Modifications in Dipole and Quadrupole Field Strengths During Ramping Because of Beam Pipe Eddy Currents

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Abstract

For SSC magnet dimensions, a beam tube with 100 μm (4 mils) of copper plating, and a ramp time of 10^3 seconds, the change in the dipole field is $\Delta b_0 = -1.5$ units at injection (-3.7×10^{-2} units at full field). For the arc quadrupoles, $\Delta b_1 = -0.67$ units at injection (-1.7×10^{-2} units at full field). These values assume the copper's RRR is 100 at injection and 50 at full field.

1. Fields from Multipole Ring of Current

For the purposes of this estimate we assume that there is a tube of longitudinal surface current $K(\varphi)$ at radius R (the beam pipe), surrounded by a cylindrical iron yoke of radius R_0. The other parts of the cold mass (coils, collars) are assumed to have negligible susceptibility. With the approximation of infinite permeability for the iron, a two-dimensional multipole surface current distribution of the form,

$$K(\varphi) = -\frac{I}{R}\cos m\varphi \tag{1}$$

produces the fields inside and outside $r = R$,

$\underline{0 < r < R}$

$$\begin{Bmatrix} B_r \\ B_\varphi \end{Bmatrix} = \frac{\mu_0 I}{2R^m}\left(1 + \frac{R^{2m}}{R_0^{2m}}\right) r^{m-1} \begin{Bmatrix} \sin m\varphi \\ \cos m\varphi \end{Bmatrix} \tag{2}$$

$\underline{R < r < R_0}$

$$\begin{Bmatrix} B_r \\ B_\varphi \end{Bmatrix} = \frac{\mu_0 I\ R^m r^{m-1}}{2\ R_0^{2m}} \left(1 \pm \frac{R_0^{2m}}{r^{2m}}\right) \begin{Bmatrix} \sin m\varphi \\ \cos m\varphi \end{Bmatrix} \tag{3}$$

where m = 1 (dipole), 2 (quadrupole), etc.

2. Effective Eddy Current from Ramping of the Primary Multipole Field

During ramping of the SSC magnets eddy currents are induced in the beam pipe. If the beam pipe has a thin copper liner (to lower the resistive wall impedance presented to the beam), the only significant current flow is in that copper layer. Consider a multipole field of the form of Eq. (2), but with coefficient $B_m(t)$, e.g.

$$B_\varphi = B_m r^{m-1} \cos m\varphi \tag{4}$$

Application of Faraday's law to this quasi-static situation yields a longitudinal electric field,

$$E_z = \frac{\dot{B}_m}{m} r^m \cos m\varphi \tag{5}$$

and a current density $J_z = E_z/\rho$, where ρ is the electrical resistivity (of the copper). For a thin-walled beam tube or a high-conductivity layer of thickness t, the product tJ_z at r = R represents the surface current $K(\varphi)$. Comparison with Eq. (1) yields the effective current I_{eff},

$$I_{eff} = -\frac{R^{m+1}}{m} \left(\frac{t}{\rho}\right) \dot{B}_m \tag{6}$$

If the (thin) wall of the beam pipe is made up of several layers, each of thickness t_i and resistivity ρ_i, the factor (t/ρ) is replaced by a sum over the individual factors (t_i/ρ_i).

3. Changes in the Dipole and Quadrupole Fields

Inside the beam pipe of radius R, the fields are given by Eq. (2) and Eq. (6) for the current I. The incremental multipole coefficient is therefore

$$\Delta B_m = -\frac{\mu_o Rt}{2\rho}\dot{B}_m \cdot \frac{1}{m}\left(1 + \frac{R^{2m}}{R_0^{2m}}\right) \tag{7}$$

Explicitly, for the dipole field (B_0) and the quadrupole gradient (G), we have

$$\Delta B_0 = -\frac{\mu_o Rt}{2\rho}\dot{B}_0\left(1 + \frac{R^2}{R_0^2}\right) \tag{8}$$

$$\Delta G = -\frac{\mu_o Rt}{4\rho}\dot{G}\left(1 + \frac{R^4}{R_0^4}\right) \tag{9}$$

For the SSC parameters ($R \approx 1.7$ cm, $R_0 \approx 5.5$ cm), the enhancement factor $(1 + R^2/R_0^2) = 1.10$ for the dipole and 1.01 for the quadrupole.

4. Numerical values

The following numerical values are relevant:

$\dot{B} = 6.6 \times 10^{-3}$ T/s

$\dot{G} = 212 \times 10^{-3}$ T/m·s

$R = 1.7 \times 10^{-2}$ m

$\rho = 1.6 \times 10^{-8}$ Ω-m/RRR

$\mu_0 = 4\pi \times 10^{-7}$

then $\Delta B_0 = -4.83 \times 10^{-6}$[RRRt] tesla

$\Delta G = -7.14 \times 10^{-5}$[RRRt] T/m

where the thickness t is in millimeters. These changes are fractionally largest at injection for the obvious reason that the fields are smaller then,

but also because RRR is larger. The multipole coefficient increments at injection are

$$\Delta b_0 = -1.46 \times 10^{-1} \text{ [RRRt] } \underline{\text{units}}$$

$$\Delta b_1 = -6.74 \times 10^{-2} \text{ [RRRt] } \underline{\text{units}}$$

Copper at 4K and 0.33 tesla has RRR $\gtrsim$ 100. Using the nominal values of RRR = 100 and t = 0.1 mm, we have $\Delta b_0 = -1.5$ units and $\Delta b_1 = -0.67$ units. [Recent measurements on various plated beam tubes indicate RRR values in the range 100 to 400 at 0.33 tesla and 50 to 100 at 6 tesla. Values much in excess of 50 at full field will require reduction in the thickness of the plating in order to keep the magnetic forces during a quench to reasonable values. [RRRt] values of order 3 to 6 mm at full field are reasonable. This translates to roughly 6 to 20 mm at injection.]