RF and Longitudinal-Phase-Space Parameters in the SSC

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The radio-frequency acceleration system of the SSC is to satisfy the following conditions:

1. $f_{RF} = h\, f_o = 374.741$ MHz[1]
   $f_o = \beta c/2\pi R = 3.6144...$kHz (revolution frequency)
   (circumference = 82.944 km)
   $h = 103{,}680$ (harmonic number)
   β = beam velocity in units of c (= 1.0 - 4.4×10^{-7} at 1 TeV)

2. accelerating ramp = 1,000 seconds
   $E = 1 \rightarrow 20$ TeV (beam energy)
   thus $\Delta E = 5.26$ MeV per turn

3. beam longitudinal emittance ε_L (= $\sigma_E \cdot \sigma_t$) = 0.035 eV-sec at 1 TeV
   and 0.233 eV-sec at 20 TeV[2]

4. σ_z (= $\beta c \sigma_t$) $\cong$ 7 cm at 20 TeV

5. Ratio of RF bucket area to rms beam longitudinal phase-space area ($R_{bk} = A_{BK}/\pi\varepsilon_L$) ≥ 18 is desirable during storage at 20 TeV in order to suppress longitudinal emittance growth due to RF noise. This is an approximate rule based on observations at the SppS at CERN.[3]

The 1-TeV longitudinal emittance (specified in the RDS because of the transverse mode-coupling instability) now is conservative by a factor of 5 because the present SSC design has a smaller peak current and different lattice parameters. The 20-TeV longitudinal emittance (specified to control transverse-emittance growth due to intra-beam scattering) is probably about right because the smaller peak current here is compensated by the smaller transverse emittance due to radiation damping, which was not invoked in the RDS estimate. The 1-TeV longitudinal-emittance specification should be reviewed and possibly modified.

1. SSC-N-117, Rev.
2. RDS, p.165. Careful! RDS uses "95% area emittance". I.e., $\varepsilon_{RDS} = 6\pi\, \varepsilon_L$
3. RDS, p.139

The pertinent equations are:

1. bucket half height $(\Delta E/E)_{BK}$

$$(\Delta E/E)_{BK} = \pm\left[\frac{2\,eV}{\pi\, h\, \eta\; E\, \beta^2}\right]^{1/2} F(\phi_s) \tag{1}$$

where $F(\phi_s) = |\cos\phi_s - (\pi/2 - \phi_s)\sin\phi_s|^{1/2}$ (1a)

ϕ_s = synchronous phase = $\sin^{-1}(\Delta E/\,eV)$

V = peak RF voltage

η = slippage factor = $(df/\,f)/\,(dp/\,p)$

$= \alpha - \gamma^{-2} \cong \alpha$

α = compaction factor = 2.227×10^{-4} (SSC-N-139)

2. bucket area A_{BK}

$$A_{BK} = \frac{4\beta}{\pi f_{RF}}\left[\frac{2\,eV\,E}{\pi\, h\, \eta}\right]^{1/2} M(\phi_s) \qquad \text{(eV-sec)} \tag{2}$$

where $M(\phi_s)$ = moving-bucket factor, plotted in Figure 1. (Tabulated in UCID 1030, F.T.Cole and P.L.Morton, Sept. 1964)

The rms bunch length σ_z (= $\beta c\, \sigma_t$) and energy spread σ_E can be expressed in terms of the rms longitudinal emittance and/ or the RF bucket parameters:

3. $$\sigma_z = \left[\frac{\varepsilon_L\, c^2}{f_o}\right]^{1/2}\left[\frac{\eta}{2\pi\, h\, E\, eV\cos\phi_s}\right]^{1/4} \tag{3a}$$

or $$= \frac{2\,\lambda_{RF}}{\pi}\left[\frac{\beta\, M(\phi_s)}{\pi\, R_{bk}\cos^{1/2}\phi_s}\right]^{1/2} \tag{3b}$$

and 4. $$\sigma_E/\,E = \left[\frac{\beta^2 f_o\, \varepsilon_L}{E}\right]^{1/2}\left[\frac{2\pi\, h\, eV\cos\phi_s}{\eta\, E}\right]^{1/4} \tag{4a}$$

or $$= \left[\frac{8\,\beta^3 M(\phi_s)\, eV\cos^{1/2}\phi_s}{R_{bk}\,\pi^2\,\eta\, E\, h}\right]^{1/2} \tag{4b}$$

where R_{bk} is the bucket-to-beam area ratio:

$$R_{bk} = A_{BK} / \pi \varepsilon_L \tag{5}$$

Equations (3) and (4) follow from the basic synchrotron oscillation equation for the rate of change of the phase ø relative to the synchronous particle:

$$\phi_{max} = \frac{\eta h \omega_o}{\beta^2} (\Delta E/E)_{max} = \nu_s \omega_o \Delta\phi_{max} \tag{6}$$

where the synchrotron tune ν_s (phase oscillations per turn) is given by

$$\nu_s^2 = \frac{h \eta eV \cos \phi_s}{2 \pi E} \tag{7}$$

Since the desired increase in longitudinal emittance at 20 TeV, relative to that at 1 TeV(a factor of 6.66), is not too far from the increase in bucket area in going from 1 to 20 TeV at constant voltage ($\sqrt{20} = 4.47$), a linear increase in emittance plus a constant 20 MV peak RF voltage turns out to be a satisfactory arrangement. In this scenario

$$E\,(\text{TeV}) = 1 + 19\,t, \quad 0 < t < 1 \quad (t = \text{time in units of 1,000 sec}) \tag{8a}$$

$$\varepsilon_L(\text{eV-sec}) = 0.035 + 0.198\,t, \quad 0 < t < 1 \tag{8b}$$

$$\phi_s = 0, \quad t < 0 \text{ and } t > 1 \quad \text{(injection and storage)} \tag{8c}$$
$$= \sin^{-1}(5.26/20) = 15.24^o, \quad 0 < t < 1 \quad \text{(during acceleration)}$$
$$F(0) = 1 \;;\; F(15.24^o) = 0.789$$
$$M(0) = 1 \;;\; M(15.24^o) = 0.582$$

$$\nu_s = 8.54 \times 10^{-3} / \sqrt{E(\text{TeV})}, \quad \phi_s = 0 \tag{8d}$$
$$= 8.39 \times 10^{-3} / \sqrt{E(\text{TeV})}, \quad \phi_s = 15.24^o$$

$$A_{BK}(\text{eV-sec}) = 2.53 \sqrt{E(\text{TeV})}, \quad \phi_s = 0 \tag{8e}$$
$$= 1.474 \sqrt{E(\text{TeV})}, \quad \phi_s = 15.24^o$$

$$(\Delta E/E)_{BK} = \pm 7.45 \times 10^{-4} / \sqrt{E(\text{TeV})}, \quad \phi_s = 0 \tag{8f}$$
$$= \pm 5.88 \times 10^{-4} / \sqrt{E(\text{TeV})}, \quad \phi_s = 15.24^o$$

$$\sigma_z(m) = 0.320\,[\varepsilon_L(\text{eV-sec})]^{1/2}\,[E(\text{TeV})]^{-1/4}\ ,\quad \phi_s = 0 \tag{8g}$$
$$= 0.323\,[\varepsilon_L(\text{eV-sec}]^{1/2}\,[E(\text{TeV})]^{-1/4}\ ,\quad \phi_s = 15.24^{o}$$

$$\sigma_E/E = 9.35 \times 10^{-4}\,[\varepsilon_L(\text{eV-sec})]^{1/2}\,[E(\text{TeV})]^{-3/4}\ ,\quad \phi_s = 0 \tag{8h}$$
$$= 9.27 \times 10^{-4}\,[\varepsilon_L(\text{eV-sec})]^{1/2}\,[E(\text{TeV})]^{-3/4}\ ,\quad \phi_s = 15.24^{o}$$

The parameters following from this scenario during injection, acceleration, and storage are listed in the following table and plotted in Figures 2-a, -b, and -c.

Table of RF and Longitudinal Phase-Space Parameters during Injection, Acceleration, and Storage

t	E	ε_L	A_{BK}	$A_{BK}/\pi\varepsilon_L$	ν_s	f_s	$(\Delta E/E)_{BK}$	σ_E/E	σ_z	$\Delta E_{BK}/\sigma_E$
10^3sec	TeV	eV-sec	eV-sec		x 10^{-3}	Hz	x 10^{-4}	x 10^{-4}	cm	
< 0	1.0	.035	2.53	23.0	8.54	30.9	±7.45	1.75	5.99	4.26
0^+	1.0	.035	1.47	13.4	8.39	30.3	5.88	1.73	6.04	3.38
0.05	1.95	.0449	2.06	14.6	6.01	21.7	4.21	1.19	5.79	3.53
0.1	2.9	.0548	2.51	14.6	4.93	17.8	3.45	0.98	5.79	3.53
0.2	4.8	.0746	3.23	13.8	3.83	13.8	2.68	0.78	5.96	3.43
0.3	6.7	.0944	3.82	12.9	3.24	11.7	2.27	0.69	6.17	3.31
0.4	8.6	.1142	4.32	12.1	2.86	10.3	2.01	0.63	6.37	3.22
0.5	10.5	.1340	4.78	11.4	2.59	9.36	1.81	0.58	6.57	3.11
0.6	12.4	.1538	5.19	10.7	2.38	8.61	1.67	0.55	6.75	3.03
0.7	14.3	.1736	5.58	10.2	2.22	8.02	1.56	0.53	6.92	2.97
0.8	16.2	.1934	5.93	9.8	2.08	7.53	1.46	0.51	7.09	2.89
0.9	18.1	.2132	6.27	9.4	1.97	7.13	1.38	0.49	7.23	2.82
1.0^-	20.0	.233	6.59	9.0	1.88	6.78	1.32	0.47	7.37	2.77
> 1	20.0	.233	11.3	15.5	1.91	6.90	1.67	0.48	7.31	3.49

The conclusion is that the proposed RF scenario (linear increase in longitudinal emittance and constant RF voltage of 20 MV) is probably satisfactory, in that the bucket area is always at least 9 times the beam area during acceleration and 15.5 times the beam area during storage at 20 TeV. Although the ratio of 15.5 is somewhat less than the approximate empirical factor of 18 quoted in the RDS, the difference is probably within the uncertainties involved. Since the longitudinal emittance requirement at 20 TeV was derived for a different set of parameters in the RDS, this analysis should be redone for the present parameters before iterating on the RF system.

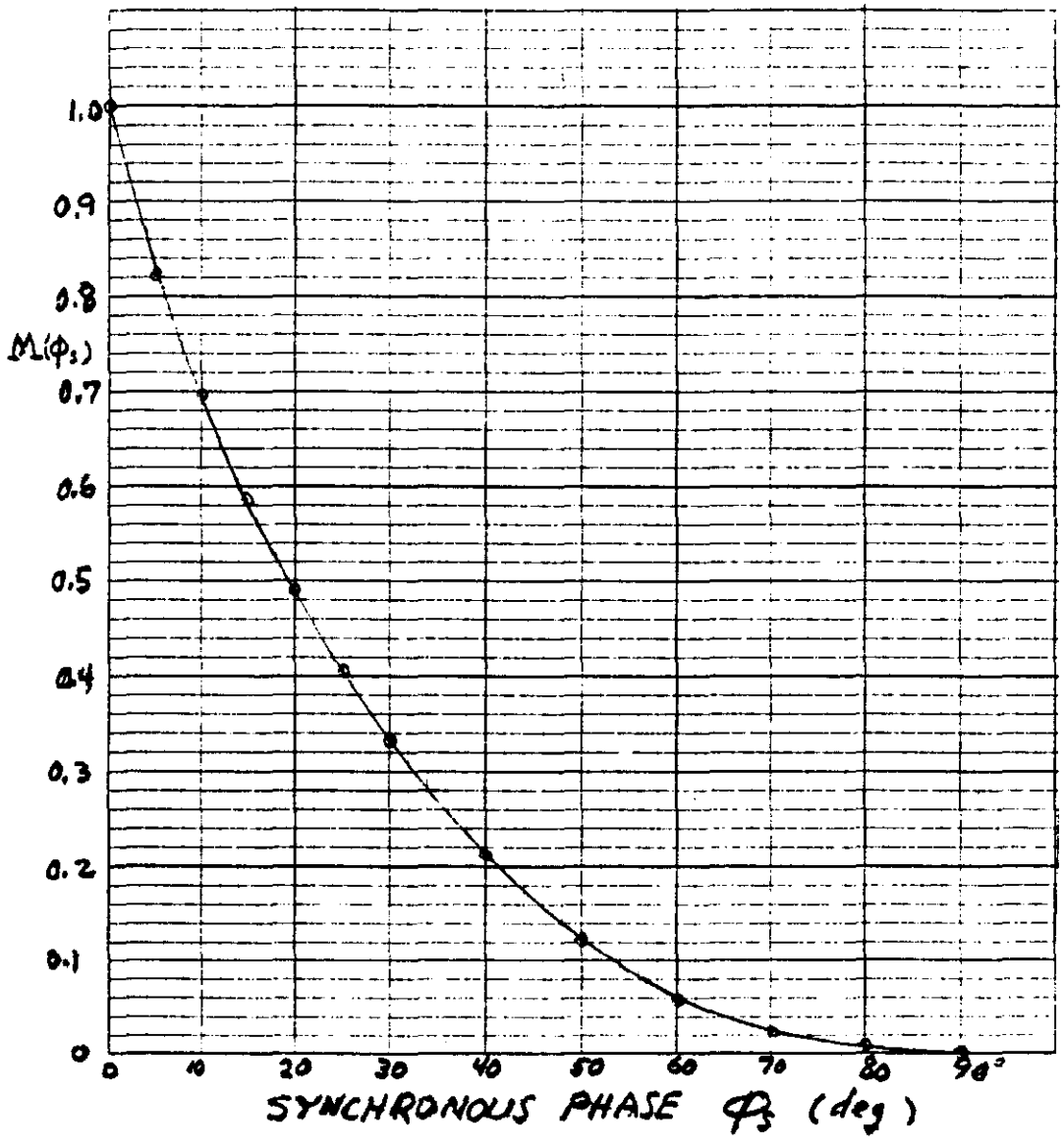

Figure 1. Plot of $M(\phi_S)$, the Moving Bucket Factor

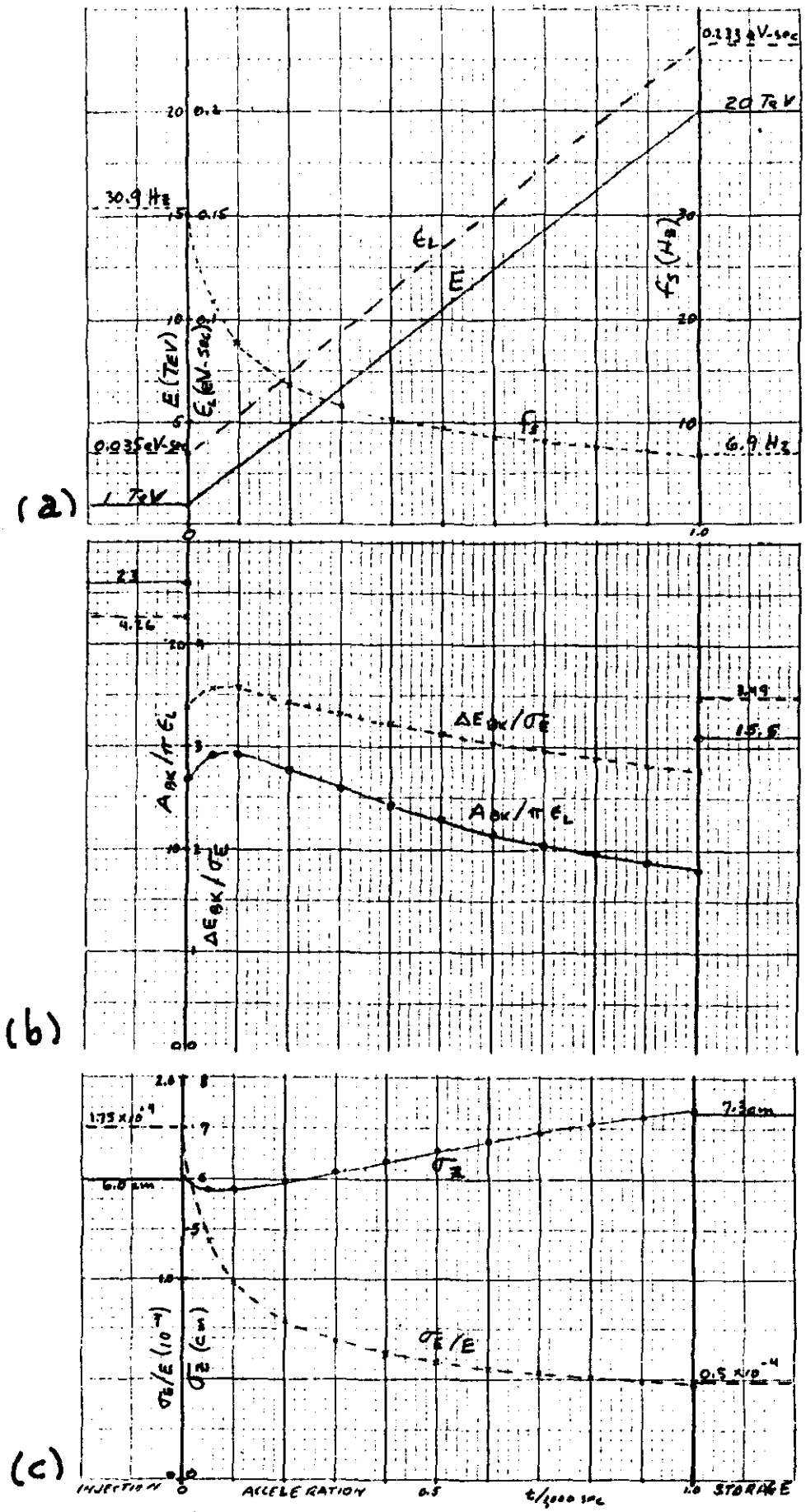

Figure 2. Variation of RF bucket and longitudinal-phase-space parameters during injection, acceleration, and 20 TeV storage:
(a) Beam energy, longitudinal emittance, and synchrotron frequency,
(b) Ratio of bucket area to beam phase-space area and ratio of bucket half-height to beam rms energy spread,
(c) RMS beam bunch length and fractional energy spread.