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SSC Technical Note No. 46

Cooldown of SSC Magnets

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SUMMARY

Cool-down of magnets represents at least a two-dimensional problem involving time t and location x along a series of magnets. Partial differential equations 14 and 15 state the problem in its simplest form. The problem becomes considerably more complex if one wishes to take into account all relevant physical properties of the helium cooling fluid and heat diffusion along individual magnets. This is quite possible but requires a sizeable computer program. In practice, the helium pressure is not allowed to change very much as the helium passes through the system, and an estimate shows that heat diffusion times are long compared to cool-down times. Nevertheless, one of the calculations discussed here shows that the cooldown time for an individual magnet does not depend strongly on heat diffusion along the magnet material.

In order to avoid large stresses in the stainless steel magnet support shell at the magnet entrance, it is advisable to keep the temperature difference between entering helium and magnet entrance at $\Delta T \leq 50^{\circ}$ K. ΔT can be increased as the magnet temperature decreases because thermal expansivities decrease and material strengths increase. Keeping the helium temperature constant during a given stage of the cool-down, of course, increases the cooldown time because the magnet temperature approaches the helium temperature approximately exponentially. In one of the calculations discussed here the helium entrance temperature is introduced as a function of the magnet temperature.

From a stepwise solution of equations 14 and 15 temperature gradients along single magnets or long strings of magnets are obtained. These calculations assume constant helium entrance temperature and therefore overestimate cooldown times somewhat. Since heat exchange increases with the difference between magnet and helium temperatures and since the lowest helium operating temperature is fixed, requiring the mentioned final exponential approach of the magnet temperature, cooldown times do not depend strongly on a particular chosen target temperature.

For a mass flow of M = 70 g/sec we find a cooldown time of 7 hours from room temperature to ~ 100° K for a single dipole. Thereafter the particular operating characteristics of the refrigerator become soon dominant. Doubling M would approximately halve this time. For cool-down from room temperature to 170° K, the maximum temperature gradient is found at the magnet entrance, when cooldown is completed there, after 1.2 hours: $dT/dx \approx 60^\circ$ K/m. Cool-down of 17 cells between refrigerators is found to take 21 days, but all cool-down times depend on the details of programming M and the helium entrance temperature.

Considerations of the cool-down (and also warm-up) of superconducting magnets can provide predictions of the time required to cool a series of magnets from room to operating temperature, or to cool a single magnet for testing. One can also obtain information on expected temperature gradients, of importance especially in a single magnet or the first magnet of a series.

To be taken into account are (1) the thermal properties and geometry of the magnet and (2) the cooling fluid (helium). Besides heat exchange between magnet components and helium, heat diffusion along the magnet may be of importance, which could equalize longitudinal gradients. Assume for the moment that heat diffusion along yoke and coil is very good. As a consequence, the magnet temperature would be almost constant longitudinally. We call

ρ_1	=	average yoke iron/coil density.
a_1	=	magnet cross section.
$c_1(T)$	=	average heat capacity of magnet material in temperature range considered.
T_a	=	initial magnet temperature.
$\rho_o(T,p)$	=	helium density at temperature T and given pressure p.
a_o	=	total cross section of cooling passages.
$c_o(T,p)$	=	helium heat capacity.
ℓ	=	magnet length.
A_o	=	total surface area for heat exchange in cooling passages.
T_{oe}	=	entrance temperature of helium.
$T_o(t)$	=	average temperature of helium in magnet.
h_o	=	heat exchange function.
v	=	average helium velocity.

Then, during time element dt

$$\rho_o a_o v c_o (T_o - T_{oe}) dt = - \rho_1 a_1 c_1 \ell dT \tag{1}$$

$$\rho_o a_o v c_o (T_o - T_{oe}) = h_o A_o \ell (T - T_o) \tag{2}$$

Using only average values between T_a and T_{oe} for c_1 and ρ_o (c_o is almost constant over most of the cool-down range) and solving eqs. 1 and 2, one obtains simply

$$T_o = T_{oe} + \frac{\alpha}{1 + \alpha} \left[T - T_{oe})\right] \tag{3}$$

$$T = \left[T_a - T_{oe}\right] e^{-\beta t} + T_{oe} \tag{4}$$

where

$$\alpha = \frac{K_2 \ell}{v} = K_2 \rho_o a_o \ \ell \ / \ M$$

with $M = \rho_o a_o v$ for the total helium mass flow,

$$\beta = K_1/(1 + a)$$

$$K_1 = \frac{h_o A_o}{\rho_1 a_1 c_1} \tag{5}$$

$$K_2 = \frac{h_o A_o}{\rho_o a_o c_o} \tag{6}$$

h_o is given by

$$h_o = 0.023 \ \frac{k_o}{4r_h} \left[\frac{4r_h \ M}{\mu_o a_o}\right]^{0.8} \left[\frac{\mu_o c_o}{k_o}\right]^{0.4} \tag{7}$$

where $k_o(T_o, p)$ = helium heat conductivity

$\mu_o(T_o, p)$ = helium viscosity

r_h = hydraulic radius of cooling passage (= 1/2 of radius if circular)

Rewrite eq. 7:

$$h_o = \frac{0.023}{4} \phi \, r_h^{-0.2} \left(\frac{4M}{a_o}\right)^{0.8}$$

with

$$\phi = k_o^{0.6} \left(\frac{c_o}{\mu_o}\right)^{0.4}$$

Using the data given in the NBS Table on thermophysical properties of helium, one finds that $\phi \approx$ constant = 1.16 for the range of temperatures and pressures of interest here. For the SSC dipoles:

a_1	=	602 cm^2
c_1	≈	0.4 Joule/g ^oK (between 170 and 293° K)
ρ_1	=	8 g/cm^3
a_o	=	26.8 cm^2
r_h	=	0.73 cm
A_o	=	36.7 cm^2/cm length
c_o	=	5.2 J/g^o K
ρ_o	=	3 x 10^{-3} g/cm^3
ℓ	=	1660 cm

Thus

K_1	=	3.04 x 10^{-5} $M^{0.8}$
K_2	=	0.14 x $M^{0.8}$
v	=	12.4 M

We consider presently only the temperature range between T_a = 293 and T_{oe} = 170° K because (a) cooldown proceeds most slowly here since magnet material heat capacities decrease more and more rapidly as temperature decreases and (b) temperature gradients are most important at the higher temperatures because thermal expansivities are largest here. For a range between 170 and 80° K, new averages for c_1 and ρ_o should be used. Using M = 70 g/sec for the helium mass flow, Table I results:

Table I

t (hours)	0	1.4	2.8	4.2	5.6	6.9	9.7	13.9
T (^oK)	293	244	215	197	186	180	174	170.8
T_o (^oK)	280	236	210	194	184	179	173	170.7

Obviously, cool-down proceeds (exponentially) reasonably fast for the first few hours, but then slows down because heat exchange depends on $(T - T_o)$ (see eq. 2).

Applying an initial temperature difference of $(T_a - T_{oe}) = (293 - 170)$, as used for the Table, at the start is excessive because of the large stresses in the yoke support shell that would be encountered at the magnet inlet. A value of $(T_a - T_{oe}) = 50^o$ K could be tolerated, which would result in an initial stress of 24 kpsi in the shell. But now the cooldown would take an even longer time. At lower temperatures large magnet entrance temperature differences can be tolerated. Thus T_{oe} should be decreased in time ("programmed") so that reasonable temperature differences at the magnet entrance can be maintained. Thus, in this simplified calculation, we shall prescribe that

$$T - T_{oe} = g(T) \tag{8}$$

and we shall use a linear function for g(T):

$$g(T) = \eta + \zeta T \tag{9}$$

where

$$\eta = \frac{g_b T_a - g_a T_b}{T_a - T_b} = 118.8$$

$$\zeta = \frac{g_a - g_b}{T_a - T_b} = -0.235$$

with

$$g_a(T_a) = T_a - T_{oe} = 50^o \text{ K}$$
$$g_b(T_b) = T_b - T_{oe} = 100^o \text{ K}$$
$$T_a = 293^o \text{ K}$$
$$T_b = 80^o \text{ K}$$

From equations 1, 2, and 8 we must now find $T(t)$, $T_o(t)$, and $T_{oe}(t)$. We shall now also take into account that the magnet heat capacity $c_1(T)$ decreases with temperature. Within 293 and 80° K one can obtain an adequate fit with

$$c_1(T) = \alpha' + \beta' T + \gamma' T^2 \tag{10}$$

where

$$\alpha' = -0.105$$
$$\beta' = 3.76 \times 10^{-3}$$
$$\gamma' = -6.43 \times 10^{-6}$$

Eqs. 1, 2, 8 result in

$$T_o - T_{oe} = \frac{\alpha}{1 + \alpha} g(T) \tag{11}$$

$$\int_T^{293} \frac{c_1(T)}{g(T)} dT = 0.4\beta t \tag{12}$$

The factor of 0.4 in eq. 12 is due to the fact that K_1 was calculated for a fixed value of $c_1 = 0.4$. Using eqs. 9 and 10 (or any other expressions or graphs), eq. 12 can be integrated. The results are displayed in Fig. 1 or can be expressed by

$$0.4\ \beta t = A\ \ell n(g(T)) + Bg(T) + C(g(T))^2 \tag{13}$$

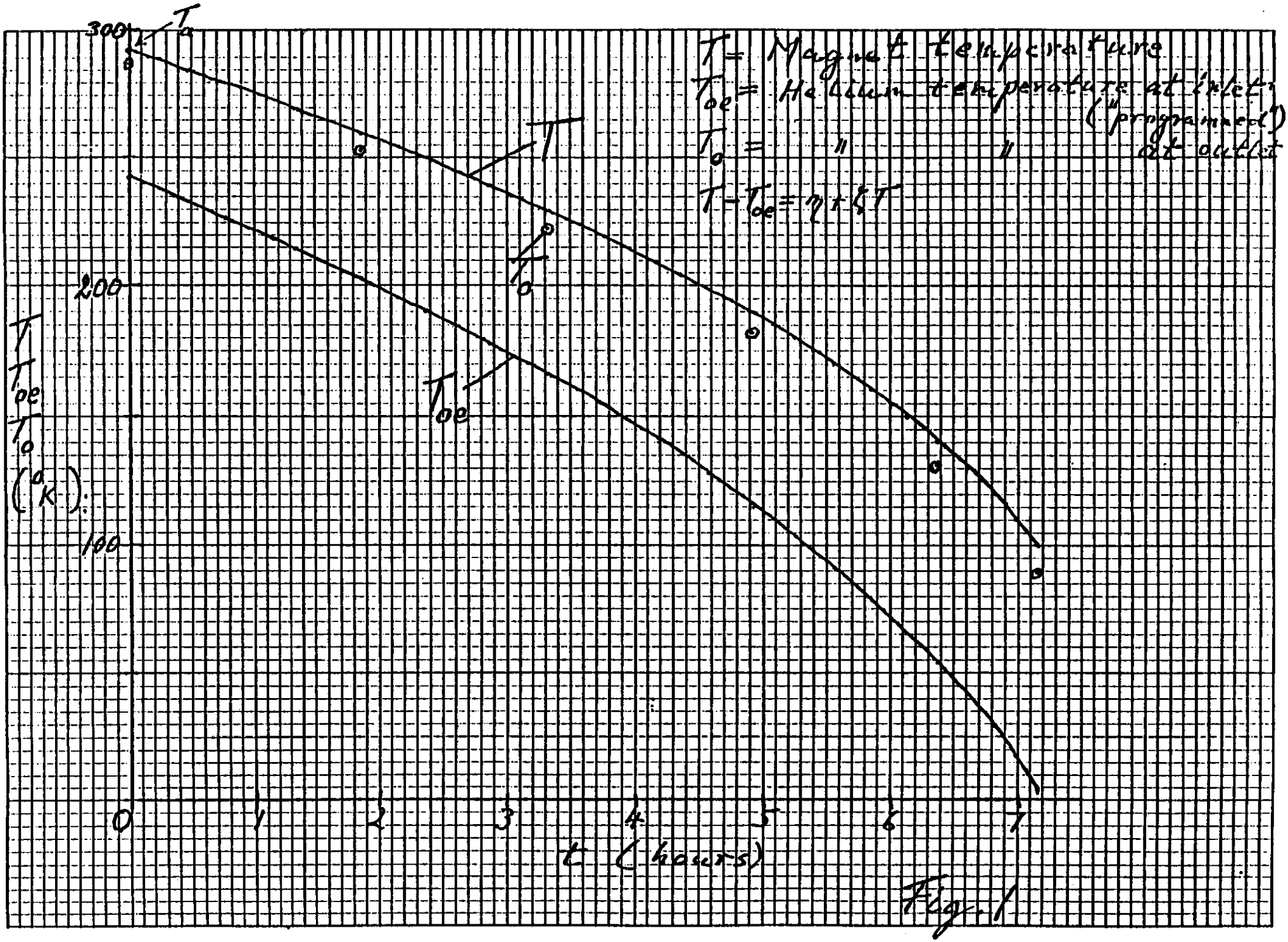
T = Magnet temperature
T_{oe} = Helium temperature at inlet (programmed)
T_o = " " at outlet
$T - T_{oe} = \eta \times \delta T$
300
200
100
0
1
2
3
4
5
6
7
t (hours)
T
T_o
T_{oe}
T, T_{oe}, T_o (°K)

Fig. 1

$A = -0.652$

$B = -0.0496$

$C = 2.48 \times 10^{-4}$

Cool-down of a dipole from room temperature to 100° K, according to this calculation, is expected to be accomplished in about 7 hours if T_{oe} proceeds from 243 to about 5° K. Average helium temperature T_o would be $\leq 11^{\circ}$ K less than T throughout the cool-down.

Below $T = 100^{\circ}$ K, details of operation of the helium refrigerator become dominant if a large value for g(T) is to be maintained. (Obviously $g(T) = T - T_{oe}$ must decrease now since $T_{oe} > 0$.) On the other hand, $c_1(T)$ continues to decrease rapidly. According to this simple calculation, cool-down to magnet operating temperature should be completed in 8 to 9 hours.

So far we have assumed that heat diffusion along the magnet is so good that T varies very little with location x. We can test this assumption by means of an expression for the diffusion time

$$\tau = \frac{x^2 \rho_1 c_1}{4\, k_1}$$

in which a diffusion wave would arrive at x if it started at $x = 0$. k_1 is the heat conductivity along the magnet. $k_1 \approx 1$ W/cm$^{\circ}$ K if the yoke were made of solid iron. Since it is laminated, we can expect k_1 to be considerably smaller. For the temperature range of interest we shall again use $c_1 = 0.4$. Thus

$$\tau = 2.2 \times 10^{-4}\, x^2 \text{ (hours)}$$

For $x = 1/4\, \ell = 415$ cm

$$\tau = 38 \text{ hours}$$

or for $x = \ell$

$$\tau = 606 \text{ hours}$$

These times are much longer than the cool-down times calculated so far. About 60% of the coil consists of copper with $k_1 = 4$:

$$\tau = 1.0 \times 10^{-4}\ x^2$$

which still results in large values for τ. Besides, the total copper cross section is very small compared to a_1.

Without heat diffusion T depends on x as well as t. Equations 1 and 2 now must be written

$$\rho_1 a_1 c_1 \frac{\partial T}{\partial t} = \rho_o a_o c_o v \frac{dT_o}{dx} = - h_o A_o \left(T - T_o\right) \tag{14}$$

and since the temperature of the helium flow depends on x and t, $\left(v = \frac{dx}{dt}\right)$,

$$\rho_o a_o c_o v \frac{dT_o}{dx} = \rho_o a_o c_o \left[\frac{\partial T_o}{\partial t} + v\frac{\partial T_o}{\partial x}\right] = h_o A_o\left(T - T_o\right) \tag{15}$$

A solution for these two partial differential equations can be obtained if one sets

$$T = S(x,t')\ e^{-xK_2/v}\ e^{-K_1(t - x/v)} \tag{16}$$

$$\frac{\partial^2 S}{\partial_x \partial t'} - \frac{K_1 K_2}{v} S = 0 \tag{17}$$

$$t' = t - x/v$$

A derivation for equation 17 and its solution was given previously, in ISA Technical Note No. 17. The solution is obtained by means of the Bessel function $J_o(iz)$, where $z = 2\left[\frac{K_1 K_2}{v}\ x\ t'\right]^{1/2}$, $i = (-1)^{1/2}$. Below we shall state results for $z > 0.1$. First, we give results for $0 \le z \le 0.1$ for which $J_o(iz) \approx 1$. Then

$$T = T_a - \left(T_a - T_{oe}\right) e^{-xK_2/v} \left(1 - e^{-K_1(t - x/v)}\right) \tag{18}$$

which has good validity only if

$$t_{min} = \frac{x}{v} \leq t \leq \frac{7280}{x} + \frac{x}{v} = t_{max} \tag{19}$$

Thus, at the magnet entrance, $x = 0$, eq. 18 is always valid, but for $x > 0$ the allowed time-span decreases fast. t_{min} is the time when a helium element arrives at x, having entered the magnet at $t = 0$. Equation 18 allows us also to calculate temperature gradients along the magnet near the entrance. Table II gives results for $x = 0$, $T_{oe} = 170^{o}$ K.

Table II

t(sec)	0	100	200	400	1000	2000	3000	4000	5000
T(^{o}K)	293	283	273	256	220	190	178	173	171
dT/dx ^{o}K/cm	0	0.05	0.10	0.18	0.34	0.50	0.56	0.58	0.59

Thus, in 1.4 hours the magnet entrance would be cooled down to almost 170° K, if T_{oe} were held constant. The temperature gradient near $x = 0$ reaches 59° K/meter, starting out very small.

For $z > 0.1$, or $t > t_{max}$, $x > 0$, we shall here simply use some of the results in the above-given reference. We list for the arrival time of the diffusion wave at x,

$$t_w = \frac{x}{v_c} \tag{20}$$

where

$$v_c = \frac{Mc_o}{\rho_1 a_1 c_1 + \rho_o a_o c_o} \tag{21}$$

is the diffusion wave velocity.

$$t_c = \left[\left(\frac{K_2 x}{v}\right)^{1/2} + 2\right]^2 \frac{1}{K_1} + \frac{x}{v} \tag{22}$$

is the time when cool-down at x is completed within 0.5%. Here the (usually negligible) term x/v is the arrival time of the helium at x, having started at x = 0, t = 0. The time interval during which cool-down proceeds vigorously at x is

$$\Delta t_c = \frac{4}{K_1}\left(\frac{K_2 x}{v}\right)^{1/2} \tag{23}$$

The total length of the diffusion wave, when it arrives at x = L, at time

$t_w = \frac{L}{v_c}$, is

$$\lambda = L - \left[\left(\frac{LK_1}{v_c}\right)^{1/2} - 2\right]^2 \frac{v}{K_2} \tag{24}$$

At $x = L - \lambda$, cool-down is thus just completed. Therefore $t_w(L) = t_c(L - \lambda)$, having neglected the term x/v. The term in brackets in eq. 24 must be >0. Therefore

$$L \geq \frac{4v_c}{K_1}$$

For v_c we obtain

$$v_c = 0.189 \text{ cm/sec}$$

Therefore

$$L \geq 830 \text{ cm}$$

Indeed, when L = 830, λ = 830 and therefore $t_w(830)$ (= 1.2 hours) must be equal to $t_c(0)$ ($\approx$ 1.2 hours, see Table II). If L were less than 830 cm,

cool-down at x = 0 could not yet have been completed. Table III shows results obtained with eqs. 20 to 24. Here, again, M = 70 g/sec, T_a = 293° K, T_{oe} = 170° K. v_c = 0.19 cm/sec. Also listed are the average gradient, $(T_a - T_{oe})/\lambda$, and the magnet length $(L - \lambda)$ that is completely cooled down at time t_c. Finally, n is the number of dipoles cooling down.

Table III

x, L (m)	t_w (hours)	t_c	Δt_c	λ (m)	$\frac{T_a - T_{oe}}{\lambda}$ (°K/m)	$L - \lambda$ (m)	n
8.3	1.2	4.9	2.4	8.3	14.8	0	1/2
16.6	2.4	7.1	3.5	15.2	8.1	1.4	1
33.2	4.9	11.0	4.9	24.9	4.9	8.3	2
49.8	7.3	14.6	6.0	32.4	3.8	17.4	3
66.4	9.8	17.9	6.9	38.8	3.2	27.6	4
83.	12.2	21.2	7.7	44.3	2.8	38.7	5
166	24.4	36.6	10.9	66.2	1.9	99.8	10
332	48.8	65.6	15.5	97.4	1.3	235	20
498	73.2	93.6	18.9	121	1.0	377	30
830	122	148	24.4	160	0.8	670	50
1660	244	280	34.6	231	0.5	1429	100
2490	366	411	42.3	285	0.4	2205	150
3104	456	506	47.3	321	0.4	2783	187

For $x = 8.3$ m the average temperature gradient between $x = 0$ and 8.3 m is 14.8° K/m, in Table III. For $x = 0$ we found in Table II a gradient of $\sim 58^{\circ}$ K/m at $t \approx 1.2$ hours, meaning that the temperature distribution begins to rise more steeply at $x = 0$. This is to be expected.

The last row of Table III gives the approximate length of all magnets and other hardware between refrigerators (17 cells). Complete cool-down time t_c is 506 hours, or 21 days for $M = 70$ and $T_{oe} = 170$. This is an overestimate since T_{oe} would not be kept constant during cool-down until operating temperature is reached. Doubling M would approximately halve t_c. Considerations similar to those leading to Figure 1 must be introduced. If M and T_{oe} are given as functions of T or t, the calculations discussed here can be used to give more accurate estimates.

Still better accuracy for cooldown calculations can be obtained by solving partial differential equations 14 and 15 with available computer programs, also introducing all relevant temperature dependences for ρ, ρ_o, c_o, etc. The best, and probably unnecessary, accuracy can be obtained by also introducing all the relevant helium properties in more detail, requiring more equations to be solved, as mentioned earlier in this Note.

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