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MAGNET QUENCH EFFECTS ON COPPER-PLATED BEAM TUBES IN SSC DIPOLES

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Magnet Quench Effects on Copper-Plated Beam Tubes in SSC Dipoles

Summary and Conclusions

New calculations are given for stresses in and deflections of SSC dipole beam tubes during a magnet quench. During a quench these tubes can be exposed to a considerable external pressure in addition to the bending stresses calculated here. Recent calculations by J.G. Cottingham show that the main current decay, and therefore field decay during a quench will take only about 0.1 sec. The resulting current induced in the copper plating inside the beam tube can be very large due to the very low resistivity of the copper at low temperature. A moderating effect on the induced currents is due to (1) the magneto-resistance effect and (2) the increase of the copper resistivity with rising temperature, due to heat produced in the copper plating. The latter effect becomes important here because of the anticipated fast decay of the magnetic field. Calculated stresses and deflections depend very much on how well the beam tube is supported. Rigid support (assumed at or near the midplane) will not be possible. For an unsupported tube the calculations result in bending stresses that could be as high as 84 kpsi in the stainless steel beam tube and 63 kpsi in the copper plating which would yield the copper. With rigid support, one would find 20 kpsi in the tube and 16 in the copper, with a radial deflection of only 0.002" in the polar region. Considering several uncertainties (e.g., initial clearances or the elastic modulus of the main coils in a radial direction) in obtaining a rigid support, actual maximum bending stresses in the beam tube of 35 to 50 kpsi may be obtained and correspondingly somewhat less in the copper plating which, again, most likely would yield, with an accompanying increase of the low temperature resistivity. Due to additional considerations, the results given here supersede those discussed in a talk recently given to members of the SSC CDG and others.

In a Magnet Division Note (No. 141-20) stresses and deflections were calculated for the beam tubes in SSC dipole magnets, when a magnet quench occurs. During a quench the magnetic field decays rapidly and thus currents are induced in the 0.01 cm thick high-conductivity copper plating on the inside of the stainless steel beam tubes. Depending

on the magnitude of the field B, its decay derivative $\dot{B}$, the conductivity of the copper–plating, the wall thickness of the beam tube, and the support given to the tube, stresses and deflections can become quite large. When combined with the helium pressure build–up around the tube, due to the quench, and with the (stability–balanced, but present) compressive hoop stress in the tube wall due to Kevlar strand wound around the tube, available margins against yielding of the tube may become too small or non–existent. Some permanent distortion of the tube could result. The tube carries the field correction (trim) coils whose function would thus be impaired. Furthermore, proper circulation of the cooling helium may be affected. In addition, the soundness of the inner main coil may suffer, and there might be effects on the beam dynamics.

The above-mentioned previous calculations made use, for an approximation, of a linear field decay from full to zero field, lasting 0.3 seconds. More recently, J.G. Cottingham (private communication) has determined, from measurements on 4.5 m long magnets, that actually most of the decay occurs during a period of 0.1 sec. Figure 1 shows main coil quench current I_q vs. time. The transfer function for the dipoles is about 10. Therefore the initial field B_0 here is about 6.6 Tesla. With some further improvements of conductors, this may in fact become the magnet operating field. It may also be attempted to test the magnets at lower than operating temperatures. In these cases quench fields as high as 7.5 T may be reached (in some tests at lowered temperatures 8 T had been reached in 4.5 m long magnets). At such high fields the transfer function decreases, but not very much (for our purpose). Maximum forces are reached at fields considerably below B_0, as expected from the shape of the $I_q(t)$ curve.

The conductivity of the copper plating is affected by the magnetic field B and the temperature T to which it is exposed. In the previous calculations, maximum forces occurred at time $t = 0$. Therefore T was the operating temperature. Here the temperature will have increased when maximum forces are reached. The conductivity also decreases with increasing field B ("magneto–resistance effect"). The resistivity can be expressed by

$$\rho(B,T) = \rho(0,273)\frac{(0.003R(T)B + 1)}{R(T)} \tag{1}$$

with good approximation. R(T) is the "relative resistivity ratio" which remains constant up to a temperature well above the magnet operating temperature, but then decreases rapidly, depending on the initial magnitude of R(T) at operating temperature.

It was shown in CBA Technical Note No. 52 that the azimuthal current density $j_1(\theta)$ induced by the field decay $\dot{B}$ is

$$j_1(\theta) = \frac{I}{2}\frac{\cos\theta}{r_{cu}} \; (A/cm) \tag{2}$$

where the total induced current

$$I = \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} j_1 \, r_{cu} \, d\theta$$

is given by

$$I = 2 \times 10^{-4} \frac{\dot{B} \, t_{cu} r_{cu}^2}{\rho(B,T)} \; (A) \tag{3}$$

Azimuth θ is measured from the magnet midplane. $r_{cu}(cm)$ is the average radius of the copper plating and $t_{cu}(cm)$ its thickness.

The temperature increase in the copper, in principle, varies with θ since j_1 is a function of θ. Azimuthal heat diffusion tends to distribute the heat with a time constant

$$\tau_d = \frac{s^2 D \, c(T)}{4 \, k(B,T)} \tag{4}$$

where D = density, c(T) = heat capacity, k(B,T) = heat conductivity, and s = azimuthal distance. For the copper plating, for average temperatures and average heat capacities, one estimataes that τ_d is smaller than the decay time of quench current I_q, in fact much smaller near operating temperature. In order to avoid much complexity in the calculations we can therefore assume that the heat produced is distributed uniformly over the copper plating. The heat dq produced per element, per unit time, and per unit tube length, is

$$dq = j_1^2 (r_{cu} d\theta)^2 \frac{\rho}{t_{cu}(r_{cu} d\theta)}$$

and the temperature increase

$$\dot{T} = \frac{\int_o^\pi dq}{c_{cu}(T) D_{cu} t_{cu}}$$

Making use of eqs. 2 and 3,

$$\dot{T} = \frac{10^{-8} \dot{B}^2 r_{cu}^2}{2\rho(B,T) D_{cu} c_{cu}(T)} \tag{5}$$

This would apply if only the copper were heated. However, heat diffuses into the stainless steel tube. Using eq. 4 one can show that τ_d for this radial diffusion is considerably smaller

than the field decay time, so that all or at least a large fraction f_s of the tube wall thickness receives heat. Therefore we should multiply the right side of eq. 5 by the factor

$$\frac{t_{cu} r_{cu} c_{cu}(T) D_{cu}}{(t_{cu} r_{cu} c_{cu}(T) D_{cu} \; + \; f_s t_s r_s c_s(T) D_s)} \tag{6}$$

$t_s, r_s, c_s(T)$, D_s refer to the stainless steel tube. Numerically, one finds that

$$t_{cu} c_{cu} D_{cu} \; \ll \; f_s t_s c_s(T) D_s.$$

Therefore

$$\dot{T} \; \approx \; \frac{10^{-8} \dot{B}^2 \; r_{cu}^2 t_{cu}}{2 \; \rho(B,T) \; f_s t_s c_s(T) D_s} \tag{7}$$

neglecting the small difference between r_{cu} and r_s. Calculations have been performed for $t_{cu}/f_s t_s = 0.1 (t_{cu} = 0.01cm, \; t_s = 0.1cm, \; f_s = 1)$, $0.05 (f_s = 0.5)$, and $0.15 (f_s = 1.5$: heat penetration into the trim coil assembly).

The magnetic flux density through the tube consists of the field $B_f(t)$ due to the main coils and the field B_i due to the current induced in the copper plating.

$$B \; = \; B_f \; + \; B_i \tag{8}$$

(Inside the tube B_i is parallel to B_f since $j_1 \sim \cos\theta$.)

In CBA Tech. Note No. 52 it was shown that

$$B_i \; = \; \dot{B} \tau \tag{9}$$

where

$$\tau \; = \; 2\pi \; \times \; 10^{-9} \frac{t_{cu} r_{cu}}{\rho(B,T)} \tag{10}$$

Combining eqs. 1, and 7 to 10, we finally obtain

$$\dot{T} = \frac{10^{-8} \dot{B}^2 r_{cu}^2 t_{cu} R(T)}{2\rho(0,273)(0.003 R(T) B \; + \; 1) \; f_s t_s c_s(T) D_s} \tag{11}$$

$$B \; = \; B_f \; + \; \frac{0.4\pi \; f_s t_s c_s(T) D_s}{r_{cu}} \; \frac{\dot{T}}{\dot{B}} \tag{12}$$

For our case, the term containing $\dot{T}/\dot{B}$ in eq. 12 will turn out to be considerably smaller than B_f. Therefore one can merely calculate $\dot{T}$, setting $B = B_f$, from eq. 11 and enter

$\dot{T}/\dot{B}_f$ into eq. 12. This will result in a first approximation for B. Repeating this procedure results in very fast convergence. $R(T)$ and $c_s(T)$ must be given. The copper plating is reported to have a resistivity ratio of 400 at operating temperature. From the BNL Selected Cryogenic Data Notebook we infer the following Table I.

The force distribution is given by

$$f(\theta) \;=\; 7.2 \times 10^{-3}\, \frac{BI\,\cos\theta}{r_{cu}} (psi) \tag{13}$$

acting horizontally outward (excepting a small vertical component due the fact that B_i inside the copper plating is not everywhere parallel to B_f.

Thus approximately $f(\theta) \sim \frac{B\dot{B}}{\rho}$. In CBA Tech. Note No. 364 and Magnet Division Note No. 141-20 expressions for radial deflections w were derived. They take the form

$$w = \sum_{2,3,\cdots} a_n \,\cos(n\theta) \tag{14}$$

Also, for azimuthal deflections:

$$v = \sum_{2,3,\cdots} \frac{a_n}{n} \,\cos(n\theta) \tag{15}$$

Expressions for the stresses were also given in the references. R. B. Meuser from LBL has kindly pointed out that a factor of $0.91 = (1 - 0.3^2)$, where 0.3 is Poisson's ratio, was omitted previously. This has been included here.

Results are given in Table II for maximum stresses in the stainless steel tube ($\sigma_{ss\ max}$) and copper plating (σ_{cumax}) and for maximum radial deflections w_{max}. It is assumed that the tube is unsupported or supported rigidly in the midplane.

The last row in Table II repeats the results (multiplied by 0.91) calculated for a linear decay in 0.3 sec. There the maximum stress and deflection occurs at $t = 0$. Therefore no value for f_s is given. $R(4)$ was here only 200.

Table II shows that, for rigid support in the midplane, maximum stresses and deflections (here occurring at $\theta = 90$ deg) are very much smaller than without support, although even here some of the values for σ_{cumax} are large enough that the copper plating may yield. Rigid support is not really possible because (1) there will be some initial clearance

(0.001 to 0.002") between beam tube assembly and inner coil, (2) the tube supports are intermittent, presently 3" length per 6", although this may be decreased to 0.25" per 0.5" for this and other reasons, (3) the radial elastic modulus of the main coil assembly is relatively small, probably below 5×10^5 *psi*, although a reliable value has not been established, (4) some of the somewhat porous epoxy-fiberglass structure appearing at the inner main coil surface may be crushed by a radial pressure. Adding all of these effects, one may end up with a maximum possible motion at the supports of 0.006 to 0.01", which, of course, would result in considerably larger stresses, in the range between 35 and 50 kpsi, depending on the value for f_s. If a measured value for the radial elastic modulus (as a function of applied pressure) were available, and also more details on the other 3 points mentioned above, it would be worthwhile to extend the present calculations.

f_s was defined as a measure for the depth to which heat diffuses radially into or beyond the wall thickness of the beam tube. Table II shows that the maximum stresses and deflections <u>increase</u> with increasing f_s. The reason is that the larger f_s the cooler the copper plating remains, and therefore the resistivity ratio $R(T)$ remains larger during a quench, resulting in larger induced currents.

It was estimated (see eq. 4) that azimuthal heat diffusion along the copper plating during a quench is fast enough that the tube would assume a uniform temperature during a quench. Including diffusion in the calculation (adding the appropriate partial differential equation to the calculations) would show that stresses and deflections in the tube are somewhat smaller than calculated here, the reason being that current density j_1 is largest at $\theta = 0$, thus producing the largest amount of heat, and therefore having the largest effect on decreasing $R(T)$. Thus j_1 would increase at a lower relative rate near $\theta = 0$ than for larger θ. In addition, j_1 near $\theta = 0$, also has the largest effect on the tube stresses because of the horizontally outward direction of the force distribution $f(\theta)$.

Dipole Quench Current I_q vs. time.

From calculations by T. G. Cottingham.

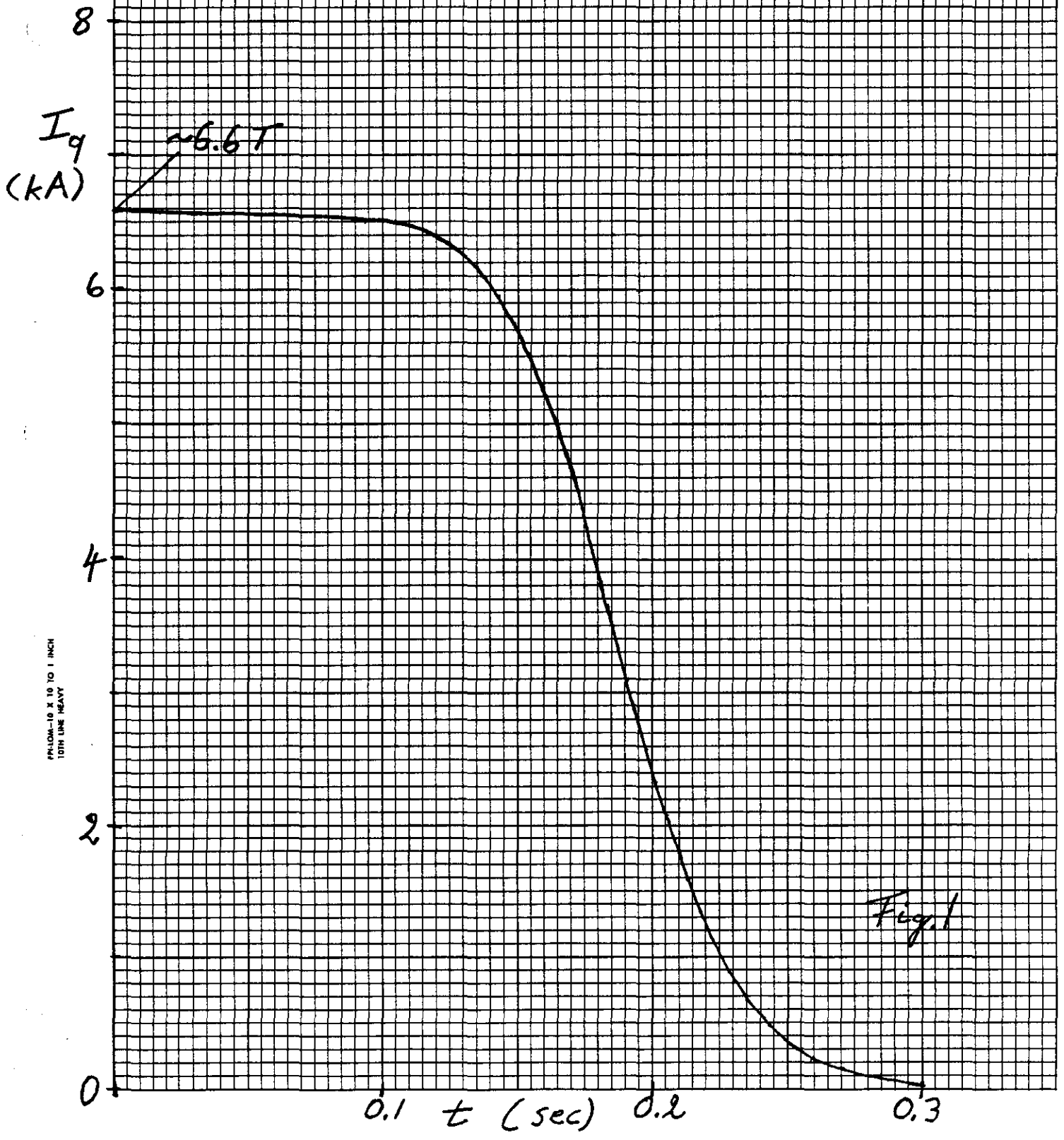

Fig. 1

PN-LOM-10 X 10 TO 1 INCH
10TH LINE HEAVY

Table I

$T(^{o}K)$	$R(T)$	$c_s(T)(J/g^{o}K)$
4	400	0.0022
6	400	0.0033
8	400	0.0043
10	400	0.0053
12	400	0.0063
14	400	0.0073
15	400	0.0084
17	370	0.010
20	313	0.013
25	222	0.019
30	143	0.032
35	91	0.048
40	61	0.068
45	38	0.087
50	27	0.104
55	19	0.130
50	15	0.140
65	11	0.155
70	8	0.170
80	7	0.200

TABLE II

			Unsupported			Supported		
B_o (T)	f_s	R(4)	σ_{ss} max (kpsi)	σ_{cu} max (kpsi)	w max (inch)	σ_{ss} max (kpsi)	σ_{cu} max (kpsi)	w max (inch)
7.5	1	400	71.7	54.6	0.019	17.2	14.0	0.0015
6.6	1	400	63.8	48.5	0.016	15.3	12.5	0.0014
7.5	0.5	400	55.2	42.0	0.015	13.2	10.8	0.0012
7.5	1.5	400	82.8	63.0	0.022	19.8	16.2	0.0018
7.5	---	200	41.2	31.9	0.011	10.0	8.2	0.0009