Tune Shifts Due to Systematic Decapole Field Errors After Correction by Two Families of Lumped Decapoles

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The systematic field error in the SSC dipoles due to the persistent currents mainly contains the sextupole and decapole components. The Conceptual Design Report assumes that these systematic field errors are compensated by distributed coils inside the dipole magnets and around the vacuum pipe. A more economical way is to replace the distributed coils by lumped correctors in the primary spools. However, the compensation is then no longer perfect and it is necessary to examine the effectiveness of lumped corrections. SSC-N-145 (Chao, Forest, Peterson) examined the effect of lumping the sextupole corrections. The tolerable systematic sextupole field was found to be $b_2=3\times10^{-4}cm^{-2}$. In this note, we examine the corresponding effect of the decapole component.

A systematic decapole, specified by coefficient b_4, contributes to tune shifts that depend on $\delta=\Delta E/E$ and betatron emittances $\epsilon_{x,y}=A_{x,y}^2/\beta_{x,y}$, where $A_{x,y}$ are the betatron amplitudes of the particle under consideration. The tune shifts, to first order in b_4, are

$$\begin{aligned}\Delta\nu_x &= 2b_4\langle\beta_x\eta^3\rangle\delta^3 + 3b_4\langle\beta_x^2\eta\rangle\epsilon_x\delta/2 - 3b_4\langle\beta_x\beta_y\eta\rangle\epsilon_y\delta \\ \Delta\nu_y &= -2b_4\langle\beta_y\eta^3\rangle\delta^3 - 3b_4\langle\beta_x\beta_y\eta\rangle\epsilon_x\delta + 3b_4\langle\beta_y^2\eta\rangle\epsilon_y\delta/2\end{aligned} \tag{1}$$

where η is the dispersion function and $\langle .. \rangle$ means averaging over the dipole magnets.

We assume the lattice is made of a string of FODO cells. For lumped correction of the tune shifts, we assume two families of decapoles D_F and D_D, located at the focusing and defocusing quadrupoles. Let $D_{F,D}=N(B_y'''''L/B\rho)_{F,D}/24$, where N is the total number of cells, the tune shifts due to the lumped decapoles are

$$\Delta\nu_x = (D_F\beta_F\eta_F^3+D_D\beta_D\eta_D^3)\delta^3/\pi + 3(D_F\beta_F^2\eta_F+D_D\beta_D^2\eta_D)\varepsilon_x\delta/4\pi$$
$$- 3\beta_F\beta_D(D_F\eta_F+D_D\eta_D)\varepsilon_y\delta/2\pi$$

$$\Delta\nu_y = -(D_F\beta_D\eta_F^3+D_D\beta_F\eta_D^3)\delta^3/\pi - 3\beta_F\beta_D(D_F\eta_F+D_D\eta_D)\,\varepsilon_x\delta/2\pi$$
$$+ 3(D_F\beta_D^2\eta_F+D_D\beta_F^2\eta_D)\,\varepsilon_y\delta/4\pi \qquad (2)$$

We assume that the correction procedure is to use the two decapole families to remove the δ^3 terms in the tune shifts caused by the systematic field error (1). This gives the decapole strengths

$$D_F = -2\pi b_4(\beta_F^2-\beta_D^2)^{-1}\eta_F^{-3}[\beta_F\langle\beta_x\eta^3\rangle-\beta_D\langle\beta_y\eta^3\rangle]$$
$$D_D = -2\pi b_4(\beta_F^2-\beta_D^2)^{-1}\eta_D^{-3}[\beta_F\langle\beta_y\eta^3\rangle-\beta_D\langle\beta_x\eta^3\rangle] \qquad (3)$$

Since the tune shifts contain also amplitude dependent terms that are not compensated by the two families of lumped decapoles, there are residual terms in the tune shifts given by substituting (3) into (1). This gives

$$\Delta\nu_x = a\,\varepsilon_x\delta + b\,\varepsilon_y\delta$$
$$\Delta\nu_y = b\,\varepsilon_x\delta + c\,\varepsilon_y\delta \qquad (4)$$

where, to first order in b_4,

$$a = 3b_4/2\,\{\langle\beta_x^2\eta\rangle - (\beta_F^2-\beta_D^2)^{-1}[(\beta_F^3/\eta_F^2-\beta_D^3/\eta_D^2)\langle\beta_x\eta^3\rangle$$
$$+ \beta_F\beta_D(\beta_D/\eta_D^2-\beta_F/\eta_F^2)\langle\beta_y\eta^3\rangle]\}$$

$$b = -3b_4\,\{\langle\beta_x\beta_y\eta\rangle - \beta_F\beta_D(\beta_F^2-\beta_D^2)^{-1}[(\beta_F/\eta_F^2-\beta_D/\eta_D^2)\langle\beta_x\eta^3\rangle$$
$$+ (\beta_F/\eta_D^2-\beta_D/\eta_F^2)\langle\beta_y\eta^3\rangle]\}$$

$$c = 3b_4/2\,\{\langle\beta_y^2\eta\rangle - (\beta_F^2-\beta_D^2)^{-1}[\beta_F\beta_D(\beta_D/\eta_F^2-\beta_F/\eta_D^2)\langle\beta_x\eta^3\rangle$$
$$+ (\beta_F^3/\eta_D^2-\beta_D^3/\eta_F^2)\langle\beta_y\eta^3\rangle]\}$$

(5)

Note that the contributions from the lumped decapoles tend to cancel the contributions from the systematic decapole component in the dipole magnets. Cancelling the chromatic dependence of tune shifts thus also helps substantially in reducing the amplitude dependence of the tunes.

Whether a systematic decapole component in the dipole magnets can be compensated by two families of lumped decapole magnets, will be determined by whether the residual tune shifts exceeds 0.005 in a resonable range of δ and $\epsilon_{x,y}$. This is discussed later.

We now compute the various lattice quantities assuming FODO cells with thin-lens quadrupoles. Let the quadrupole focal length be f, the cell length be 2L, and betatron phase advance per cell be μ. Let the space between the quadrupoles be filled by uniform bending with bending angle per cell 2θ. The beta functions and the dispersion function at position s ($0<s<L$, $s=0$ and $s=L$ are locations of the defocusing and focusing quadrupoles, respectively) are given by

$$\beta_x(s) = 2L\,(1 + s/f - L/2f - sL/2f^2 + s^2/2f^2)\,/\,\sin\mu$$

$$\beta_y(s) = 2L\,(1 - s/f + L/2f - sL/2f^2 + s^2/2f^2)\,/\,\sin\mu$$

$$\eta(s) = \theta(4f^2/L)\,(1 - L/4f + s/2f + s^2/8f^2 - Ls/8f^2) \qquad (6)$$

Specific values at the decapoles are

$$\beta_F = 2L\,(1 + L/2f)\,/\,\sin\mu$$
$$= [L/\sin(\mu/2)]\,\{[1+\sin(\mu/2)]\,/\,[1-\sin(\mu/2)]\}^{1/2}$$

$$\beta_D = 2L\,(1 - L/2f)\,/\,\sin\mu$$
$$= [L/\sin(\mu/2)]\,\{[1-\sin(\mu/2)]\,/\,[1+\sin(\mu/2)]\}^{1/2}$$

$$\eta_F = \theta(4f^2/L)\,(1 + L/4f)$$
$$= \theta L\,[2+\sin(\mu/2)]\,/\,2\sin^2(\mu/2)$$

$$\eta_D = \theta(4f^2/L)\,(1 - L/4f)$$
$$= \theta L\,[2-\sin(\mu/2)]\,/\,2\sin^2(\mu/2)$$

$$\sin(\mu/2) = L/2f \qquad (7)$$

The averaged quantities $\langle \ldots \rangle$ can be written as $(1/L)\int_0^L ds \ldots$ Using the explicit expressions (6) and using MACSYMA, we find

$$\langle \beta_x^2 \eta \rangle = A(-3L^6+98f^2L^4-280f^4L^2+13440f^6)/13440f^6$$

$$\langle \beta_x \beta_y \eta \rangle = A(-3L^6+182f^2L^4-3640f^4L^2+13440f^6)/13440f^6$$

$$\langle \beta_y^2 \eta \rangle = A(-L^6+70f^2L^4-840f^4L^2+4480f^6)/4480f^6$$

$$\langle \beta_x \eta^3 \rangle = B(L^8-63f^2L^6+1512f^4L^4+26880f^6L^2+645120f^8)/645120f^8$$

$$\langle \beta_y \eta^3 \rangle = B(L^8-99f^2L^6+3528f^4L^4-134400f^6L^2+645120f^8)/645120f^8 \tag{8}$$

where

$$A = (2L/\sin\mu)^2 [48f^2/L]$$

$$B = (2L/\sin\mu) [48f^2/L]^3$$

Substituting (7) and (8) into (5) gives

$$a = (3b_4A/2)(1 - L^2/16f^2)^{-2} L^2 (-L^{10}+57f^2L^8-1084f^4L^6 +29760f^6L^4-172032f^8L^2)/41287680f^{12}$$

$$b = (-3b_4A)(1 - L^2/16f^2)^{-2} L^2 (L^{10}-105f^2L^8+4532f^4L^6 -123840f^6L^4+602112f^8L^2)/41287680f^{12}$$

$$c = (3b_4A/2)(1 - L^2/16f^2)^{-2} L^2 (-L^{10}-7f^2L^8+5732f^4L^6 -212928f^6L^4+5505024f^8L^2-27525120f^{10})/41287680f^{12} \tag{9}$$

In case of 60° cells, L=f, Eq.(9) reduces to

$$a = -b_4 \theta L^3\, 1433/11340 = -0.126\, b_4 \theta L^3$$

$$b = -b_4 \theta L^3\, 1609/1890 = -0.851\, b_4 \theta L^3 \qquad (10)$$

$$c = -b_4 \theta L^3\, 24697/1260 = -19.6\, b_4 \theta L^3$$

In case of 90° cells, $L=2^{1/2}f$, we have

$$a = -b_4 \theta L^3\, 14551/82320 = -0.177\, b_4 \theta L^3$$

$$b = -b_4 \theta L^3\, 15489/13720 = -1.13\, b_4 \theta L^3 \qquad (11)$$

$$c = -b_4 \theta L^3\, 1082567/82320 = -13.2\, b_4 \theta L^3$$

The c coefficient is much larger than the a and b coefficients. This feature seems to agree with tracking studies.[Leemann, report to appear] For given θ and L, 90° cells have somewhat smaller tune shifts.

For the SSC Conceptual Design lattice, which has L=96 m, θ=0.008181 and μ=60°, yielding

$$a = (-912\ m^3)\, b_4$$

$$b = (-6160\ m^3)\, b_4 \qquad (12)$$

$$c = (-1.42\times10^5\ m^3)\, b_4$$

A linear aperture criterion that $|\Delta\nu_{x,y}| < 0.005$ in the rangle $|\delta| < 0.001$ and $\epsilon_{x,y} < 7.6\times10^{-8}$ m (amplitude of 5 mm at β=330 m) requires $|a,b,c| < 6.6\times10^7 m^{-1}$, which in turn sets the limit $b_4 < 0.047\times10^{-4}\ cm^{-4}$. This is to be compared with the value of $b_4 = 0.11\times10^{-4} cm^{-4}$ for the case expected of 2 μm filaments. The tight tolerance can be relaxed by one of the following methods (or a combination of them):

(1) Increase the number of decapole families. In principle, five families can remove completely the first order tune shifts. The remaining effects are of second order in the decapole strengths and require more study, especially by particle tracking. Perhaps three families suffice.

(2) Shortening the cells. The tune shifts are proportional to L^4. Two-micron filaments fulfill the linear aperture criterion if cell length is decreased from 192 m to 155 m.

(3) Use 90° cells. The tolerance on b_4, assuming L=96 m, becomes 0.07×10^{-4} cm^{-4}.

(4) Impose a tighter constraint on the linear aperture. The horizontal amplitude can be tolerated with little tune shifts. Reducing the need on δ from 0.001 to 0.0007 and A_y from 5 mm to 4 mm are sufficient to make 2-μm filaments tolerable.

Whether these methods are practical remain to be studied. Tentatively, it seems that as far as the tune shift criterion is concerned, the persistent-current decapole field error due to 2 μm filaments can be tolerated provided a combination of proper measures mentioned above are taken.