SSC-N-143

BUNCH-LENGTH DEPENDENCE OF POWER LOSS FOR THE SSC

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ABSTRACT

We evaluate the power lost by several mechanisms in the SSC as function of bunch length. The items considered are: RF cavities, inner bellows, sliding-contact bellows, beam position monitors plus kickers, resistive wall, and coherent synchrotron radiation

1. Introduction

We assume that the beam has M bunches, each with an electric charge $N_B e$, moving in an orbit of length $2\pi R$ with frequency f_o. As they circulate around the machine they lose power due to the retarding effect of the longitudinal impedance $Z_{\|}(\omega)$. For gaussian bunches the power loss is given by[1]

$$P = M(N_B e f_o)^2 \sum_{p=-\infty}^{+\infty} Re[Z_{\|}(p\omega_o)] \ \exp\left(-p^2\omega_o^2\sigma_z^2/c^2\right) \tag{1.1}$$

where σ_z is the *rms* bunch length and $\omega_o = 2\pi f_o$.

In practice we replace the summation by an integral in the above formula. This replacement is a valid approximation provided: a) that $Z_{\|}(\omega)$ varies little (and smoothly) over a frequency interval of size ω_o, and b) that $\omega_o\sigma_z/c \ll 1$. Since the average radius is given by $R = c/\omega_o$, this last assumption means that $\sigma_z \ll R$, which is well-satisfied in practice. As for condition a), we assume it to be valid. Thus our starting formula is

$$P = M(N_B e f_o)^2 \int_{-\infty}^{+\infty} \frac{d\omega}{\omega_o} Re[Z_{\|}(\omega)] \ \exp\left(-\omega^2\sigma_z^2/c^2\right) \tag{1.2}$$

A particular expression for the impedance, which is often used for many structures, is that of a simple resonator,

$$Z_{\|}(\omega) = \frac{R_S}{1 + iQ\left(\frac{\omega_R}{\omega} - \frac{\omega}{\omega_R}\right)} \tag{1.3}$$

in which case Eq.(1.2) yields,[2] assuming that $\omega_R/\omega_o \gg Q$,

$$P = M(N_B e f_o)^2 R_S \left(\frac{2\pi R}{\sigma_z}\right) \frac{Re(zw(z))}{\sqrt{4Q^2 - 1}} \tag{1.4}$$

where

$$z = \frac{\pi f_R \sigma_z}{Qc}\left(\sqrt{4Q^2 - 1} + i\right) \tag{1.5}$$

In Eq.(1.4) $w(z)$ is the complex error function[3] and $f_R = \omega_R/2\pi$. In the limit

when $Q \gg 1$, Eq.(1.4) reduces to

$$P = M(N_B e f_o)^2 R_S \left(\frac{\pi\omega_R}{Q\omega_o}\right) \exp\left(-\omega_R^2\sigma_z^2/c^2\right) \tag{1.6}$$

In what follows we use certain assumed forms for the impedances for the different structures, and then use Eqs. (1.2) or (1.4) to calculate the power loss. We also assume nominal SSC values for the parameters, namely $M = 17,280$ bunches per ring, $N_B = 7.3 \times 10^9$ particles per bunch, a revolution frequency $f_o = 3.62$ KHz (corresponding to $\omega_o = 2.27 \times 10^4$ sec^{-1}) and an average orbit radius $R = c/\omega_o = 13.2$ km. The charge of the proton is $e = 1.6 \times 10^{-19}$ C. Note that the factor $N_B e f_o = 4.2 \times 10^{-6}$ A, which appears squared in these expressions, is the average bunch current. All results presented for the power loss are per ring.

2. Impedances

2.1 RF Cavities

Each one of the 40 RF cavities is assumed to be represented by a superposition of 19 resonators with parameters as given in Table (1).[4] In practice, due to cavity construction errors and temperature variations, the resonant frequencies vary slightly from cell to cell, effectively broadening the resonant peaks. A crude way to take this effect into account is to make the following changes to the numbers in Table (1):

$$\begin{aligned} R_S &\rightarrow 40R_S && \text{for n=1} \\ R_S &\rightarrow \ \ 2R_S && \text{for n=2, ... ,19} \\ Q &\rightarrow \ \tfrac{1}{20}Q && \text{for n=2, ... ,19} \end{aligned}$$

and then use Eq.(1.4) for each mode. We evaluate the power loss for the fundamental mode (n=1) separately from the others (n=2, ... ,19), which are added together.

2.2 Inner Bellows

These are represented by a superposition of two resonators[5] with parameters given by Table (2), and the power is computed from Eq.(1.4). The values of the shunt impedances R_S take into account the total of 4.54×10^5 corrugations for the whole ring.

2.3 Beam Position Monitors and Kickers

The 850 BPMs and two kickers (one for injection plus one for abort) are represented by one single broad-band resonator as per Table (3).[6]

2.4 Sliding Contact Bellows

The impedance for the 5,000 units is represented[7] by the superposition of two resonators plus a frequency-independent term. The resonator parameters are presented in Table (4) (the values of R_S include the appropriate factor of 5,000.) The resonant contribution to the power is computed from Eq.(1.4). The constant part of the impedance is

$$\begin{aligned} Re[Z_{\|}(\omega)] =& \frac{Z_o}{2\pi} \log\left(\frac{b+\Delta}{b}\right) \\ \simeq& \frac{Z_o}{2\pi}\left(\frac{\Delta}{b}\right) \end{aligned} \tag{2.1}$$

where $Z_o = 4\pi/c \simeq 377\ \Omega$ is the impedance of the vacuum, $\Delta = 0.1$ mm is the gap size, and $b = 1.65$ cm is the pipe radius. From Eq.(1.2) we get the contribution to the power loss per unit,

$$P = M(N_B e f_o)^2 \frac{Z_o}{\sqrt{\pi}} \left(\frac{\Delta}{2b}\right)\left(\frac{R}{\sigma_z}\right) \tag{2.2}$$

which is then multiplied by 5,000.

2.5 Resistive Wall

Here the impedance is[8]

$$Z_{\|}(\omega) = (1-i)\frac{R}{b\sigma\delta(\omega)} \tag{2.3}$$

where $\sigma = 1.6 \times 10^{19}$ sec^{-1} is the cold copper conductivity, and

$$\delta(\omega) = \frac{c}{\sqrt{2\pi\sigma\omega}} \tag{2.4}$$

is its skin depth. From Eq.(1.2) we get

$$P = \Gamma(\tfrac{3}{4}) M (N_B e f_o)^2 Z_o \left(\frac{\delta_o}{2b}\right) \left(\frac{R}{\sigma_z}\right)^{3/2} \tag{2.5}$$

where $\delta_o \equiv \delta(\omega_o)$ and $\Gamma(\frac{3}{4}) \simeq 1.23$ is the gamma function.

2.6 Coherent Synchrotron Radiation

The power radiated by the bunches in a circular orbit of radius ρ in the absence of any shielding is[9]

$$P = \frac{\pi\Gamma(\frac{2}{3})}{3^{1/3}\Gamma(\frac{1}{3})} M (N_B e f_o)^2 Z_o \, \sigma_\phi^{-4/3} \tag{2.6}$$

where σ_ϕ is the angular *rms* bunch size. This expression must be modified to account for the shielding and for the fact the the orbit is not circular. This last modification is approximately accounted for by a factor ρ/R if we identify ρ to be the radius of curvature of the orbit in the dipole field magnets, $\rho = 10.1$ km (the factor $\rho/R \simeq 0.765$ can be thought of as the fraction of the orbit that is circular). Thus the appropriate expression is

$$P = 1.101 \; M (N_B e f_o)^2 Z_o \left(\frac{\sigma_z}{\rho}\right)^{-4/3} \times \left(\frac{\rho}{R}\right) \times \textit{shielding factor} \tag{2.7}$$

The *shielding factor* is given by[10]

$$\textit{shielding factor} = 3^{-1/6} \, 2^{-1/3} \frac{b}{(\rho\sigma_z^2)^{1/3}} \tag{2.8}$$

This last expression is in fact an upper bound since it is obtained from a very simplified geometry in which the shielding is provided by two infinite parallel metallic plates, which is less effective than a pipe.

3. Results and Comments

Results for resonant power loss are presented in Fig.(1). Here *RF(fund)* corresponds to the cavity's fundamental mode, *RF(higher)* is the sum of all higher modes contributions, *IB* corresponds to inner bellows, *SCB* to sliding contact bellows (resonant), and *BPMK* to beam position monitors plus kickers. Nonresonant losses are presented in Fig.(2), where *SCB* corresponds to sliding contact bellows (nonresonant), *RW* to resistive wall, and *CSR* to coherent synchrotron radiation. In all cases we have *not* made allowance for the change in the number of particles per bunch which must accompany a change in bunch length in order to maintain a constant luminosity. This would not change our numbers significantly.

ACKNOWLEDGEMENTS

I am grateful to Alex Chao for many discussions.

APPENDIX : RESONATOR PARAMETERS

Table 1. PEP cavity parameters[4]

mode n	f_R [GHz]	R_S [MΩ]	$Q \times 10^{-3}$
1	0.3589	11.84	42.9
2	0.5072	2.03	36.5
3	0.8302	0.025	44.4
4	0.8834	0.532	54.5
5	0.9261	1.482	35.8
6	1.1671	0.622	50.6
7	1.2157	0.332	61.9
8	1.4401	0.434	91.5
9	1.4474	0.555	60.7
10	1.5330	0.018	53.5
11	1.5860	0.238	70.8
12	1.6685	0.282	55.3
13	1.7377	0.298	56.3
14	1.8522	0.017	58.3
15	1.9589	0.302	77.8
16	2.065	0.794	71.5
17	2.077	0.130	72.1
18	2.153	0.046	83.6
19	2.290	0.116	68.9

Table 2. Inner bellows parameters

f_R [GHz]	R_S [MΩ]	Q
12.3	6.1	4
49.0	1.6	20

Table 3. BPM's plus kickers parameters

f_R [GHz]	R_S [MΩ]	Q
2.9	0.12	1

Table 4. Sliding contact bellows parameters

f_R [GHz]	R_S [MΩ]	Q
6.2	0.10	6
16	0.15	1

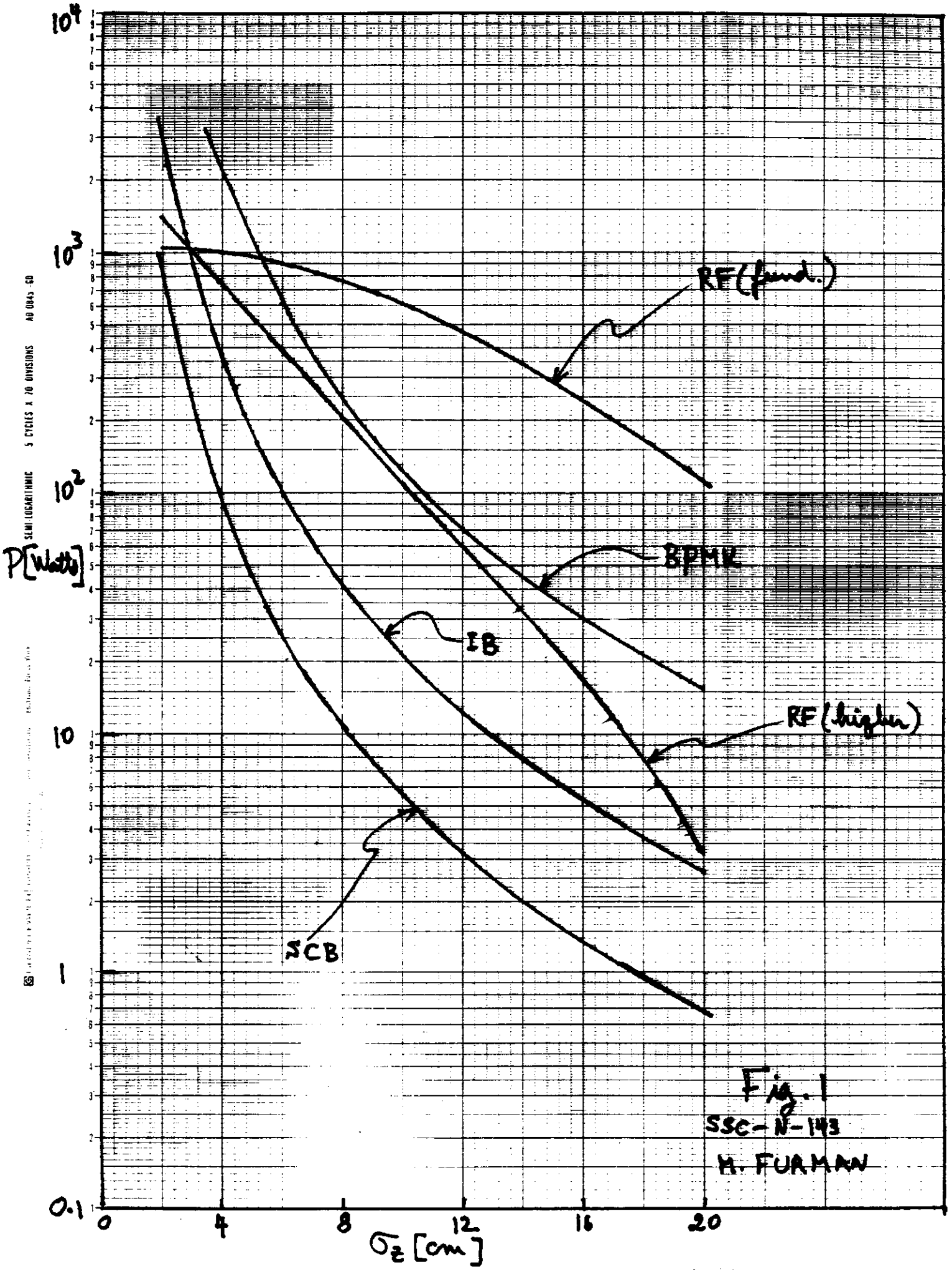

Fig. 1
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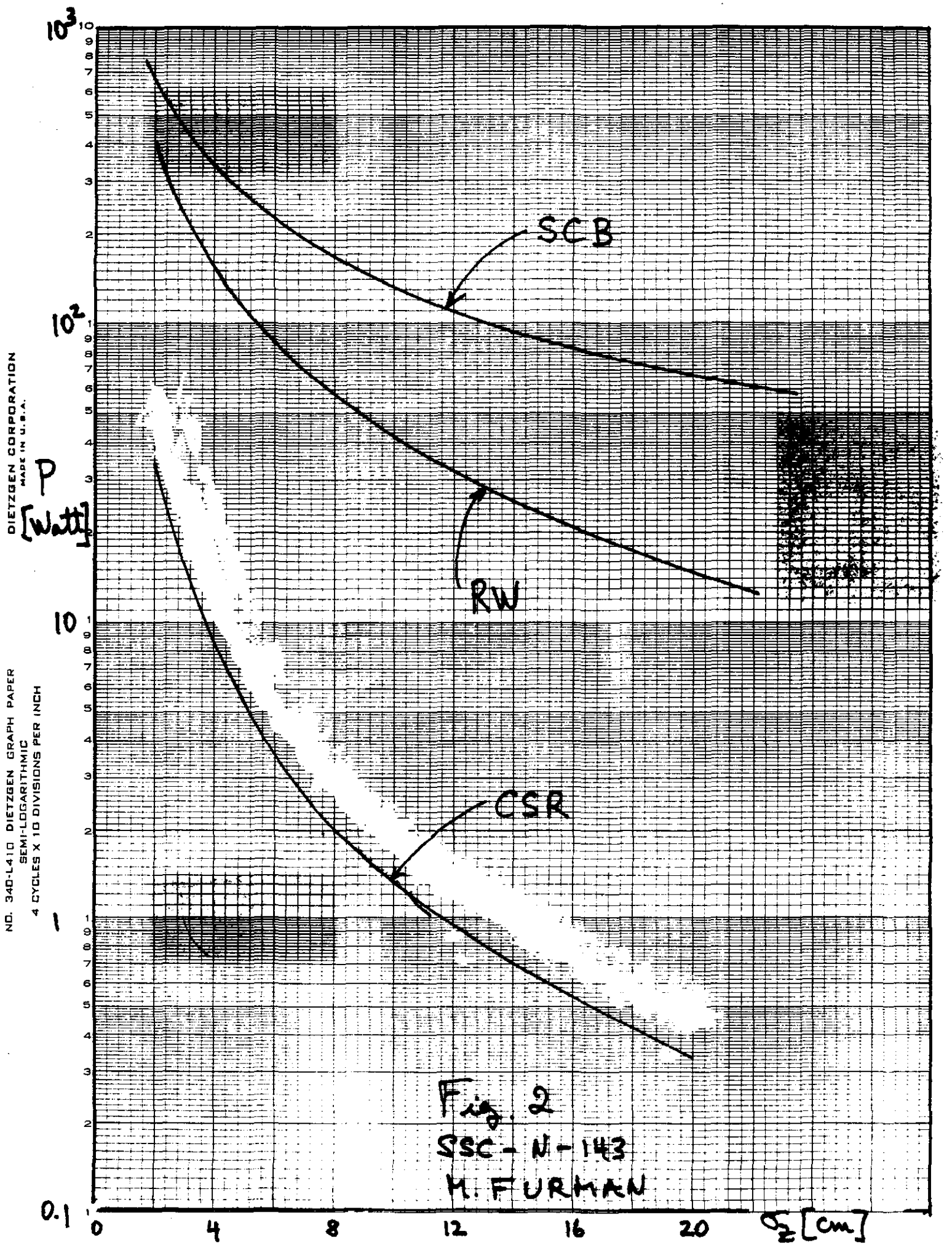

Fig. 2
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REFERENCES

1. Alex Chao, in *Physics of High Energy Particle Accelerators* (SLAC Summer School, 1982), ed. by M. Month, AIP Conf. Proc. no. 105, p.353.
2. M. A. Furman, SSC-N-142.
3. Handbook of Mathematical Functions, ed. by M. Abramowitz and I. A. Stegun, Dover(1965), p. 297.
4. These are PEP cavity modes found from URMEL; Z. D. Farkas and K. Bane, SSC-N-124.
5. K. Bane and R. Ruth, SSC-SR-1017 (1985); J. Bisognano and K. Y. Ng, *ibid.*; R. Ruth, in *Accelerator Physics Issues for a Superconducting Super Collider*, Ann Arbor, 1983, ed. by M. Tigner, Univ. of Michigan rep. UM HE 84-1; J. M. Wang and C. Pellegrini, proc. XI Int. Conf. on High Energy Accelerators, Geneva, CERN (1980); R. Ruth and J. M. Wang, IEEE Trans. Nucl. Sci., NS-**28**, 2405 (1981).
6. R. Shafer, SSC-SR-1017; E. Colton and T. S. Wang, *ibid.* and SSC-N-144.
7. K. Y. Ng, SSC-SR-1017.
8. J. D. Jackson, *Classical Electrodynamics*, 2nd. ed., J. Wiley & Sons, 1975.
9. L. I. Schiff, Rev. Sci. Inst., **17**, 6 (1946).
10. J. Schwinger, Phys. Rev. **75**, 1912 (1949); "On Radiation by Electrons in a Betatron", c.1945 (unpublished); J. S. Nodvick and D. S. Saxon, Phys. Rev. **96**, 180 (1954); M. A. Furman, SSC-N-178.

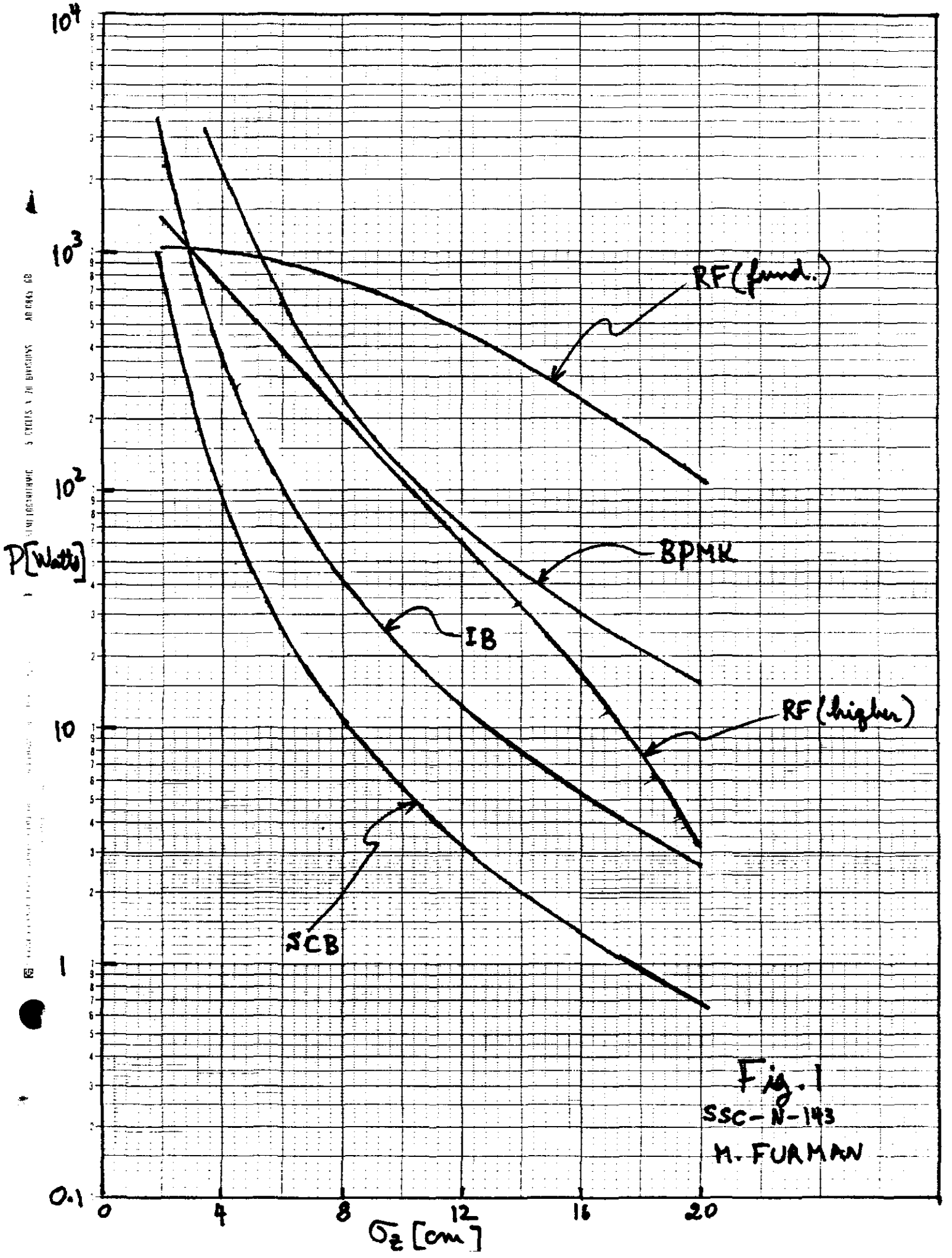

Fig. 1
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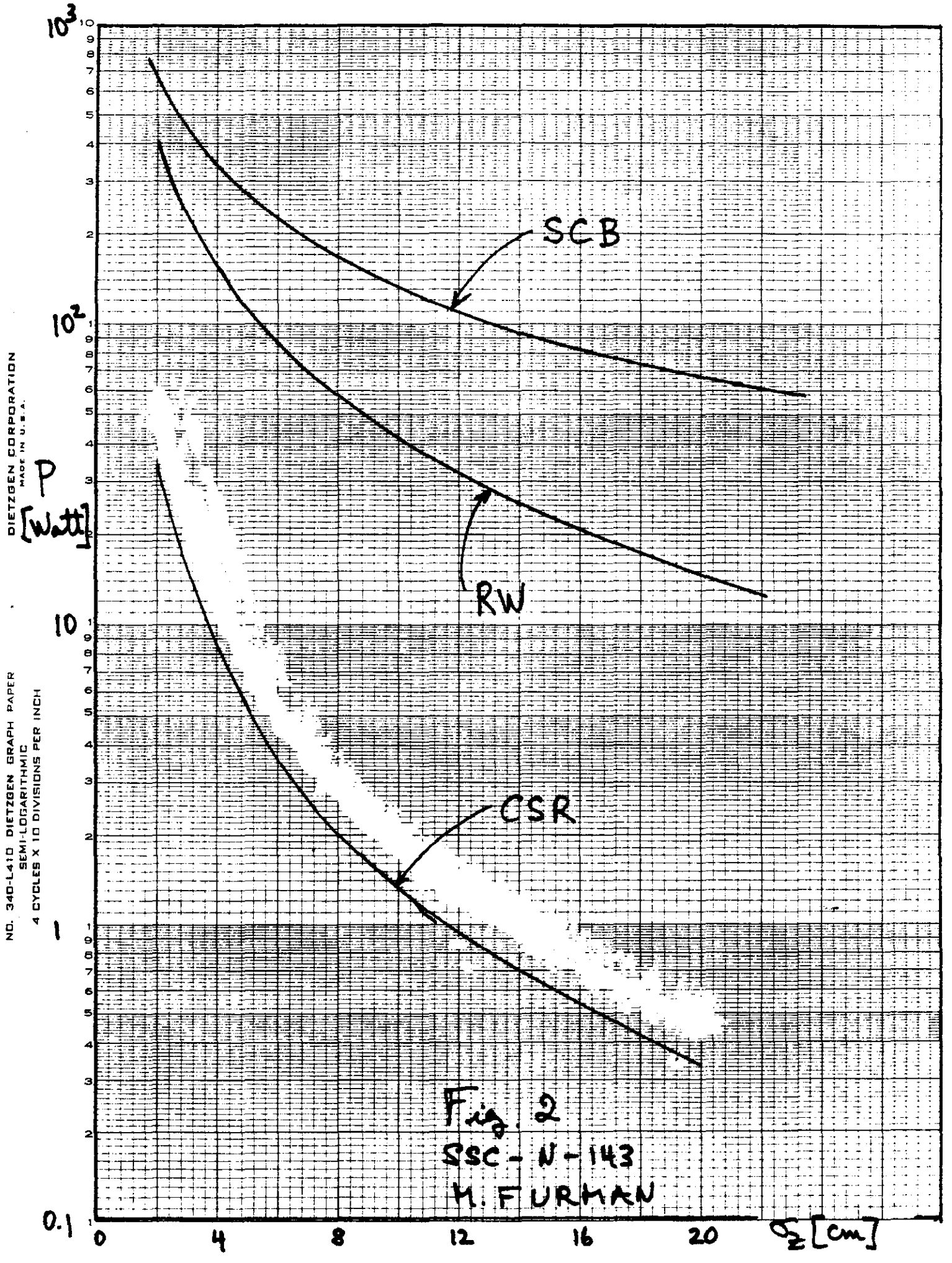

Fig. 2
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