Tolerances for Residual Systematic Octupole Components in SSC - Dipole Magnets .

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Introduction

In the Conceptual Design Report[1] for the SSC the dipole magnets are assumed to have neither normal nor skew systematic octupole components . The goal of this note is to determine the tolerable amounts of any residual octupole moments with regard to the dynamic and chromatic behaviour of the TLD124B test lattice .

Tolerance Criteria

It is well known for the SSC , that the dynamic aperture is a poor tool to probe the effects of systematic multipoles , since the concept of a dynamic aperture becomes fairly useless without the inclusion of the random part of the magnetic errors , however the presence of random errors masks any systematic effects[2),3)] . Instead the effect of systematic octupoles on the momentum and amplitude dependent tune shifts have been explored . In order to determine what are tolerable, rsidual systematic octupole strengths , the criteria for the maximum acceptable tune shifts , that were established for the investigation of the clustered IR option[4)] , have been adopted once again . The relevant criteria for this study can be summarized in the following way :

$$|\Delta Q_x| \leq 0.005 \quad \text{and} \quad |\Delta Q_y| \leq 0.005$$

are the maximum permissible tune shifts for

(A) $\delta = \pm 0.001$ and $x = y = 0.0$ mm ;

(B) $\delta = \pm 0.001$ and

$r_{arc} = \sqrt{x^2_{arc} + y^2_{arc}} = 5$ mm for $\beta = 330$ m and $\eta = 3$ m ;

(C) $\delta = 0.0$ and

$r_{arc} = \sqrt{x^2_{arc} + y^2_{arc}} = 7$ mm for $\beta = 330$ m and $\eta = 3$ m .

Since one of the interaction points (IP) served as the tracking point, the amplitudes for conditions (B) and (C) have to be translated from the arc to the IP with

$$\beta^*_x = \beta^*_y = 1 \text{ m}, \quad \eta^* = 0 \text{ m}, \quad \alpha^*_x = \alpha^*_y = 0,$$

where they become :

$$r^* = r_{IP} = \sqrt{x^2_{IP} + y^2_{IP}} = .276 \text{ mm} \quad \text{for} \quad \delta = \pm 0.001 \quad \text{(B)}$$

and

$$r^* = r_{IP} = \sqrt{x^2_{IP} + y^2_{IP}} = .386 \text{ mm} \quad \text{for} \quad \delta = \pm 0.0 \quad \text{(C)} .$$

The actual tracking was done on the real space diagonal, $x = y$, and the initial coordinates for the tracked particles are (always keeping $x' = y' = 0$) :

$$x_{IP} = y_{IP} = .195 \text{ mm} \quad \text{for} \quad \delta = \pm 0.001 \quad \text{(B)}$$

and

$$x_{IP} = y_{IP} = .273 \text{ mm} \quad \text{for} \quad \delta = \pm 0.0 \quad \text{(C)} .$$

Estimate of Permissible Octupole Strength

The only magnetic errors of the dipole magnets considered in this study are octupoles . The horizontal variation of the vertical field component caused by octupole moments a_3 , b_3 is

$$B_z/B_o = b_3 (x^3 - 3xz^2) + a_3 (z^3 - 3x^2z) .$$

A particle with $\delta = \Delta p/p$ moves around the off-momentum closed orbit $x_o = \eta\delta$. If the horizontal coordinate with respect to x_o is ξ , the field variation B_z/B_o can be expressed as

$$B_z/B_o = b_3 \{ \xi^3 - 3\xi z^2 + 3x_o(\xi^2 - z^2) + 3x_o^2\xi + x_o^3 \}$$

$$+ a_3 \{ z^3 - 3\xi^2 z - 6x_o\xi z - 3x_o^2 z \} .$$

This shows : Only the normal octupole moment b_3 produces for an off-momentum particle a quadrupole feeddown leading in first order to a tune shift . Furthermore, for a one-dimensional horizontal motion the tune-shift caused by a systematic octupole b_3 is given by[5)]

$$\Delta Q = \beta\, b_3 < \cos\Phi\, (\eta\, \delta + A_\beta \cos\Phi)^n > / A_\beta \ ,$$

where A_β is the betatron amplitude , η is the dispersion at the location of the multipoles and the brackets imply an average over the betatron phase angle Φ , which provides

$$\Delta Q = 3\, \beta\, b_3\, (\eta^2 \delta^2 / 2 + A_\beta^2 / 8) \ .$$

Using the figures quoted above for the tune shift criteria one arrives at the following limits for b_3 :

$$|\, b_3^{max} \,| = \begin{array}{llll} 1.68\ m^{-3} & \text{for}\ \ \delta = \pm .001 & \text{and}\ \ r_{arc} = 0\ \text{mm} & (A)\ , \\ 1.15\ m^{-3} & \text{for}\ \ \delta = \pm .001 & \text{and}\ \ r_{arc} = 5\ \text{mm} & (B)\ , \\ 1.24\ m^{-3} & \text{for}\ \ \delta = 0 & \text{and}\ \ r_{arc} = 7\ \text{mm} & (C)\ . \end{array}$$

Tracking

The octupole moments of the bending magnets have been represented by 5 equally spaced thin lens elements per half cell . The fractional tunes were obtained from a Fourier transform[6)] of the horizontal and vertical betatron motion observed in 500 turn tracking runs performed with "RACETRACK" [7)]. The resolution achieved in these runs has been determined by a series of runs with identical initial conditions but differing in the number of turns tracked . For the range of values for a_3 and b_3 studied here it was concluded, that the fractional part of the tunes obtained from the betatron motion of 500 turns had 4 significant digits . Comparison of the tune shifts provided by tracking data and by analytic results (expansion around the off-momentum fix point, using "RACETRACK" and "MARYLIE") showed agreement of better than 20 % for the amplitude dependent tune shift .

Discussion of Results

In Fig. 1 the chromatic behaviour of the bare , chromaticity corrected test lattice TLD124B is presented . The dotted rectangular boxes indicate the acceptable tune shifts within the required range of momentum .The fractional tunes for $\delta = +.001$ and $\delta = -.001$ and for zero transverse amplitudes as a function of the normal (skew) octupole strength b_3 (a_3) are shown in Fig. 2a (2b). Again the dotted lines indicate the acceptable tune ranges . As was pointed out, the skew octupole produces for an off-momentum particle a skew quadrupole feeddown ,which in first order does not produce a tune shift . This is clearly demonstrated by the tracking results of Fig. 2b . From Fig. 2a one finds the limits for the normal octupole strenth b_3 :

$$-1.0\ m^{-3} \leq b_3 \leq +1.5\ m^{-3}$$

The asymmetry in the normal octupole limits corresponds exactly to the difference in the tune values for $\delta = -.001$ and $\delta = +.001$. These values for the limits of b_3 are 1/3 tighter than the prediction for the tune shift of the one-dimensional horizontal motion . As it turns out, it is this set of initial conditions (criterion A), that imposes the most restrictive limitations on the octupole strength .

Figs. 3 - 5 summarize the off-momentum, finite amplitude tune shift as a function of the octupole strengths (criterion B) . The tune dependence on the individual octupoles b_3 or a_3 is documented in Fig. 3a and Fig. 3b, respectively . Comparison with the zero amplitude case of Figs. 2a and 2b shows very little difference . For all tracking runs the particles were launched with $x' = y' = 0$. The effect of the launching position in phase space on the amplitude dependent, off-momentum tune shift was investigated by analytic calculations in form of an expansion around the off-momentum fix point . The error bar associated with the dependence on the initial phase space coordinates { x,x'; y,y' } was found to be $\delta Q \sim 0.002$.

Fig. 4 and Fig.5 present the combined effect that normal and skew octupole have on the off-momentum, finite amplitude tune shift . In Fig.4 this is explored along the axis $a_3 = b_3$. This data provides less stringent limits for a_3 and b_3 ,which are symmetrical :

$$-1.5\ m^{-3} \leq a_3 , b_3 \leq +1.5\ m^{-3} .$$

Fig. 5 presents the effect when b_3 and a_3 are varied along the axis $b_3 = a_3$. Here again the limits are less restrictive :

$$-2.0\ m^{-3} \leq b_3 \leq +1.5\ m^{-3} \quad \text{and} \quad -1.5\ m^{-3} \leq a_3 \leq +2.0\ m^{-3}\ .$$

The remaining figures (Figs. 6 - 8) refer to the on-momentum, amplitude dependent tune shift as a function of the octupole strengths (criterion C) . Figs. 6 - 8 are analogous to Figs. 3 - 5 . As in previous studies it turns out that the off-momentum criteria (A) and (B) are much more restrictive than the on-momentum, amplitude dependence criterion (C) . In the range of the octupole strenths explored, none of the on-momentum cases shown in Figs. 6 - 8 violates this criterion (C) .

Conclusion

Of the 3 tune shift criteria studied to determine the maximum permissible systematic octupole strength for the SSC dipole magnets, the off-momentum, zero amplitude tune shift criterion (A) is the most stringent one . However, it must be pointed out that this is not a general feature but depends on the particular lattice . When the two lattices with 60 and 90 degree phase advance were compared[8)] , it was found that the 60 degree lattice was preferable with regard to the on-momentum linear aperture, while off-momentum the 90 degree lattice was slightly better . Thus for the 60 degree lattice investigated here one expects the off-momentum criterion to be the tightest one to meet . Based on this criterion the maximum, uncorrected, residual octupole strength is of the order of $1\ m^{-3}$.

References

1) SSC Central Design Group , "SSC Conceptual Design Report" , April 1986

2) B. Leemann , "Persistent Current Effects on Chromatic Lattice Behaviour with 2μ filaments" , SSC - N - 169 (April 1986)

3) B. Leemann , A. Wrulich ,"Comparison of Various Clustered Interaction Regions with Regard to Chromatic and Dynamic Behavior" , SSC - 68 (February 1986)

4) The Clustered Interaction Region Study Group , "The Clusterted Interaction Region Option for the SSC" , SSC - SR - 1014 (September 1985)

5) Ref. 1) , p. 150

6) Richard Talman, private communication

7) A. Wrulich, "RACETRACK , a Computer Code for the Simulation of Non-linear Particle Motion in Accelerators", DESY 84 - 026 , (March 1984)

8) A. Chao , E. Forest , "A Comparison of Linear Aperture Between Lattices with 60° and 90° per Cell" , SSC - N - 81 (October 1985)

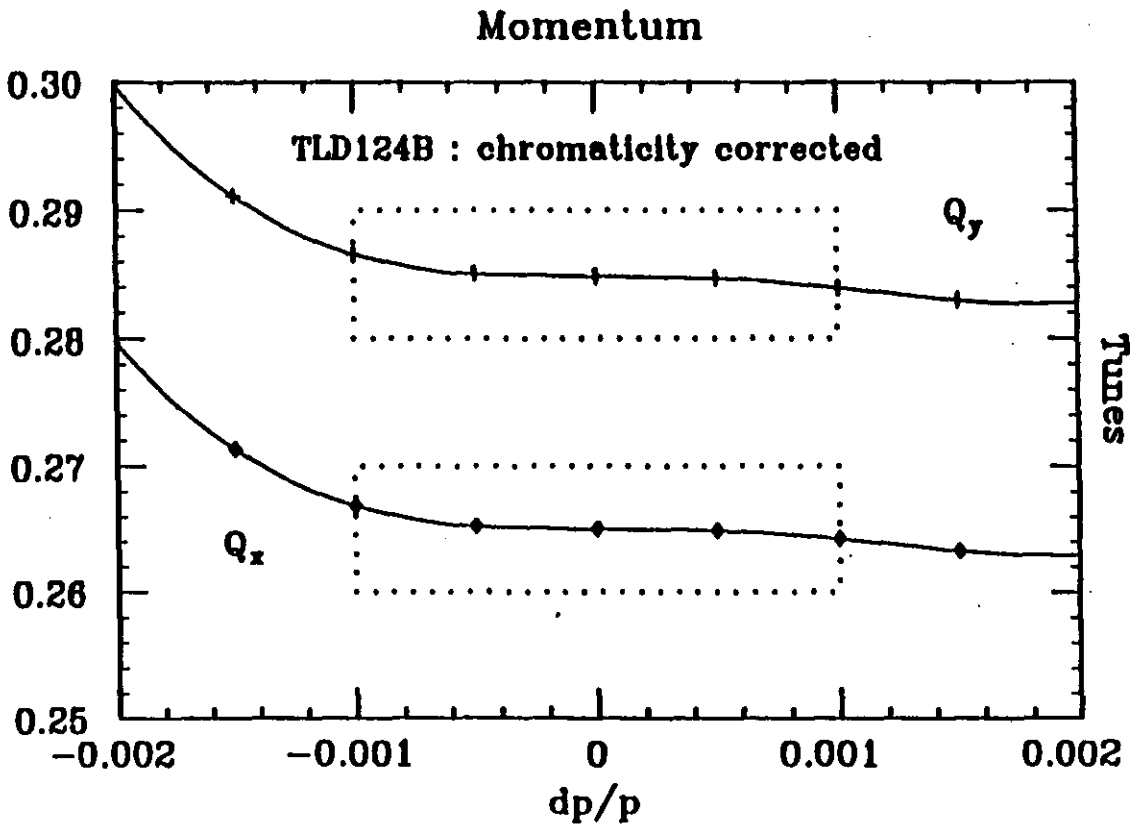

FIG. 1 : Chromatic behaviour of bare test lattice

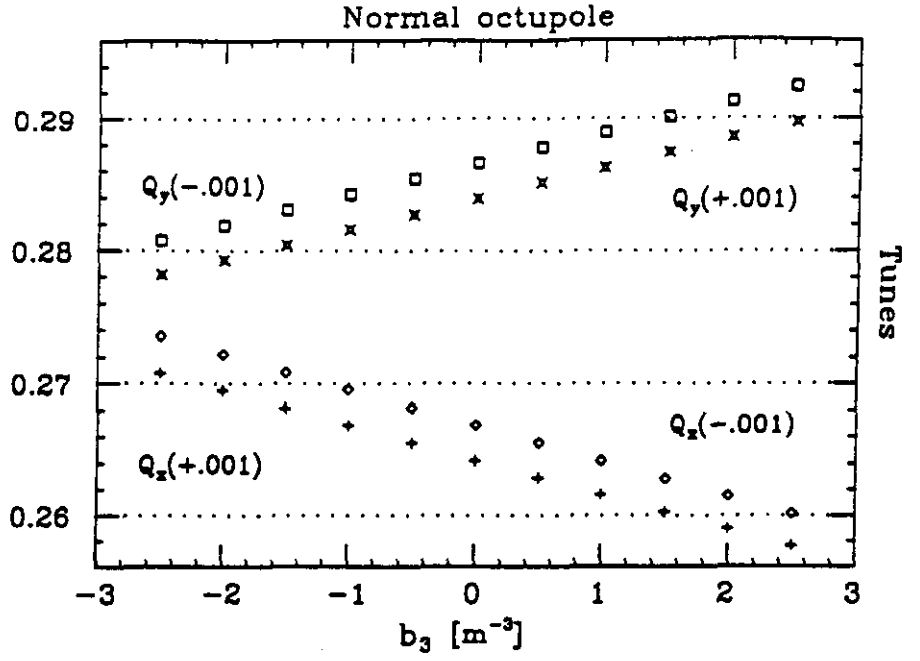

FIG. 2a : $x^*=y^*=0.000$ mm ; $dp/p = \pm 0.001$

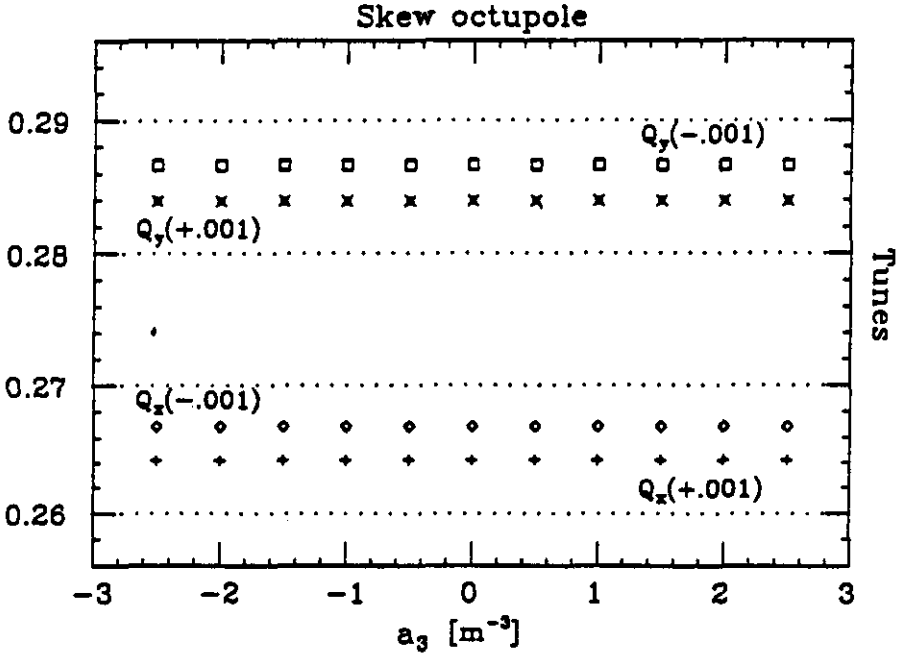

FIG. 2b : $x^*=y^*=0.000$ mm ; $dp/p = \pm 0.001$

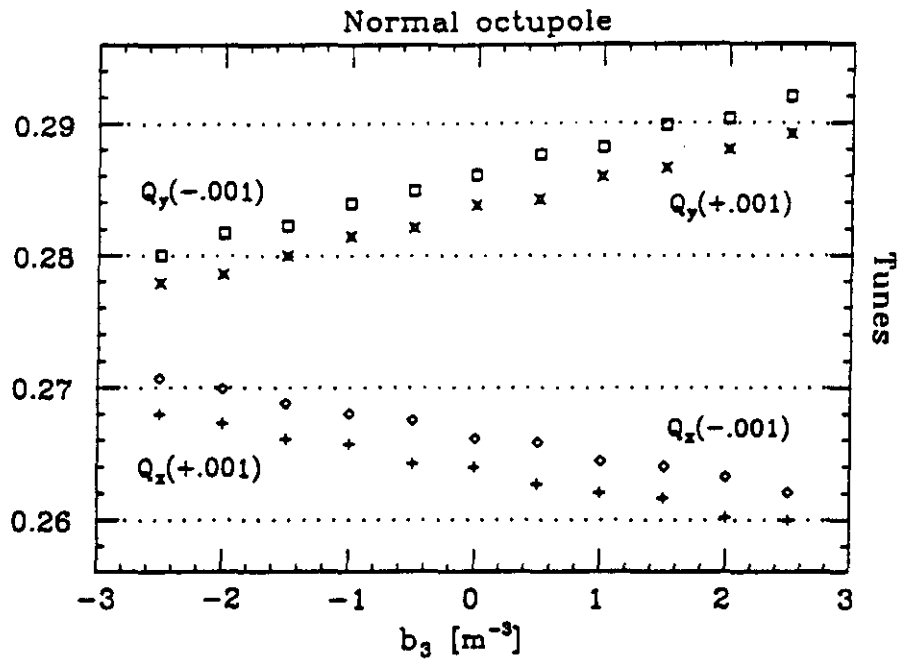

FIG. 3a : $x^*=y^*=0.195$ mm ; $dp/p = \pm 0.001$

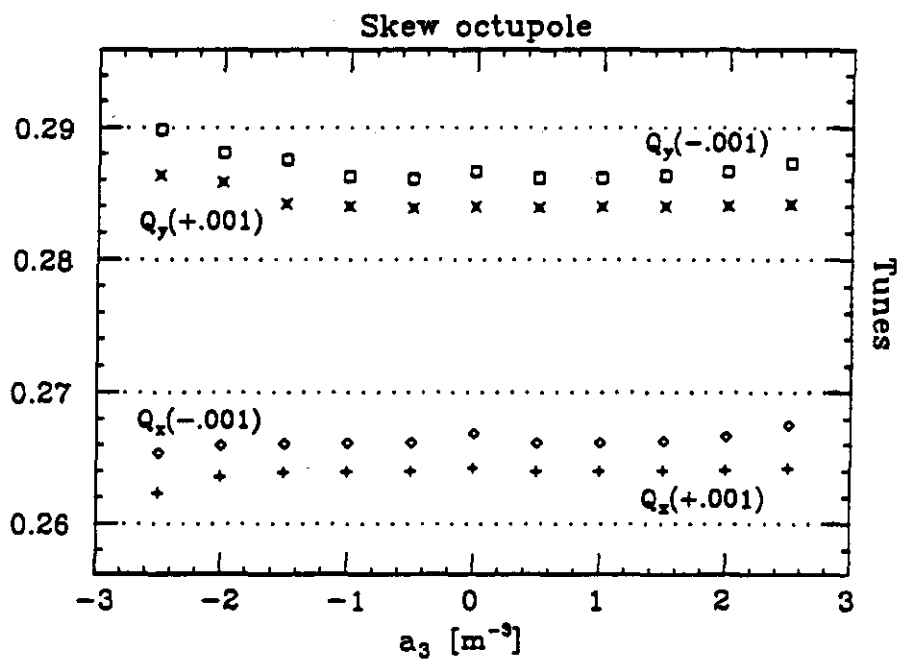

FIG. 3b : $x^*=y^*=0.195$ mm ; $dp/p = \pm 0.001$

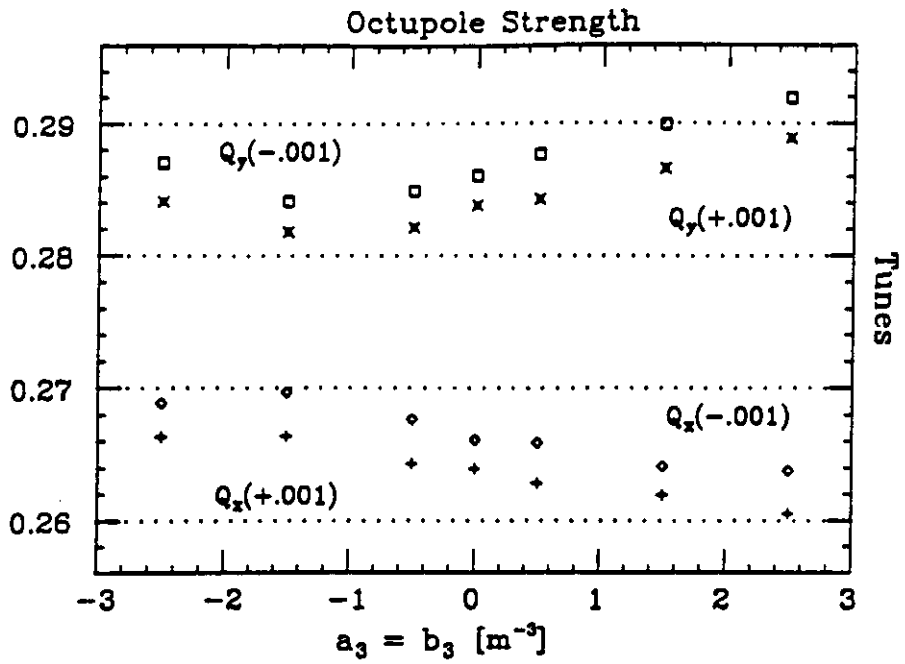

FIG. 4 : $x^*=y^*=0.195$ mm ; $dp/p = \pm 0.001$

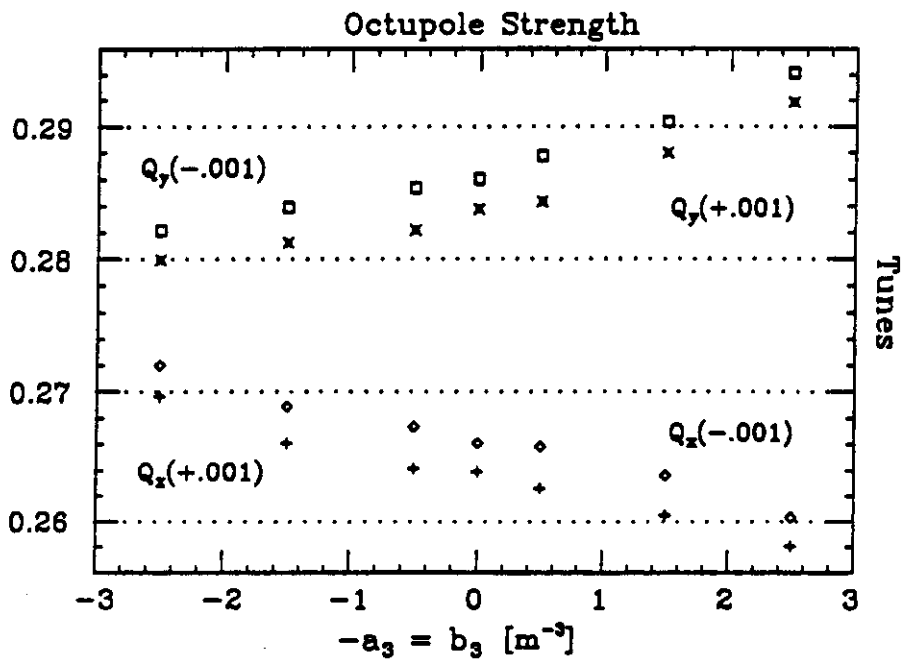

FIG. 5 : $x^*=y^*=0.195$ mm ; $dp/p = \pm 0.001$

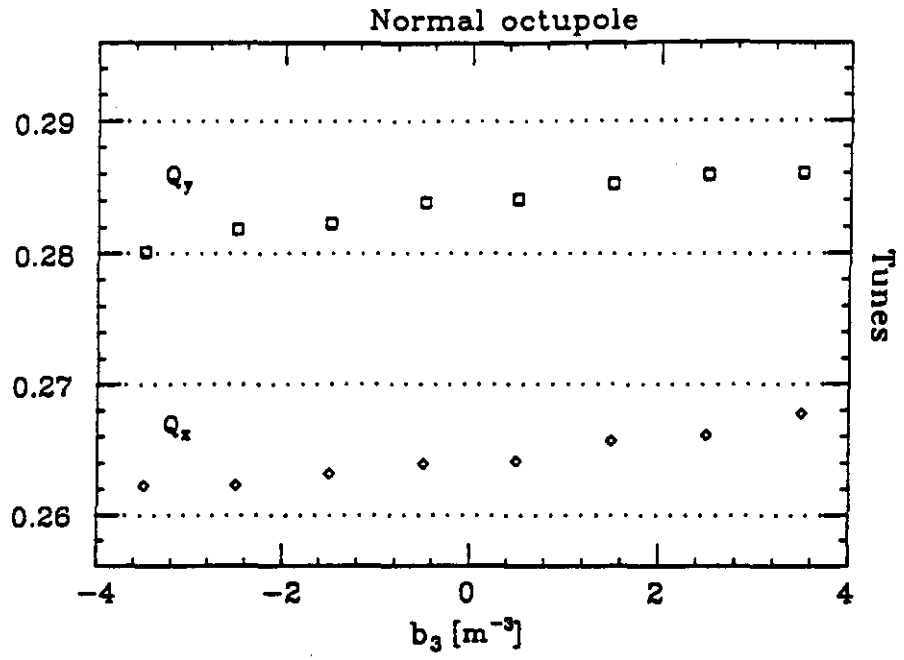

FIG. 6a : $x^*=y^*=0.272$ mm ; dp/p = 0.0

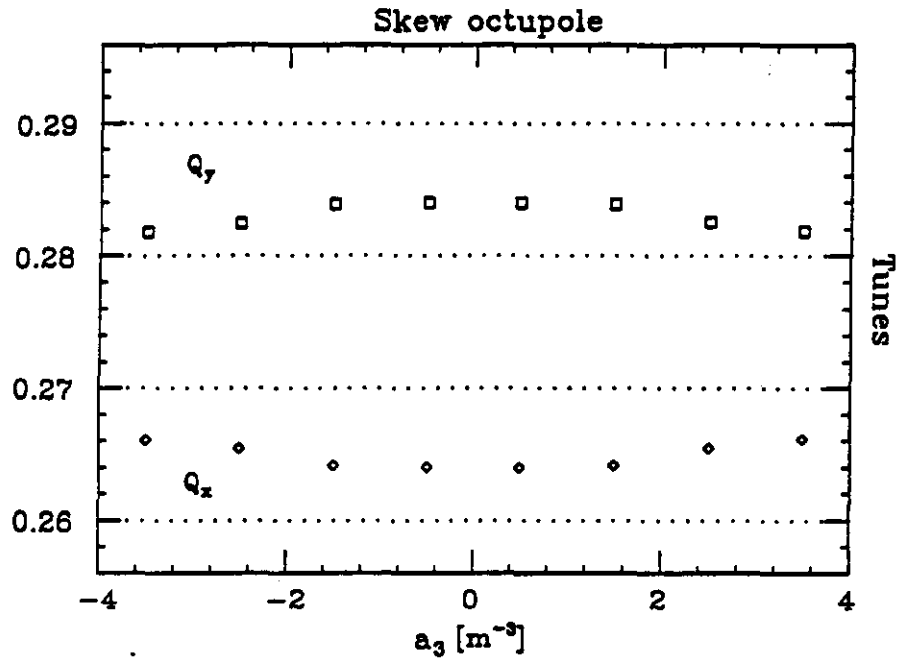

FIG. 6b : $x^*=y^*=0.272$ mm ; dp/p = 0.0

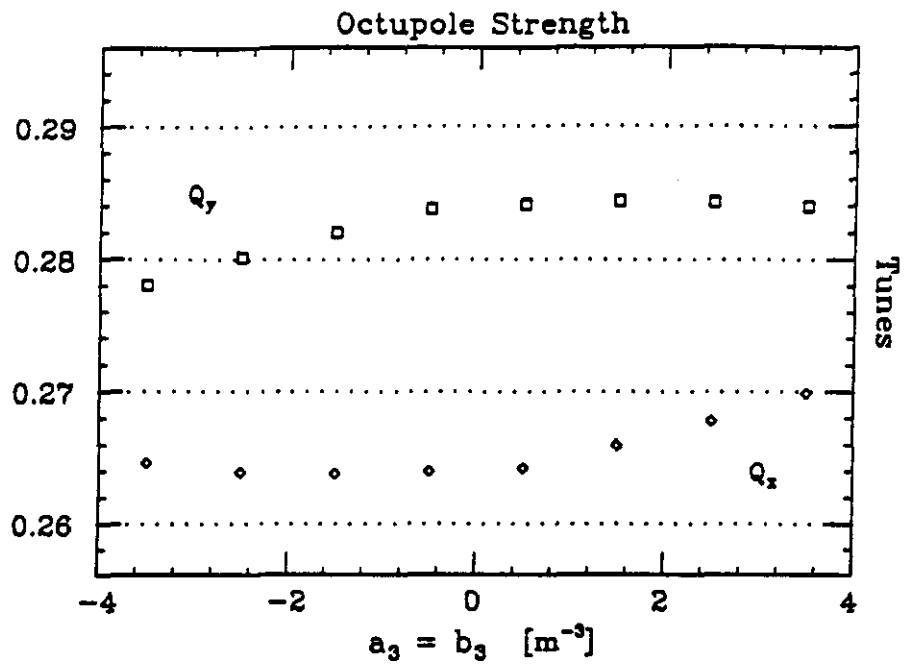

FIG. 7 : $x^*=y^*=0.272$ mm ; dp/p = 0.0

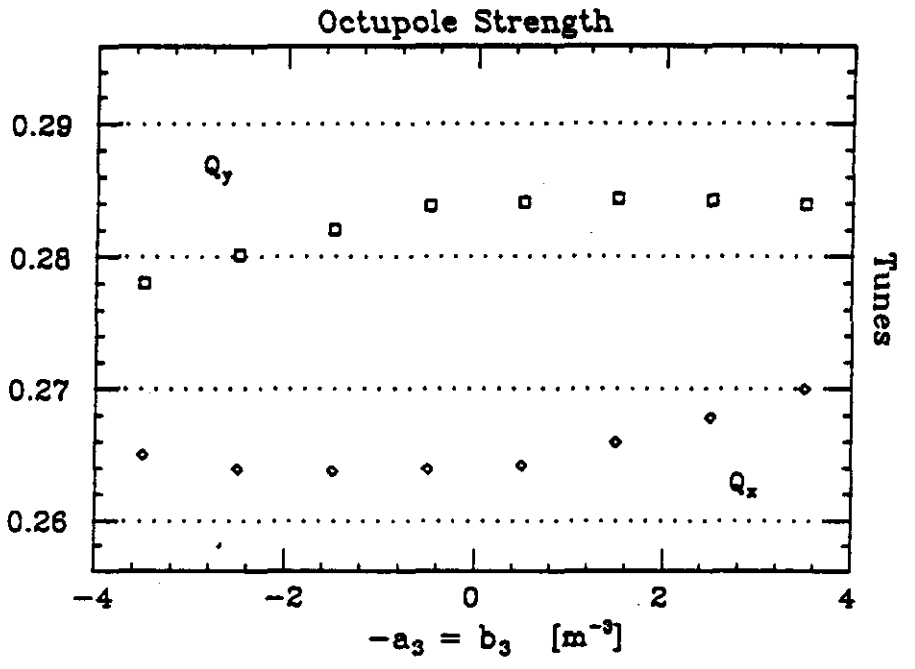

FIG. 8 : $x^*=y^*=0.272$ mm ; dp/p = 0.0