SSC-N-146

Collimator Wake Fields and Impedances in SSC

Huan Lee

March 1986

INTRODUCTION

To protect the superconducting quadrupoles from the spray of particles after collision at each interaction point, it has been proposed to insert a collimator along the beam line before the first triple quadrupole. There will be eight such collimators in the SSC ring. In this note we report the results of calculating the wake fields and impedances due to these scraping collimators. We use the code TBCI[1] to compute both the transverse and longitudinal wake fields generated by a truncated Gaussian bunch. The impedances are extracted from the fast Fourier transform of the bunch wakes.

IMPEDANCE OBTAINED FROM BUNCH WAKES

Ideally we would like to compute the wake fields due to a single charge, denoted by $W_\perp$ and $W_\parallel$ for the transverse and longitudinal, respectively, from which the corresponding impedances are simply given by

$$iZ_\perp(\omega) = \int_{-\infty}^{\infty} W_\perp(t)e^{i\omega t}dt \tag{1}$$

and

$$Z_\parallel(\omega) = \int_{-\infty}^{\infty} W_\parallel(t)e^{i\omega t}dt \tag{2}$$

The code, however, computes the wake fields due to a bunch of finite length, which we denote by $\bar{W}_\perp$ and $\bar{W}_\parallel$. Hence $\bar{W}_\perp$ ($\bar{W}_\parallel$) is actually the convolution of $W_\perp$ ($W_\parallel$) with the bunch distribution $\rho(z/c)$, i.e.,

$$\bar{W}(t) = \int_{-\infty}^{\infty} W(t'-t)\ \rho(t')\ dt' \tag{3}$$

Let $i\bar{Z}_\perp$, $\bar{Z}_\parallel$, and $\tilde{\rho}$ be the Fourier transforms of $\bar{W}_\perp$, $\bar{W}_\parallel$, and ρ, respectively. In principle, we can caluculate the impedance Z by

$$Z(\omega) = \bar{Z}(\omega)/\tilde{\rho}(\omega). \tag{4}$$

We take ρ to be a Gaussian of width (in length) σ, it follows that

$$\tilde{\rho}(\omega) = e^{-\omega^2\sigma^2/2c^2} . \tag{5}$$

In code calculation, the Gaussian bunch is truncated symmetrically to a total length 10σ, but (5) is still an excellent approximation.

It is important to notice that $\tilde{\rho}(\omega)$ is obtained by the Fourier transform with $t = 0$ defined at the bunch center. On the other hand, from (3) the bunch wakes $\bar{W}$ start at an earlier time $t = -\ell/c$, where 2ℓ is the bunch length. This time shift has to be included in the Fourier transform to ensure that the impedances have the correct physical phases.

CONSTRAINTS ON BUNCH WIDTH

In numerical computation, the rapid growth of the exponential factor ρ^{-1} in (4) at high frequencies would render $Z(\omega)$ unreliable. Roughly speaking, we expect meaningful information in $Z(\omega)$ to be limited by the frequency

$$f = \frac{\omega}{2\pi} \simeq \frac{c}{2\sigma} \quad ,$$

which is 15 GHz for $\sigma = 1$ cm. It is therefore desirable to use a bunch width as small as possible which is compatible with the code constraint.

Since the code TBCI requires that the mesh size Δ be the same in both the radial and axial directions, in modeling the actual collimator configuration we have to pick a Δ no less than 0.11 cm. The code further demands that σ be at least greater than 5Δ. It turns out that, unlike the resonance cavity problems, there are significant noises in computing the wake fields using even $\sigma = 10\Delta$. The noise become insignificant at $\sigma = 14\Delta$ for $\bar{W}_\perp$ and $\sigma = 21\Delta$ for $\bar{W}_\parallel$.

PARAMETERS

The following parameters are used in TBCI calculation:

bean tube radius = 1.65 cm
collimator radius = 0.55 cm
collimator length = 100 cm
length of beam tube = 100 cm
mesh size Δ = 0.11 cm
σ/Δ = 6, 10, 14, 21
bunch length = 10 σ

NUMERICAL RESULTS

1. Transverse Wake Fields and Impedances

The transverse bunch wakes $\bar{W}_\perp$ calculated by using $\sigma/\Delta = 10$ and 14 are plotted in Fig. 1. We see that for $\sigma = 10\Delta$, there are numerical noises which oscillate undamped following the real wake. In contrast, they are absent in Fig. 1b where $\sigma = 14\Delta$. The frequency spectrum of the noise peaks sharply around 11 GHz, as shown in Fig. 2. For $f \lesssim 9$ GHz, the impedances computed for both $\sigma/\Delta = 10$ and 14 agree very well with each other. The latter are shown in Fig. 3. We note that $Z_\perp$ is dominated by its imaginary part, with the ratio ($\mathrm{Re}Z_\perp/\mathrm{Im}Z_\perp$) being less than 4%. In the frequency range below 9 GHz, we can represent $Z_\perp$ approximately by the expression,

$$Z_\perp = -12.5\,(1 + 0.032\,f^2)\,i, \tag{6}$$

with $Z_\perp$ and f in units of kΩ/m and GHz, respectively.

2. Longitudinal Wake Fields and Impedances

For the longtitudinal bunch wakes, we again observe the numerical noise as shown in Fig. 4a, where we use $\sigma = 10\Delta$. The noise becomes insignificant only when we use a large ratio $\sigma/\Delta = 21$, as shown in Fig. 4b. The resulting longitudinal impedances are plotted in Fig. 5. From our previous discussion, we expect that $Z_{\|}$ computed with such a width is meaningful only for $f \lesssim 7$ GHz. In fact, $Z_{\|}$ in this frequency range obtained by using $\sigma/\Delta = 10$, 14, and 21 all agree with one another. The longitudinal impedances are also dominated by the imaginary parts, and can be represented approximately by

$$Z_{\|} = -13.2\ f(1 + 0.069\ f^2)i, \tag{7}$$

with $Z_{\|}$ in units of Ω.

DISCUSSION

The rms bunch length σ_z is 7 cm for the SSC, which corresponds to a frequency spectrum extended up to about $f = c/2\pi\sigma_z = 0.7$ GHz. Looking at the expressions (6) and (7), we see that just taking the first term is quite adequate for frequencies below a few GHz. With eight collimators in the ring, the total impedances are MΩ/m

$$Z_{\perp} = i\ 0.10\ (+\ 0.032\ f^2)\ \mathrm{M\Omega/m}$$

$$Z_{\|}/n = -i\ 3.7 \times 10^{-4}\ (1 + 0.069\ f^2)\ \Omega,$$

where $n = \omega/\omega_0$ with $\omega_0 = 2.27 \times 10^4$ Hz being the revolution frequency. These values are very small compared to the corresponding bellows impedances.[2, 3] Compared to the space charge impedances, again the transverse collimator impedance is much smaller, whereas the longitudinal one is comparable at 20 TeV.

I would like to thank Ruth Hinkins for her indispensable help in developing the postprocessor for the TBCI output. I have benefited from discussions with K. Bane, A. Chao, and S. Chattopadhay.

REFERENCES

1. T. Weiland, DESY 82-015 (1982) and Nucl. Inst. and Meth. 212, 13 (1983).
2. K. Bane and R. Ruth, SSC Impedance Workshop Report SSC-SR-1017, p.10 (1985).
3. J. Bisognano and K.Y. Ng, SSC Impedance Workshop Report SSC-SR-1017, p.61 (1985).

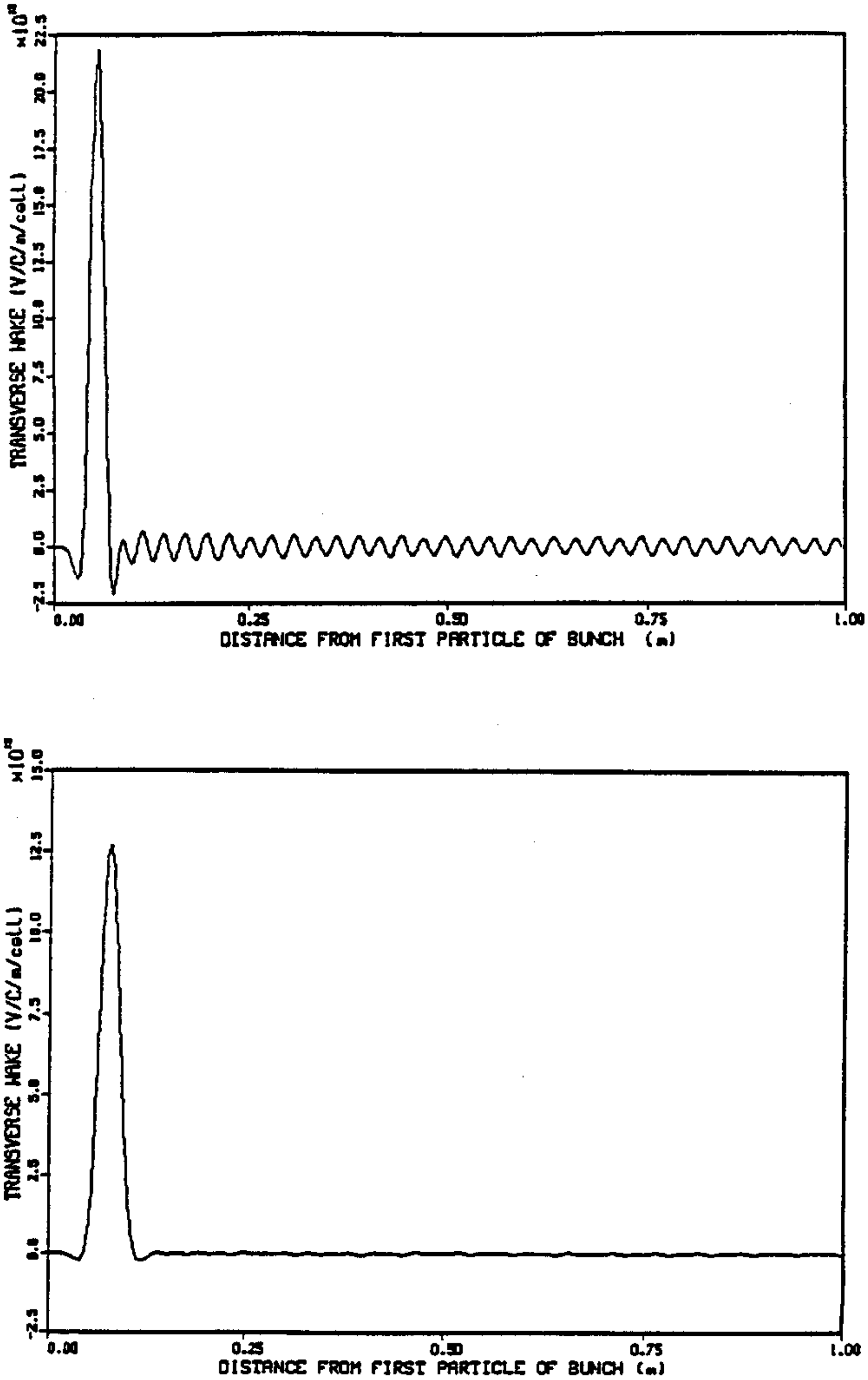

Fig. 1. Transverse bunch wakes calcualted using (a) $\sigma = 10\Delta$ and (b) $\sigma = 14\ \Delta$.

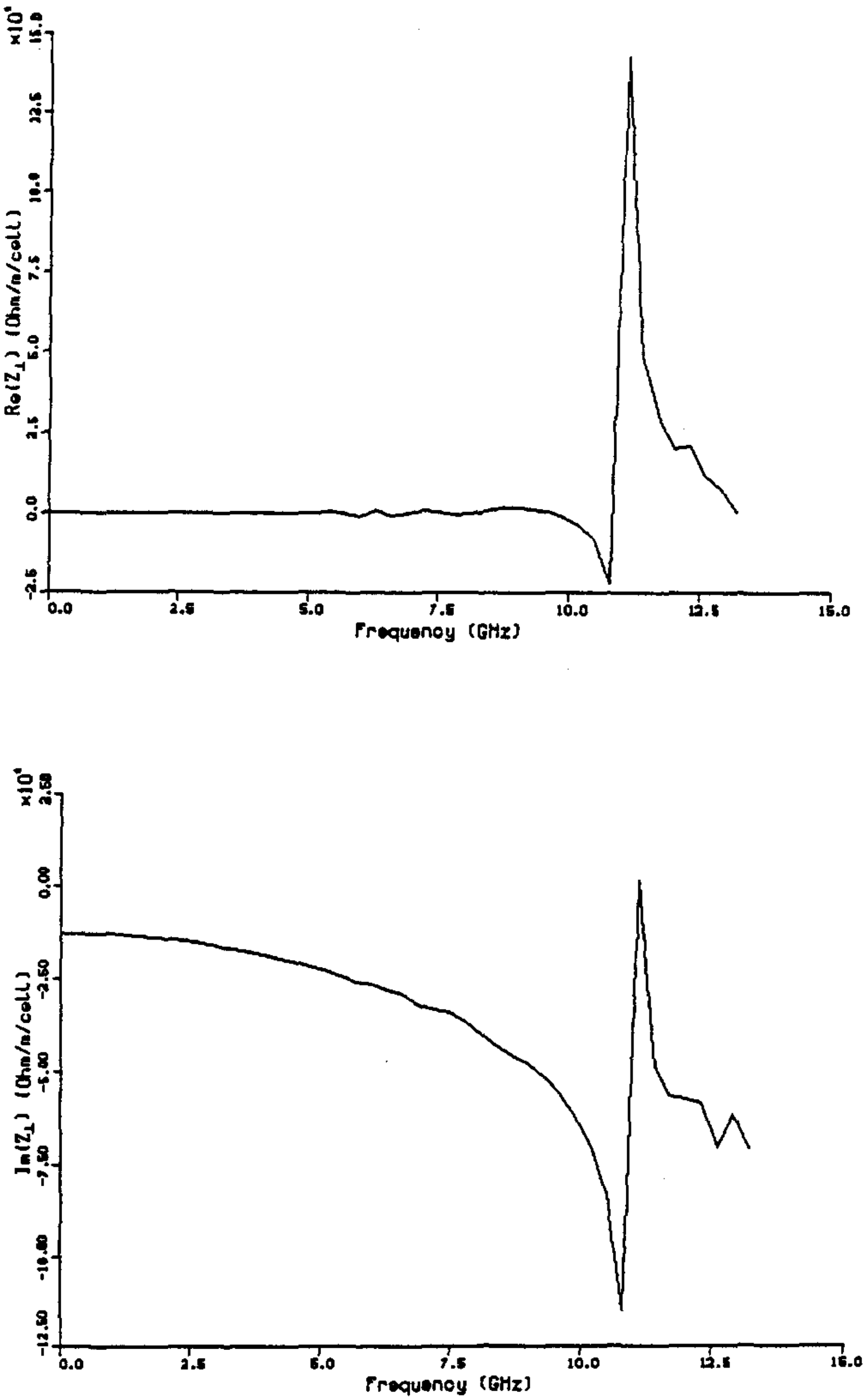

Fig. 2. (a) Real and (b) imaginary parts of the transverse impedance using $\sigma = 10\Delta$, which has a strong noise peaking around 11 GHz.

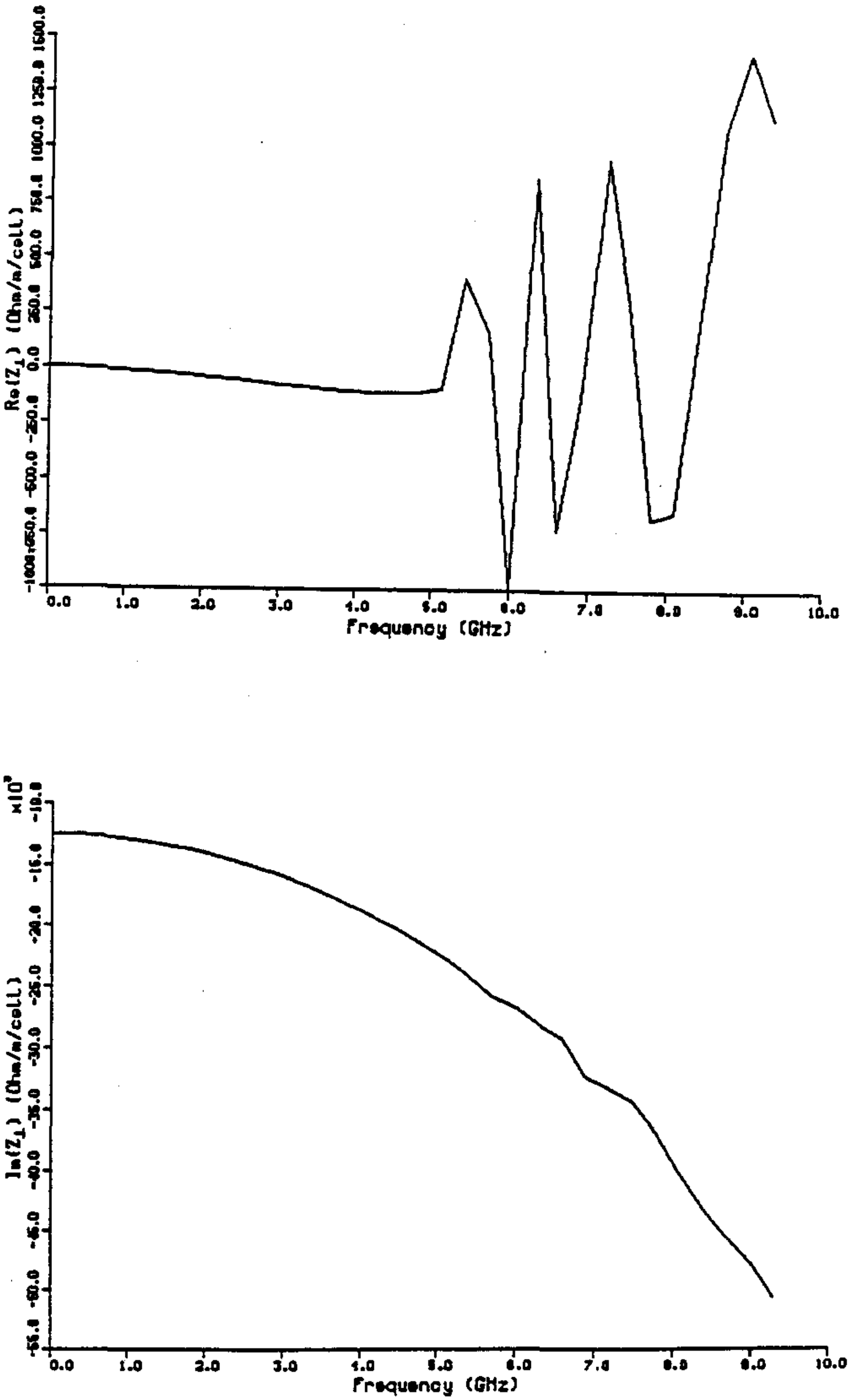

Fig. 3. (a) Real and (b) imaginary parts of the transverse impedance using $\sigma = 14\Delta$.

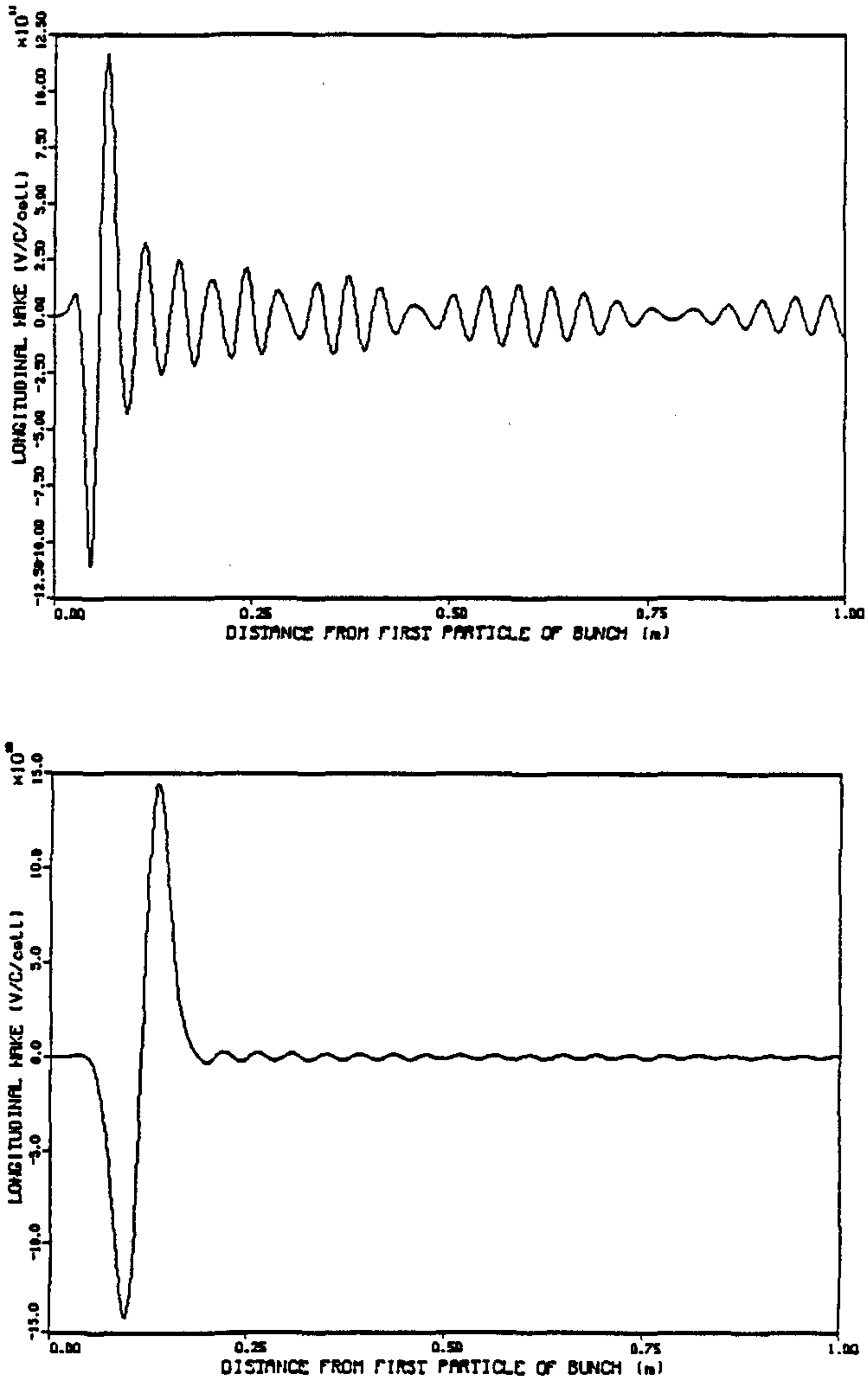

Fig. 4. Longitudinal bunch wakes calculated using (a) $\sigma = 10\Delta$ and (b) $\sigma = 21\ \Delta$.

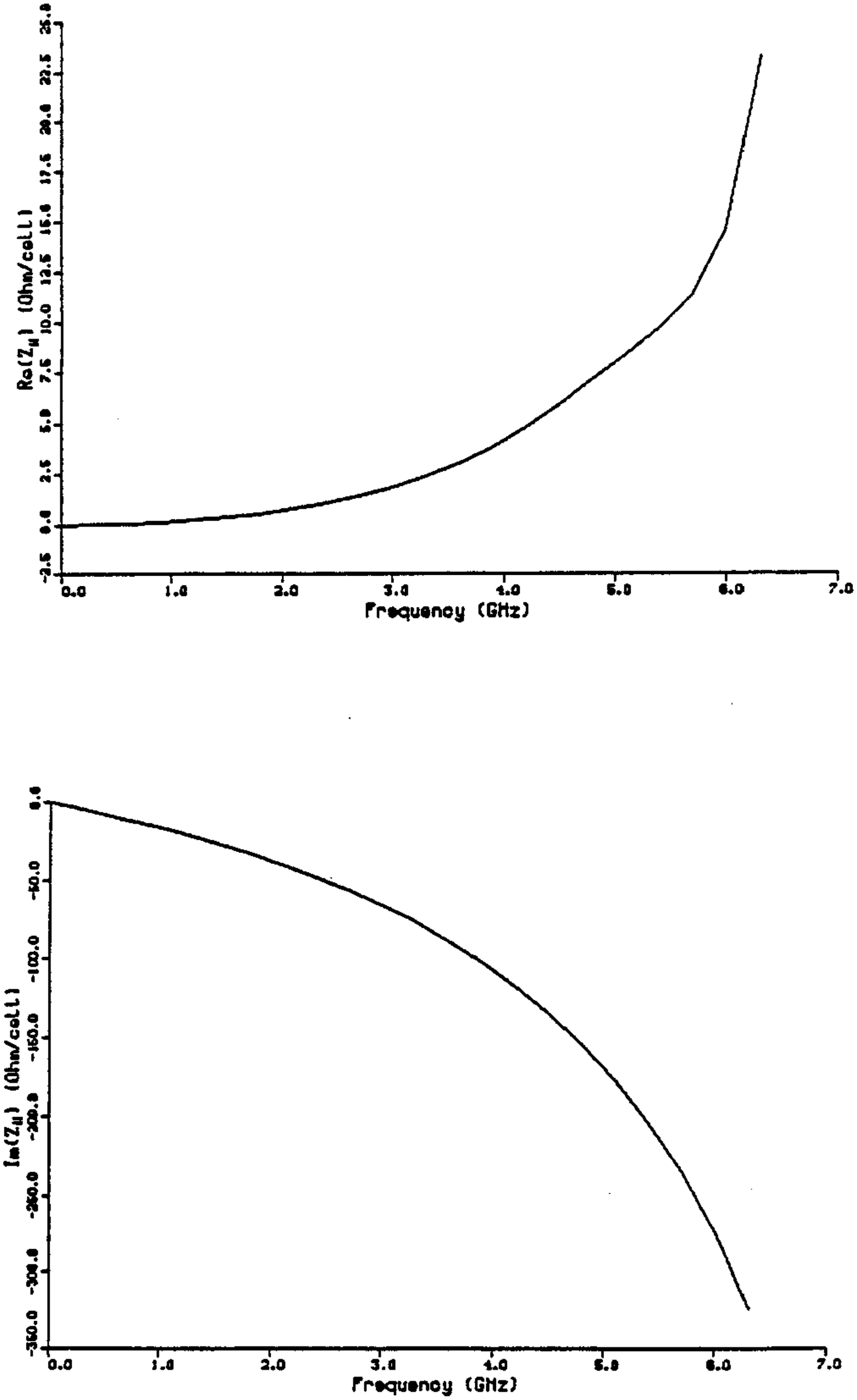

Fig. 5. (a) Real part and (b) imaginary part of the longitudinal impedance.