel to the conductor width

$$I = \frac{10^{-4}}{2}\dot{B}\frac{bd^2}{\rho_a}\left(1 - \frac{\cos h\left(\frac{2\rho_a}{\rho_p}\right)^{1/2}\frac{z}{d}}{\cos h\left(\frac{2\rho_a}{\rho_p}\right)^{1/2}\frac{\ell}{2d}}\right) \tag{18}$$

where ρ_a = resistivity along the conductors and ρ_p = resistivity between the conductors. When $\rho_a \rightarrow 0$,

$$I \rightarrow \frac{10^{-4}}{8}\frac{\dot{B}b}{\rho_p}(\ell^2 - 4z^2) \tag{19}$$

Note that, at $z = 0$, $I \sim \ell^2$. Thus a splice for superconductors should not become excessively long. For very large ℓ, ρ_a may not be small enough, even for a Type II superconductor to justify use of eq. 19.



Current I_{1z} in copper plating vs. z ($z=0$ at magnet center).
δ = assumed width without copper plating
variation of I_{1z} at $z=0$, with δ is shown.
ρ_{2x} = resistivity of stainless steel tube.
($\rho_{2x} = 7\times10^{-5}$ Ohm-cm is proper value)
$\delta=0$
0.2
0.4 cm
0.8
1.2
1.6
2.0
2.4
3.4
7×10^{-6}
7×10^{-5}
$\rho_{2x}=7\times10^{-4}$ (Ohm-cm)
3
2
1
0
I_{1z} (kA)
100
500
z (cm)
magnet center

Fig. 1

Percentages, vs. angle θ_{o}, of deflection and stress at $\theta = 0$, as a function of θ_o.

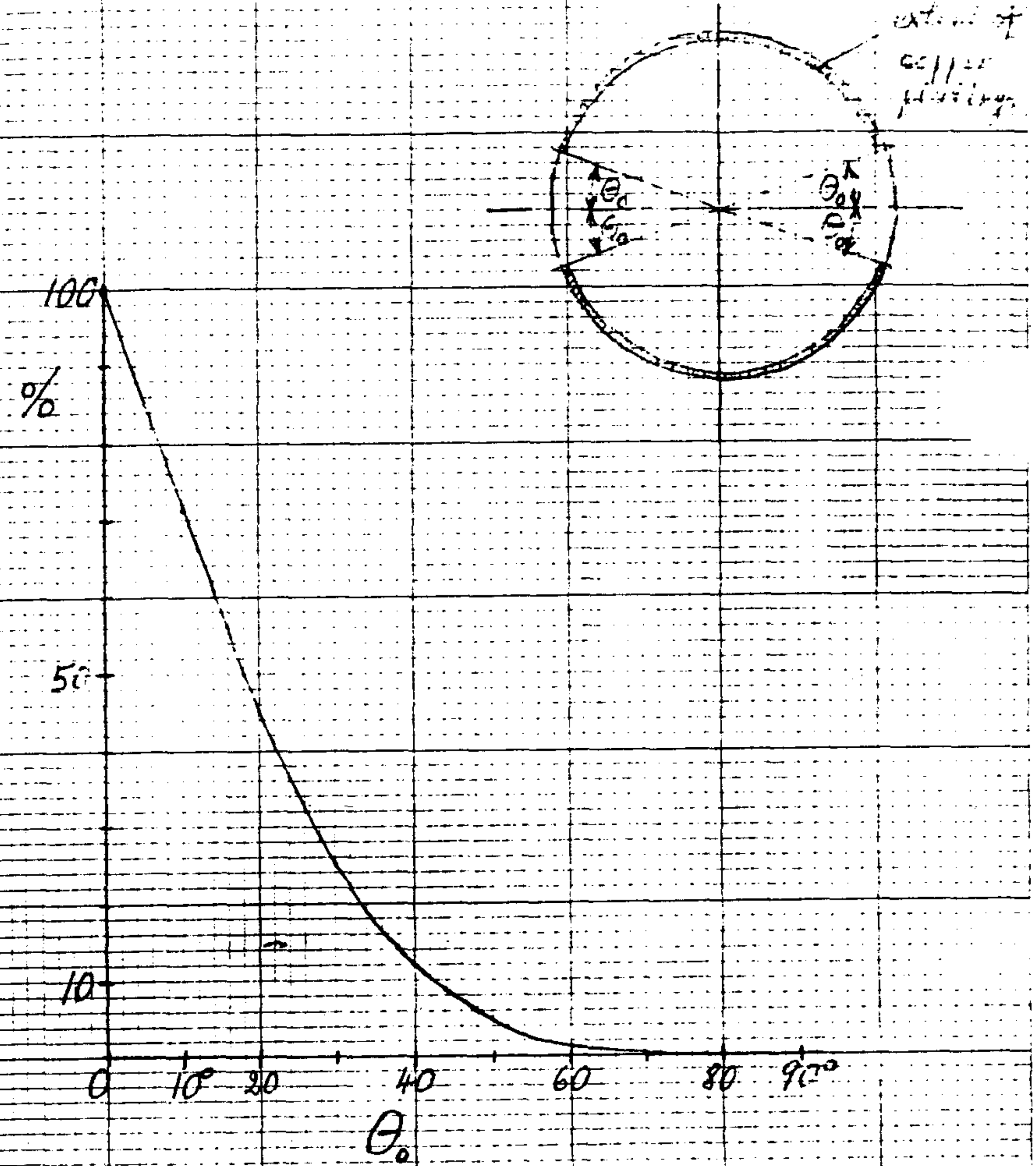


Fig. 2