Systematic Multipoles in the SSC

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The linear aperture of the SSC is seriously endangered by systematic multipole components in the SSC dipole magnets. These multipole components lead to amplitude and momentum-dependent tune shifts, and the contributions of the systematic components add coherently around the ring.

In lowest order, the tune shift due to a systematic multipole b_n in the SSC dipoles is given by*

$$\Delta\nu_x = \bar{\beta}_x \, b_n < \cos\phi \, (\eta\delta + A_\beta \cos\phi)^n > / A_\beta$$

In this expression only 1-D horizontal motion is included. A_β is the betataron oscillation amplitude, η is the dispersion function, $\delta = \Delta p/p$ and ϕ is the betatron phase. Averaging over betatron phases and amplitudes leads to the amplitude and momentum-dependent tune-shift formulas listed in Table 1.

Stability in the SSC requires that the beam be confined within a tune space free of tenth-order resonances, which at typical working points implies $\Delta\nu_{Total} < 0.02$. The head-on and long-range beam-beam tune shifts in the SSC sum to numbers of that magnitude; tune-shifts due to other effects must be made much smaller. Thus, the criteria on amplitude and momentum-dependent tune shifts is proposed.

$$\Delta\nu \lesssim 0.005$$

at $A_\beta = 0.41$ cm (at $\beta = \bar{\beta} = 220$ m), and $\delta p/p = \pm 10^{-3}$. For the CDG lattice, $\bar{\eta}$ (average) $\cong 3.0$ m.

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* A.W. Chao, private communication.

The symmetry of the SSC dipoles permits the even multipole coefficients b_{2n} to be non-zero. The currently estimated values of the multipoles in the NC515 magnet design with 5μ filaments are shown in Table 2. The tune shifts are calculated following equation 1 using the expressions in Table 1. The results, displayed in Table 2, are expressed as tolerances on the multipole components.

The b_2, b_4 coefficients anticipated for the NC515 dipole are far larger than the desired criteria. The b_2 and b_4 multipole coefficients are dominated by persistent currents. Both are expected to be reduced by a factor of 10 by bore-tube correction windings. After reduction to $b_2 \lesssim 0.5 \times 10^{-4}$ by the correction windings, the lowest order tune shifts $\Delta\nu_x$ and $\Delta\nu_y$ can be adequately reduced to zero by the chromaticity-correction sextupoles without serious loss of aperture from higher-order effects.

The bore-tube windings are at present the only correction provided in the SSC for decapole effects. Figure 1A shows the linear aperture with b_4 uncorrected ($b_4 = 0.3 \times 10^{-4}$ cm^{-4}) and with b_4 corrected to 0.03×10^{-4}. In the uncorrected case the linear aperture is severly restricted. Correction to $b_4 = 0.03 \times 10^{-4}$ does not quite reach the desired criteria but results in only limited aperture loss at large A_β and large δ. Once operation begins, b_4 can empirically be compensated. A net value less than 0.03×10^{-4} should be possible.

Similarly the anticipated value of b_6 (0.07×10^{-4} cm^{-6}) does not quite reach the desired criteria (0.04×10^{-4}), but the loss of linear aperture is confined to large δ and A_β. (See figure 1B.)

Higher-order multipoles, b_8, b_{10}, etc. are expected to meet the desired aperture criteria. With correction of b_2, b_4 the SSC magnets should allow the necessary linear aperture.

We note here that the above analysis is 1-D with $A_y = 0$. To first approximation 2-D motion can be included by replacing A_β by $\sqrt{A_x^2 + A_y^2}$. The aperture at $A_x = 0.41$ cm (0.5 cm at $\beta max = 330$ m) and $A_y = 0$ corresponds to $\bar{A}_x = \bar{A}_y = 0.29$ cm (0.35 cm at $\beta = 330$ m).

Reduction in tune shift can be obtained by changes in the CDR lattice, changing $\bar{\beta}$, $\bar{\eta}$. As the simplest such change we consider changing $\bar{\eta}$ from 3.0 m to 1.5 m, while keeping $\bar{\beta}$ at 220 m, corresponding roughly to increasing the phase shift/cell from 60° to 90°. The resulting changes in the acceptable $\langle b_{2n} \rangle$ criteria are shown in Table 3. The b_6, b_8, b_{10}, etc. safety factors are greatly increased. However bore-tube correction of b_2 and b_4 is still necessary.

Lattice modification requires consideration of an ensemble of aperture and cost considerations not included in this note, and could not be completed before the CDR deadline; however, it should be considered in post-CDR optimization.

Table 1

Multipole	Expressions for $\Delta\nu_x$
2	$\beta b_2\ \eta\delta$
4	$\beta b_4 \left(2\eta^3\delta^3 + \frac{3A_\beta^2\eta\delta}{2}\right)$
6	$\beta b_6 \left(3\eta^5\delta^5 + \frac{15}{2}\ A_\beta^2\epsilon^3\delta^3 + \frac{15}{8}\ A_\beta^4\eta\delta\right)$
8	$\beta b_8 \left(4\eta^7\delta^7 + 21A_\beta^2\eta^5\delta^5 + \frac{35}{2}\ A_\beta^4\eta^3\delta^3 + \frac{35}{16}\ A_\beta^6\eta\delta\right)$
10	$\beta b_{10}\left(5\eta^9\delta^9 + 45A_\beta^2\eta^7\delta^7 + \frac{315}{4}\ A_\beta^4\eta^5\delta^5 + \frac{525}{16}\ A_\beta^6\eta^3\delta^3 + \frac{315}{128}\ A_\beta^8\eta\delta\right)$

Table 2
SSC Dipole Magnet Multipole Tolerances

Multipole	NC515 Estimate	Multipole Tolerances (CDR)
b_2	-4.7×10^{-4}	0.0076×10^{-4}
b_4	0.3×10^{-4}	0.018×10^{-4}
b_6	0.07×10^{-4}	0.040×10^{-4}
b_8	0.01×10^{-4}	0.088×10^{-4}
b_{10}	0.011×10^{-4}	0.29×10^{-4}

Table 3
Changes in Systematic Multipole Tolerances
with Lattice Modification

Tolerance Criteria $\left\{ \begin{array}{l} \eta \rightarrow 1.5 \text{ m} \\ \beta = 220 \text{ m} \end{array} \right\}$

b_2	0.015×10^{-4}
b_4	0.051×10^{-4}
b_6	0.18×10^{-4}
b_8	0.65×10^{-4}
b_{10}	2.27×10^{-4}

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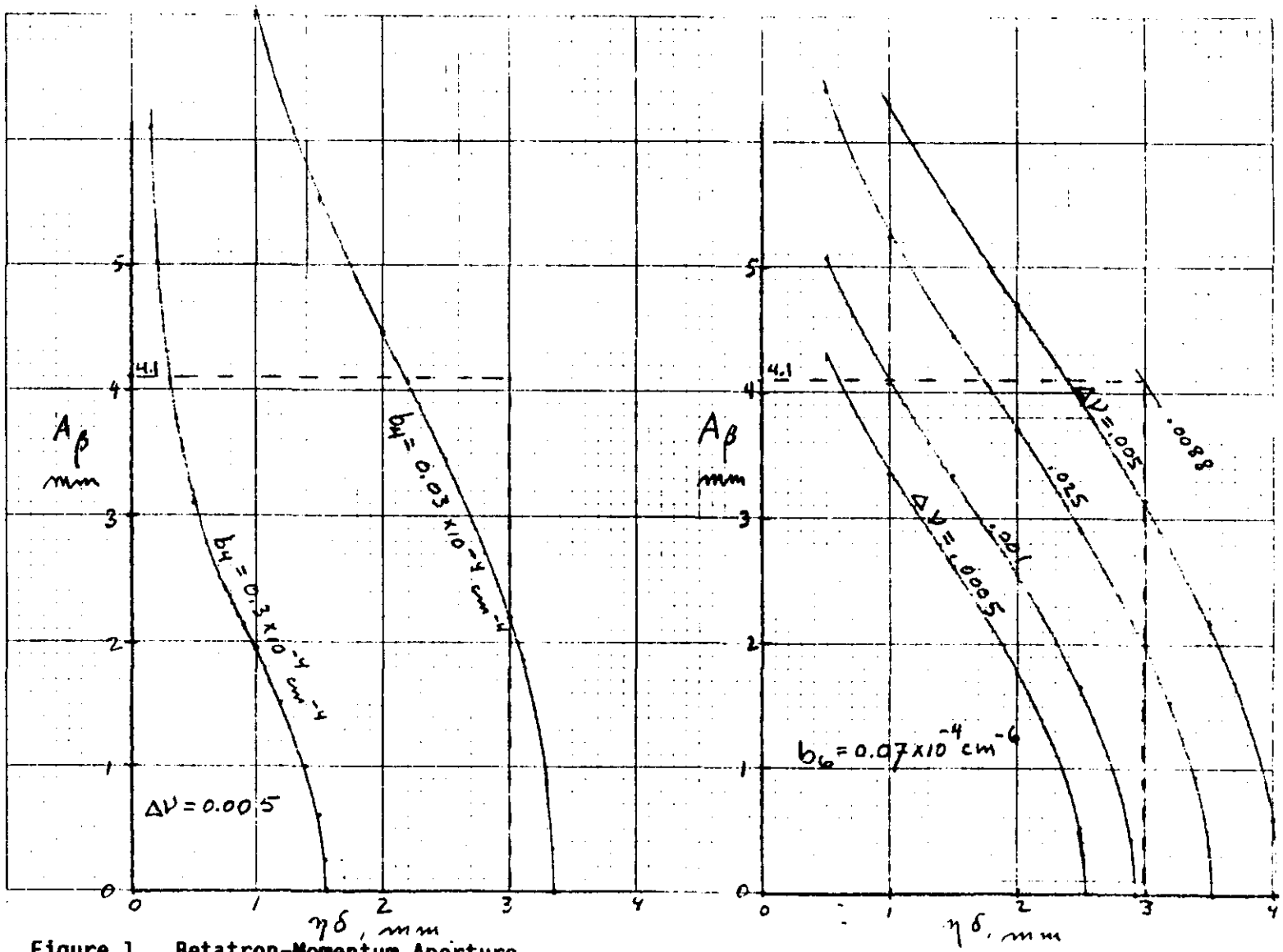

Figure 1. Betatron-Momentum Aperture

A. For anticipated by ($0.3 \times 10^{-4} cm^{-4}$) and for b_4 90% corrected, $\Delta\nu$ = .005 tune shift criterion.

B. For anticipated b_6 ($0.07 \times 10^{-4} cm^{-6}$) and several values of tune shift.

The curves in each figure differ only in the ratio $\Delta\nu/b_n$, so that the parameter variations can be interpreted in terms either of changing b_n or of changing $\Delta\nu$.