SSC-N-141
28 February 1986

Smart Power Supplies

D. E. Groom

Jackson has recently pointed out that the reliability of power supplies in the SSC correction system becomes acceptable if each has a backup, i.e. if one or both of the paralleled pair operate[SSC-N-137]. In this note we generalize the analysis to the case where the failure of up to m out of a total of n paralleled units is acceptable. Our conclusion is that more complicated design logic leads to very much better system reliability—and might even be less expensive.

For example, the "power supply" might consist of eight 20-Ampere subunits, and the operation of at least five is necessary to meet the 100-Ampere requirement. In practice, we would expect a carefully engineered design in which only the more failure-prone subsystems were replicated, and that different parts of the system had different depths of redundancy. The analysis would be more complicated, but the general pattern can be learned from the simple case stated above.

Following Jackson's analysis, we define λ as the reciprocal of the mean time between failures (MTBF) at a given epoch. Then the probability of a given sub-unit working at a given time is given by

$$W = e^{-\lambda t} \quad .$$

It is also convenient to introduce the failure probability

$$F = 1 - W \quad .$$

The probability of no more than m failures after a time t is given by*

---

* It is interesting that this rather awful looking expression can be calculated using three lines of FORTRAN, using a variant of the ususal nested polynomial algorithm:

```
      P = W**(N-M)
      DO 2 J = M,1,-1
2     P = W**(N-J+1) + (N-J+1)*F*P/J
```

$$
\begin{aligned}
P &= [\text{Prob. of 0 failures}] + [\text{Prob. of 1 failure}] + \\
&\qquad\qquad \ldots + [\text{Prob. of } m \text{ failures}] \\
&= W^n + nW^{n-1}F + \ldots + \binom{n}{m} W^{n-m} F^n \\
&= \sum_{k=0}^{m} \binom{n}{m} W^{n-k} F^k \quad .
\end{aligned}
$$

Because P is close to unity, rounding errors can be avoided by the alternative calculation of the failure probability after time t:

$$
1 - P = \sum_{\ell=0}^{n-m-1} \binom{n}{\ell} F^{n-\ell} W^{\ell}
$$

In either case, we obtain the survival probability of one unit. For a total of N units, we must also compute the probability that all are operating after a given time:

$$
P_T = P^N
$$

Once again, because of rounding error difficulties it is easier to work with $1 - P$, making a binomial expansion to evaluate P^N.

To facilitate comparison with Jackson's results, we also choose 12,500 hr for the MTBF, or $\lambda = 8 \times 10^{-5} \text{hr}^{-1}$, and choose $N = 2500$. Results for "no more than half the subunits failed" are shown in Fig. 1. (Note the changed horizontal scale.) The improvement in having more combinatorials available is quite dramatic. For example: at $\lambda t = 40$, the probability that more than 1 out of 2 available subunits have failed is 2.5186×10^{-4}, whereas the probability that more than 2 out of 4 subunits have failed is only 1.5798×10^{-5}. For 2500 units, the system survival probabilies are 0.5326 and 0.9612 for the two cases. The difference comes about because there are 6 ways a unit can keep working with 2 failed subunits.

In Fig. 2 is shown the survival probability for ($\geq m$ out of 8) subunits working. The case (≥ 2 out of 8) is already an improvement over Jackson's (≥ 1 out of 2) for times shorter than a week or so, and (≥ 3 out of 8) provides about three times the overall MTBF as does (≥ 1 out of 2). If this reliability is "excessive," then cost savings can be made in the form of subunits which carry a smaller fraction of the total required power, less dependable subunits, etc.

The general conclusion is that deeper redundancy pays off very quickly. A "smart" design might not mean a cost increment, and the power supply is very much more reliable.

Fig. 1

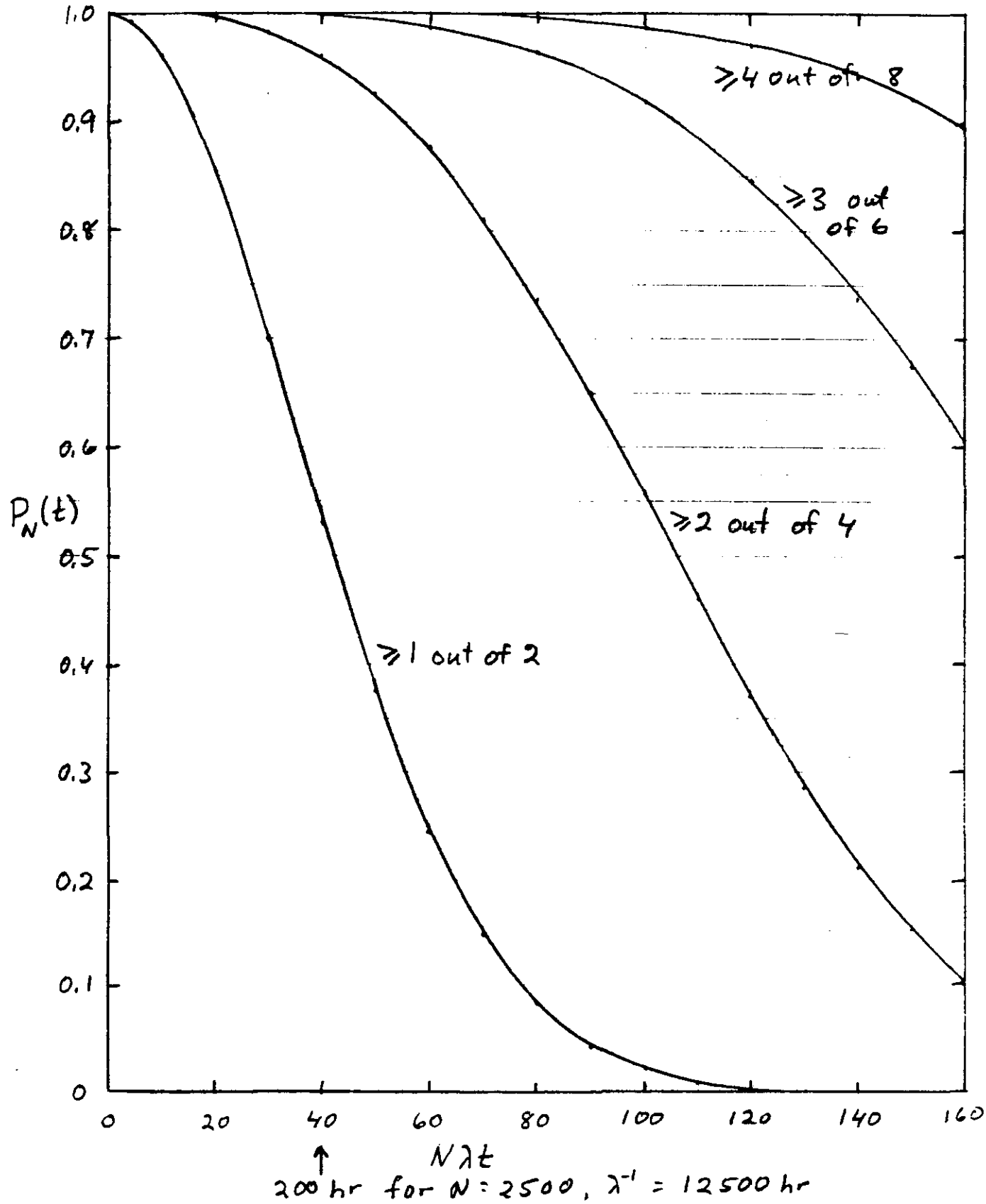
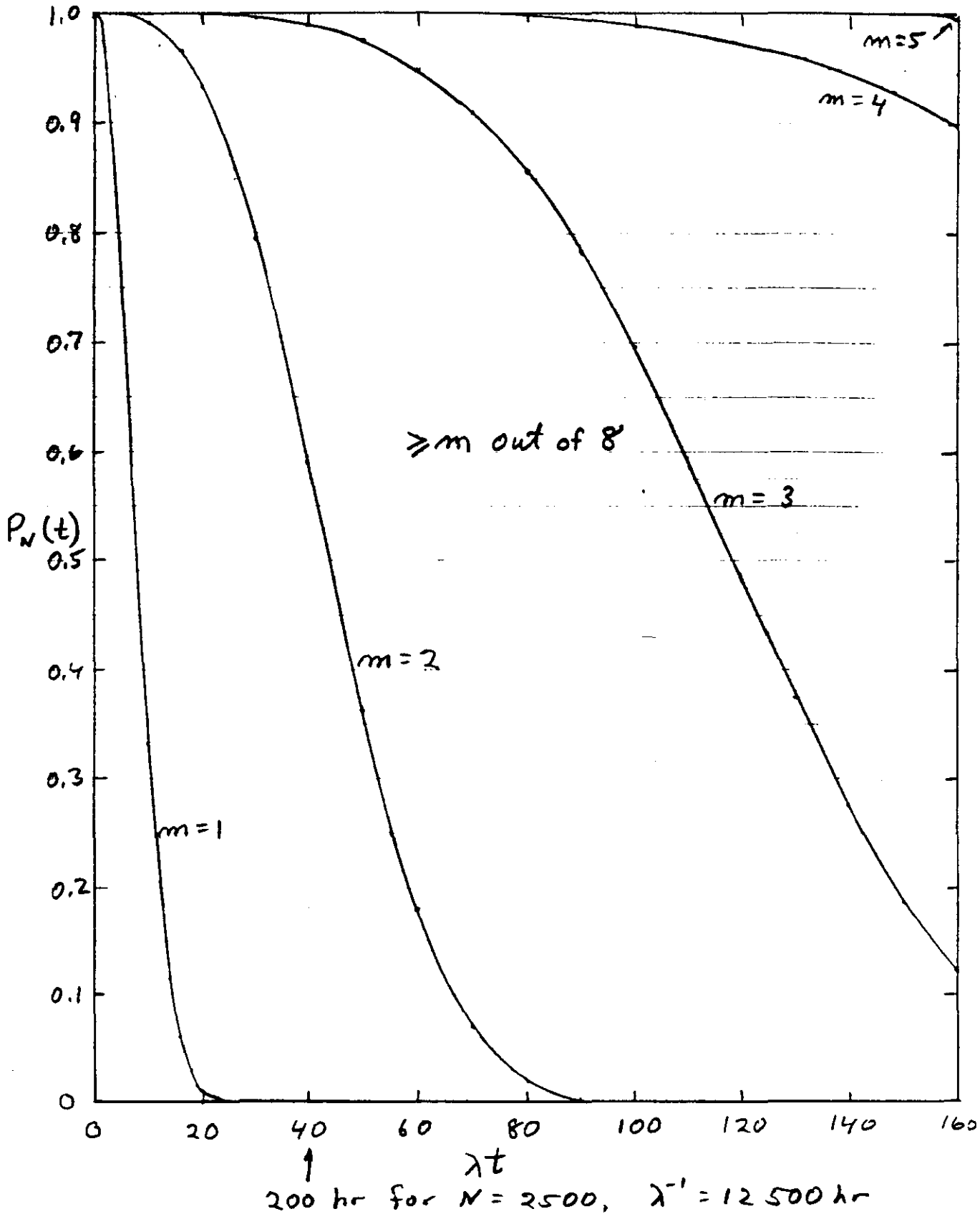

1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
$P_N(t)$
≥4 out of 8
≥3 out of 6
≥2 out of 4
≥1 out of 2
0
20
40
60
80
100
120
140
160
$N\lambda t$
200 hr for $N = 2500$, $\lambda^{-1} = 12500$ hr

Fig. 2


1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
$P_N(t)$
m=5
m=4
≥m out of 8
m=3
m=2
m=1
0
20
40
60
80
100
120
140
160
λt
200 hr for N = 2500, λ⁻¹ = 12 500 hr