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PARTITION NUMBER ERRORS IN THE SSC

Introduction

The SSC is the first proton collider in which the betatron and synchrotron damping times, due to synchrotron radiation, will be comparable with the storage time. They are given by[1]

(1) $$\tau_{h,v,e} = \tau_s / J_{h,v,e}$$

where the subscripts h, v, and e on the partition numbers J refer to horizontal, vertical, or energy oscillations. A subscript s will always refer to the real synchronous orbit of the SSC, that is, to a quantity evaluated on the unique orbit which closes on itself in all three dimensions. The fundamental damping time in equation (1) is always very close to its design value (subscript 0)

(2) $$\tau_s = k E_s^{-3} \langle G^2 \rangle_s \approx \tau_0 = k E_0^{-3} \langle G_0^2 \rangle_0$$

where k is a constant which does not concern us here, and angle brackets < > signify an average taken around the synchronous, or the design, orbit. The quantity G is the total bending strength at a given point, so that

(3) $$G^2 = (1/\rho)^2 = G_h^2 + G_v^2$$

where G_v may be due, for example, to a vertically displaced closed orbit in a quadrupole, or a non-zero a_0 in a dipole.

The partition numbers J are the quantities which are interesting here, because if any of them are zero the corresponding motion is unstable, albeit with a long rise time. Robinson has shown[2] the general result that if the magnetic fields are not time varying

(4) $$J_h + J_v + J_e = 4$$

Nominal values of the partition numbers are clearly indicated in the parameterisation

(5) $$J_h = 1 - D_h \qquad J_v = 1 - D_v \qquad J_e = 2 + D_h + D_v$$

where, in a generalisation of the results of reference [1] to include bends in the vertical plane, the fundamental quantites D_h and D_v are given by

(6.1) $$D_h = <\eta_h G (GG_h + K) >_s / < G^2 >_s$$

(6.2) $$D_v = <\eta_v G (GG_v - K) >_s / < G^2 >_s$$

in which K is the quadrupole strength on the synchronous orbit, and η is the dispersion function.

On the design orbit of a perfect, separated function, flat, SSC,

(7) $$GK = 0$$ (separated function)

$$D_v = G_v = \eta_v = 0$$

while the horizontal D-funtion is negligibly small with a tune $Q \approx 100$,

(8) $$D_h \approx \eta_{typical} G_{typical} = R/Q^2 \ . \ 1/R \approx 10^{-4}$$

or even zero[3] if the dipoles are rectangular. Also, the first, G^3, term in the numerator of (6.1) and (6.2) turns out to be negligible by comparison with the second, GK, term, in all the cases studied below, because

(9) $$G^2 \approx R^{-2} << K \approx (L_{1/2} L_{quad})^{-1}$$

where $L_{1/2}$ is the half cell length. It is therefore a good approximation, adopted hereon, to say

(10) $$D_{h[v]} = [-] <\eta_{h[v]} GK >_s / < G^2 >_0$$

where the denominator is conveniently evaluated over the design orbit. These equations will be used to study what happens to D_h and D_v when different kinds of error are present. Note that D_v tends to be much less than D_h, even in the presence of large realistic errors, if the lattice is decoupled (as is implicitly assumed), because then the vertical dispersion is (usually) relatively small.

Synchronous energy errors[4]

If the frequency of the accelerating cavity is not matched to the length of the SSC for some reason, whether by accident or on purpose, the relative synchronous energy will be non-zero,

(11) $$\delta_s = (E_s - E_0) / E_0 = (L_s - L_0) / \alpha L_0 \qquad L_s = h \, . \, \lambda_{rf}$$

$$\alpha \approx 1 / Q^2$$

where h is the harmonic number, and α is the compaction factor. The synchronous orbit is

horizontally offset by $\eta_h.\delta_s$, so that a quadrupole of design strength K_0 aquires a dipole strength

(12) $$G_h = K_0 . \eta_h \delta_s / (1 + \delta_s)$$

which, in a separated function machine with no other errors, $(\eta_v = 0)$, leads to

(11) $$D_h = 2 A \delta_s / (1 + A \delta_s^2) \approx 2 A \delta_s$$
$$D_v = 0$$

where the A is given by an average over the design orbit

(12) $$A = <\eta^2 K^2>_0 / <G^2>_0$$

Edwards[3] has shown that, for a FODO cell lattice with a phase advance of ϕ per cell, there is a strong dependence on the quadrupole length,

(13) $$A = (1 - r_Q) / r_Q . [1 + 4 / \sin^2 (\phi / 2)]$$

where r_Q is the fraction of the cell filled with quadrupoles. Appropriate SSC values give

(14) $$r_Q = L_{quad} / L_{1/2} = 3.1 / 96.0 \qquad \phi = 60 \text{ degrees}$$
$$A = 514$$

and the conditions for stable energy and horizontal betatron oscillations become

(15) $$-2 < D_h < 1$$

$$-1 / A < \delta_s < 1 / 2A$$

$$-1.94 \times 10^{-3} < \delta_s < 0.97 \times 10^{-3}$$

These numbers are comparable to the energy aperture found by tracking, and correspond to <u>systematic</u> orbit displacements of a few millimetres.

<u>Closed orbit errors</u>

Horizontal closed orbit errors x in the quadrupoles will also lead to a non-zero numerator in equation (10), since $GK=xK^2$, but there will tend to be a strong cancellation between different phases of the oscillating displacement. In a worst case analysis, assuming that the displacements in $N \approx 400$ focussing and defocussing quadrupoles in a FODO lattice SSC are all totally random,

(16) $$D_h \approx N^{1/2} [\eta_f^2 <x_f^2> + \eta_d^2 <x_d^2>]^{1/2} K^2 L_{quad} R / 2\pi$$

where R^{-2} has replaced the denominator. Using SSC values

(17) $$R \approx 13.4 \text{ kilometres}$$

$$\eta_{max} \approx 3.9 \text{ metres}, \qquad \eta_{min} \approx 2.4 \text{ metres}$$

$$<x_{min}^2> / <x_{max}^2> = \beta_{min} / \beta_{max} = 1/3$$

$$KL_{quad} \approx 2 \sin(\phi/2) / L_{1/2} = 0.0104 \text{ metres}^{-1}$$

then, with the rms orbit error $< x_f^2>^{1/2}$ measured in metres,

(18) $$D_h \approx 5.98 < x_f^2>^{1/2}$$

showing that even for an orbit scraping the vacuum chamber, this is still only a 10% effect.

Quadrupole field errors in the dipoles

If there is a systematic quadrupole error in each of the dipoles, measured by b_1 metres^{-1}, the familiar coefficient, then since $K=b_1G$ in the dipole, and assuming an otherwise perfect machine,

(19) $$D_h = <\eta\, b_1 G^2>_0 / <G^2>_0 = b_1 <\eta> \approx 3\, b_1$$

A typical value for b_1 ("b_1 of 1×10^{-4} at one centimetre") is 0.01 metres^{-1}, showing that this effect is also negligible. The effect is even smaller for random quadrupole errors in the dipoles.

Individual contributions from the IR triplet quadrupoles

So far it has been assumed that the vertical dispersion, and hence D_v, is negligible compared with the horizontal dispersion, and D_h. Furthermore, nominal (horizontal) dispersion values have been used, since this will at least give a feel for the numbers. At the IR triplet, however, both the vertical and the horizontal dispersions functions are designed to be zero. In order to see whether the very strong quadrupoles, and high betas, might nevertheless still be dangerous, consider an error dispersion, vertical or horizontal, given by

(20) $$\eta \approx q \cdot \eta_{arc\ max} (\beta / \beta_{arc\ max})^{1/2} \approx 0.22\, q\, \beta^{1/2}$$

where q is a quality factor, which is probably around 0.1 for a good machine, and as large as 0.3 or more for a bad one. There is a dipole component in the IR triplet quadrupoles, even in the absence of closed orbit errors, because of the crossing angle θ^*, so that, due to a single IR quadrupole,

(21) $$D \approx (R/2\pi) . \eta \beta^{1/2} \beta^{*\,1/2} \theta^* K^2 L_q \approx (0.035\,R) . q \beta \beta^{*\,1/2} \theta^* K^2 L_q$$

Using the extreme SSC values for the middle quadrupole, of length L_q,

(22) $\theta^* \approx 70$ microradians $\quad \beta^* = 0.5$ metres $\quad \beta = 8{,}000$ metres

$K = 0.003$ metres^{-2} $\quad L_q \approx 30$ metres

the contribution is found to be

(23) $$D \approx 0.049\ q$$

Only if all the 12 middle IR triplet quadrupoles conspire together will this effect be important. Note that the displacement due to the crossing angle, $\theta^*(\beta\,\beta^*)^{1/2} = 4.4$ millimetres, is much larger than closed orbit errors which can be reasonably expected.

Conclusions

There appears to be no particularly dangerous partition number effect lying in wait to disrupt SSC operation. The most pronounced behavior will occur on day 1, when the synchronous energy will have to be adjusted to match the true circumference of the machine. If the length error is δL, the necessary relative adjustment in the accelerating cavity frequency will be only about $\delta L/L$, which is (presumably) negligible.

References

1. Sands, The Physics of Storage Rings, SLAC-121 (1970)
2. Robinson, Radiation Effects in Circular Electron Accelerators, Phys. Rev. 111, no.2 (1958)
3. Edwards, Oscillation Damping Factors for Off-Momentum Orbits in Electron Storage Rings, FNAL TM-566 1501 (1975)
4. Peggs, PhD Thesis, ch. 5, Cornell University, (1981)