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COLD COPPER CONTRIBUTION TO SSC MACHINE IMPEDANCES: EFFECT OF ANOMALOUS SKIN EFFECT AND MAGNETORESISTANCE IN DC AND AC ELECTRIC FIELDS ON THE RESISTIVE WALL IMPEDANCES

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Introduction

The machine impedances have been discussed in the SSC reference design study.[1] The purpose of this report is to refine the estimates of the surface impedance of the copper layer on the beam pipe wall at 4.2°K by considering the anomalous skin effect and the magnetoresistance at low and high frequencies. Physical properties of the copper layer that can be important but may not be known for a particular copper layer include impurity scattering, scattering by structural defects and alterations of the scattering for large cyclotron radii of the electrons at low magnetic fields.

The contribution of a copper surface layer on the beam pipe wall to the SSC machine impedances has been estimated in the SSC reference design study by use of the "resistive-wall" impedance. The resistive wall impedance $Z_{\parallel}/n$ is given by

$$\frac{Z_{\parallel}}{n} = (1+j)\frac{Z_0}{2}\frac{\delta}{b} = \frac{1+j}{b}\sqrt{\frac{RZ_0\rho}{2n}}\ (\Omega) \qquad (1)$$

where $Z_0 = \sqrt{\mu_0/\varepsilon_0}$ is the free-space impedance, R is the effective machine radius, b is the effective beam pipe radius, ρ is the copper layer (wall) electrical resistivity and δ is the 'classical' skin depth at the excitation frequency $f = nf_0$ for f_0 the revolution frequency ≈2.5 kHz.

$$\delta = \frac{1}{\sqrt{\pi\mu_0 f\sigma}}\ (m) \quad . \qquad (2)$$

where SI units are used unless stated, the inductive reactance is positive here and $\sigma = 1/\rho$. The transverse impedance $Z_\perp$ (per unit length) has been related to $Z_\parallel$ by

$$Z_\perp = \frac{2R}{b^2} \frac{Z_\parallel}{n} \left(\frac{\Omega}{m}\right) \tag{3}$$

The resistive wall impedance $Z_\parallel$ can be related to the surface impedance Z_s of a hollow right circular cylinder with well developed skin effect by

$$Z_\parallel = \frac{R}{b} Z_s \quad , \quad Z_s = \frac{1+j}{\delta\sigma} \quad . \tag{4}$$

The surface impedance Z_s can be found by the Rayleigh formula.[2]

The broadband average[1] of the wall impedances is dominated by the high frequency portion. As an estimate of the high frequency portion, we calculate the impedances at the cutoff frequency of the beam pipe $f_c = n_c f_o \simeq f_o\, R/b = 2.39$ GHz. We can estimate the impedances at $T = 4.2°K$ if $Z_o = 377\Omega$, $b = 1.5 \times 10^{-2}$ m, $\rho = 5 \times 10^{-10}\Omega$-m, $2\pi R = 90$ km and $f = 2.39$ GHz. We find

$$\begin{aligned} \frac{Z_\parallel}{n} &= 2.89\ (1+j)\ m\Omega \\ Z_\perp &= 0.368\ (1+j)\ \frac{M\Omega}{m} \quad . \end{aligned} \tag{5}$$

For coupled-bunch instabilities[1] the excitation frequency is 2.5 kHz. The narrow band average is

$$\frac{Z_{\parallel}}{n} = 2.83\ (1+j)\Omega$$
$$Z_{\perp} = 360\ (1+j)\ \frac{M\Omega}{m} \qquad (6)$$

The effective values of the impedances in Eq. (5) will be compared with the values in the SSC design study in the conclusion. The value for $Z_{\perp}$ in Eq. (6) agrees with the value in the SSC design study in Table 4.3.1.

The above expression for the resistive wall impedance was derived for zero static magnetic field but the copper resistivity at 4.2°K was estimated by assuming that the resistivity is larger than it would be in bulk copper in a zero magnetic field. A reduction of 30 over the room temperature resistivity was taken for the design study. How good is that estimate? In this report an attempt is made to answer that question but one should note that the zero magnetic field d.c. conductivity (resistivity) of copper at 4.2°K varies considerably from sample to sample. Single crystals of exceptionally pure copper possess a much smaller d.c. resistivity at zero magnetic fields than the design value. Recently d.c. resistivity ratios of 100 to 500 for zero magnetic fields have been reported[3] for copper layers of 8 mils on cylinders that were constructed as models for the SSC beam pipes. The resistivity ratio (RR) is the ratio of the resistivities at two

temperatures; in this report the resistivity ratio is the ratio of the resisitivity at a reference temperature (room temperature ≈295°K or 273°K) to that at 4.2°K. Although reducing the bulk copper d.c. resistivity ratios in zero magnetic fields can give useful guesses of the machine impedance contribution of the cold copper in magnetic fields, the variation with frequency and magnetic field is more complicated. To estimate the corrections to the surface impedance we need estimates of certain parameters. Many are straightforward to calculate but believable values of the collisional mean free path λ of electrons or the related mean collisional time τ are more elusive. The most complete discussion of different ways of determining the mean free path in metals is given by Chopra.[4] However, for copper he quotes only one measurement at 4.2°K[5] for the mean free path of 34,000 Å for copper whiskers. Since the estimate of the electron mean free path in copper at 4.2°K is necessary, we first consider that question. Other than the approach of Isaeva[5] the two approaches commonly used for calculation of the mean free path are based on data from the surface impedance of copper at high frequencies in the anomalous skin effect regime. However, the mean free path value cannot be unfolded from the surface impedance data unless the bulk conductivity of copper is known at low temperatures. We, therefore, start with a discussion of possible values of the electrical conductivity of bulk copper at liquid helium temperatures for a d.c. electric field and zero magnetic field.

Electrical Conductivity of Bulk Copper in a d.c. Electric Field and Zero Magnetic Field

The reported range for resistivity ratios of bulk copper in a d.c. electric field and a zero magnetic field is from 2.3[6] to 8,600.[7,8] For this case $\rho(273^\circ K)/\rho(4.2^\circ K) \simeq 2.3$ and $\rho(295^\circ K)/\rho(4.2^\circ K) = 8{,}600$ but the difference between the use of iced water and room temperature is not significant. Kittel[9] (p. 238) quotes a resistivity ratio for good metals of 10^3 or more. Measurements have been made of commercial copper. Rosenberg quotes values of bulk copper resistivity ratios of about 200 for high purity samples from Johnson Mattley Spectrographically Standardized materials. Mendoza and Thomas[6] found resistance (resistivity) ratios of about 100 for copper where the reference temperature was 273°K. They used wire with less than 10^{-2}% impurities of Ca, Cd, Mg, Si, Pb, Mn and Zn. One reel of copper wire had 0.94% of Sn impurities as well as some Mn impurities. The resistivity ratio for this case was about 2.3. Berman and MacDonald[10] found electrical conductivities of 1.85×10^{10} and 1.5×10^{10} siemens for copper near liquid helium temperatures. The measured impurities were Ag < 0.0005%, Ni < 0.0003%, Pb < 0.0004%. The resistivity ratios for a reference temperature of 273°K ($\sigma = 6.45 \times 10^7$ siemens[11]) are 287 and 233 respectively. Sondheimer[12] quoted a resistance ratio (reference temperature 273°K) of 345 for a pure sample of copper but a ratio of 20 for a copper sample with 0.026% of Sn. We conclude from the experimental data that small amounts of impurities in copper can

markedly reduce the electrical conductivity of copper at liquid helium temperature.

Theoretically we expect the presence of impurity ions in copper to depress the electrical conductivity at low temperatures. At room temperatures the dominant scattering mechanism is electron-phonon collisions; this swamps any impurity scattering at room temperatures. At low temperatures only electrons in the high energy tail of a distribution function can emit phonons. Hence, electron-phonon collisions are improbable and scattering off impurity ions as well as off structural defects (dislocations, stacking faults, interstitials, vacancies and grain boundaries) is dominant. Chropa[4] claims that significant changes to resistivity come from vacancies and grain boundaries. Rosenberg put emphasis on other structural defects. Annealling can reduce the scattering by causing migration of the single vacancies and the disappearance of the grain boundary area.

Theoretical estimates of the resistivity of bulk copper at liquid helium temperature from room temperature measurements do not help much. At room temperature different specimens of copper with quite varying amounts of impurities have nearly the same electrical resistivity but those same specimens at liquid helium temperature have widely varying electrical resisitivities. The conventional formula for the temperature dependence of the electrical resisitivity[13]

$$\rho = \rho_0 [1 + \alpha (T - T_0)] \tag{7}$$

where the subscript o denotes the reference quantities only works at temperatures considerably above the liquid helium temperature. In fact for the α in Harnwell's book one finds a negative resistivity for copper at 4.2°K. A more sophisticated model employs the Block-Gruneissen formula for resistivity.[8] The approximations to the integral give

$$\frac{\rho_\ell}{\rho_h} = \frac{497.6\ T_\ell^5}{{}^4T_h}\ , \quad \theta \simeq 320^\circ K \tag{8}$$

where the subscripts ℓ(h) denote low (high) temperatures. However, at low temperatures one must add the residual resistivity to ρ_ℓ, which exceeds ρ_ℓ there, and this residual resistivity depends on the purity of the copper.[14,15]

Anomalous Skin Effect

The anomalous skin effect is important itself because the expression for the surface impedance is significantly altered in this regime. In addition the data from metals in the anomalous regime afford a common method of determining the mean free path of electrons.

The anomalous skin effect occurs when the electron mean free path is longer than the skin depth. The effective electrical conductivity decreases because only the electrons moving at small angles to the surface are in the electric field for any length of time and thereby contribute to the current. Apparently London[16] first offered a qualitative explanation. Pippard[17,18] derived an expression for the

surface resistance by means of the ineffectiveness concept. He assumed that most electrons did not contribute to the current because they did not travel parallel to the surface. He then applied the Kramers-Kronig relation to find the reactive part of the surface impedance. Sondheimer[19] contributed to the theory as well. Chambers[20] made the most detailed measurements of the surface impedance.

An approximate expression for the surface impedance can be found from the usual surface impedance Z_s by calculating an effective electrical conductivity $\sigma_{eff.}$. From a crude free electron model the electrical conductivity is given by

$$\sigma = \frac{ne^2\tau}{m_e(1-i\omega\tau)} \tag{9}$$

where n is the number density of electrons, e is the electron charge, m_e is the free electron mass and ω is the angular frequency of the electric field and τ is defined on page 4. If only a portion of the electrons are in the skin depth for any length of time, the effective density of electrons decreases to $n_{eff} = \Gamma\delta n/\lambda$ where Γ is some constant equal to 10 or so and λ is defined on page 4. For the extreme anomalous regime, $\lambda >> \delta$, the effective conductivity is considerably reduced with n_{eff} substituted for n in Eq. (9). The subsequent replacement of σ by σ_{eff} from Eq. (9) in R_s ($\omega\tau << 1$) gives an approximation to the surface resistance in the extreme anomalous limit. The surface reactance X_s can be found from

the surface resistance R_s by the Kramers-Kronig relation such that $X_s = \sqrt{3}\, R_s$.

More fundamental derivations were given where Reuter and Sondheimer[21] assumed $\omega\tau << 1$. Kittel[9] and Pippard[17] presented modified approaches based on the Boltzmann equation for solving an integro-differential equation for the electric field (planar geometry). Explicit expressions were found for purely specular or purely diffuse scattering of electrons. Surfaces that are not clean and smooth appear to exhibit diffuse scattering. Experimentally the numerical difference between the two cases is unimportant because they differ by a factor of 8/9. The surface impedance for purely specular reflection[9,17] in the extreme anomalous limit, $\lambda >> \delta$, is

$$Z_\infty = \frac{8}{9}\left(\frac{3^{1/2}\pi\omega^2\lambda}{c^4\,\sigma\mu}\right)^{1/3}\left(1 + j\sqrt{3}\right)\left(\frac{\text{sec}}{\text{cm}}\right) \tag{10}$$

where the factor of 8/9 is omitted for diffuse scattering and the subscript s is suppressed. In SI units the surface impedance for purely diffuse scattering[22] is

$$Z_\infty = 3.7897 \times 10^{-5}\left(\frac{\omega^2\lambda}{\sigma}\right)^{1/3}\left(1 + j\sqrt{3}\right)(\Omega) \tag{11}$$

in the extreme anomalous limit. Also Biquard and Septier[22] give formulas due to Chambers[20] for the surface resistance and surface reactance at temperatures T

$$R_s = R_\infty \left(1 + 0.32578\ \alpha^{-0.2752}\right)\ (\Omega)$$

$$X_s = X_\infty \left(1 + 0.13319\ \alpha^{-0.2752}\right)\ (\Omega) \tag{12}$$

for $\alpha(T) = 2\pi\omega\sigma^3\ (T)\ (\lambda/\sigma)^2$ and this $\alpha(T)$ differs from the α in Eq. (7). Qualitatively one sees that the effect of replacing σ by σ_{eff} in Eq. (1) in the surface impedance is to increase the surface impedance over its value with σ. This increase is in addition to the usual surface impedance increase due to squeezing the current into a skin layer.

The mean free path of electrons in the copper layer can be calculated by two ways from the surface impedance data. One quick way is to measure the electrical conductivity and use the electron number density n determined by Chambers[20] for the valence electrons. The other way is to determine the mean free path directly from the surface impedance data. We next discuss the latter approach. If we use the surface conductance $\Sigma = 1/R$, we can write[22]

$$\frac{\Sigma_\infty}{\Sigma_{295^\circ K}} = \frac{15.4}{f^{1/6}\ \sigma_o^{1/2}} \left(\frac{\sigma}{\lambda}\right)^{1/3} \tag{13}$$

where (σ/λ) is measured at 4.2°K, f is the excitation frequency and σ_o is the electrical conductivity of copper at 295°K and all quantities are in SI units. The conductance ratio in Eq. (13) is determined from the resistive parts of the surface impedances in Eq. (4) and Eq. (11).

Biquard and Septier have measured the surface conductance of copper cavities in the S- and L- bands. They suggested that the Joule heating losses in linear accelerators could be reduced by cooling the microwave cavities to liquid helium temperature. The microwave measurements in a TE_{011} copper cavity and copper coaxial cavity were performed at f = 1.42 GHz and 3.16 GHz which are in the desired upper range of excitation frequencies for the proposed SSC accelerator. We can calculate the mean free path from Eq. (13) if we assume σ_0 = 5.8×10^7 siemens (℧/m) at room temperature for copper and obtain resistivity ratios for copper. Since no d.c. resistivity ratios for zero magnetic fields are quoted for the copper layers by Biquard and Septier, we choose $\rho\ (295^\circ K)/\rho(4.2^\circ K) = \sigma/\sigma_0$ to be 100 and 300. The values of 100 is a conservative value for good copper wire. As the copper used in cavities had a purity of better than 99.999%, the data of Berman and MacDonald suggested a resistivity ratio of at least 300. The values in Table I for λ, δ, λ/δ, are given for the different reported surface conductance ratios and at the two resistivity ratios.

The mean free paths can be compared with the mean measured value in copper whiskers at 4.2°K which was 3.4×10^{-6} m and the calculated value from the free-electron model from Eq. (9). To find the mean free path from Eq. (9) we let $\lambda = v_F\tau$ for v_F the electron speed near the Fermi surface and let $mv_F^2/2$ equal the Fermi energy of about 6 eV (older values of 7 eV[23] and T ~ 80,000°K[8] exist). Then $v_F = 1.45 \times 10^6$ m/sec and we take the customary value for n 10^{29} m^{-3}. The corresponding mean free path is $\lambda = 2.98 \times 10^{-6}$ m.

Both these mean free paths are smaller than the calculated ones in Table I. The authors indicate some theoretical values for the conductance ratio but do not explain how they calculated those values.[14] They do state an estimate of $\lambda/\delta \sim 40$ for their case and mention that λ/δ can reach 5000. Their estimates appear to be taken directly out of Pippard's article.[17] On the other hand, Sondheimer quotes $\lambda/\delta \sim 100$ for a pure metal which suggests longer mean free paths. If the authors measured the d.c. resistivity ratio, then the mean free paths from the high conductance ratios (electrolytic polishing) and RR = 100 are closest to the data. This follows because in those cases λ/δ is nearly 40 and their theoretical and experimental values for the conductance ratios are quite close. Nonetheless the distinct possibility exists that the longer mean free paths are valid since much higher d.c. resistivity ratios have been reported for such pure copper.

Table I

f (Hz)	Σ ratio	σ/σ_0	λ (m)	δ (m)	λ/δ
3.16×10^9	3.23	100	2.53×10^{-5}	1.18×10^{-7}	215
3.16×10^9	3.33	100	2.31×10^{-5}	1.18×10^{-7}	197
3.16×10^9	5.23	100	5.96×10^{-6}	1.18×10^{-7}	50.7
3.16×10^9	5.57	100	4.94×10^{-6}	1.18×10^{-7}	42.0
3.16×10^9	3.23	300	7.59×10^{-5}	6.79×10^{-8}	1120
3.16×10^9	3.33	300	6.93×10^{-5}	6.79×10^{-8}	1020
3.16×10^9	5.23	300	1.79×10^{-5}	6.79×10^{-8}	263
3.16×10^9	5.59	300	1.48×10^{-5}	6.79×10^{-8}	218
1.42×10^9	5.30	100	8.55×10^{-6}	1.75×10^{-7}	48.7
1.42×10^9	5.30	300	2.56×10^{-5}	1.01×10^{-7}	253
2.51×10^3	-	100	-	1.32×10^{-4}	-
2.51×10^3	-	300	-	7.63×10^{-5}	-

Recently measurements of the reactive part of the surface impedance have been made.[24]

Calculation of Parameters

We are now ready to calculate the parameters needed for a discussion of the magnetoresistance in a d.c. magnetic field and the anomalous skin effect in a magnetic field. The design parameters for the SSC give $T = 4.2°K$ as the temperature of the cold copper, and $B = 6.6\ T = 6.6 \times 10^4\ G$ for the peak static magnetic flux density. The electron mass is taken to be the free electron mass. The free electron approximation is supported by the number of electrons per atom of copper found by Chambers to be nearly one. The electron gyrofrequencies ω_c and f_c[25] are

$$\omega_c = \frac{eB}{m_e c} = 2\pi f_c = 1.76 \times 10^7\ B\ (\text{rad/sec}) \tag{14}$$

$$f_c = 2.80 \times 10^6\ B\ (\text{Hz}) \quad . \tag{15}$$

For $B = 6.6\ T$; $\omega_c = 1.16 \times 10^{12}$ rad/sec; $f_c = 185$ GHz; for $B = 0.33\ T$; $\omega_c = 5.81 \times 10^{10}$ rad/sec; and $f_c = 9.24$ GHz.

In Table II we find the values of the collision time for selected mean free paths λ from Table 1. The values are selected to cover the range of mean free path values.

Table II

f (GHz)	Σ ratio	σ/σ_0	λ (m)	δ (m)	τ (sec)
3.16	5.57	100	4.94×10^{-6}	1.18×10^{-7}	3.41×10^{-12}
3.16	3.23	300	7.59×10^{-5}	6.79×10^{-8}	5.23×10^{-11}
1.42	5.30	100	8.55×10^{-6}	1.75×10^{-7}	5.90×10^{-12}
1.42	5.30	300	2.56×10^{-5}	1.01×10^{-7}	1.77×10^{-11}

The first value of τ in Table II is similar to the $\tau = 3.8 \times 10^{-12}$ sec quoted by Pippard.[17] Kittel, on the other hand, estimated τ to be 10^{-10} sec for a good conductor which gives a certain latitude.

The electron cyclotron radius $r_c = v_F/\omega_c = 1.25 \times 10^{-6}$ m for B = 6.6 T and $r_c = 2.50 \times 10^{-5}$ m for B = 0.33 T. The thickness of the copper layer is about 8 mils = 2.03×10^{-4} m. The layer was chosen to be about as thick as the skin depth at 2.5 kHz at a resistivity ratio of about 30. In Table III are given the length ratios for the λ in Table II where λ_1 corresponds to the top value of the λ column in Table II and the others follow in order.

Table III

	r_c/λ		λ/δ	λ/t	r_c/δ	
	6.6 T	0.37 T			6.6 T	0.33 T
λ_1	2.53×10^{-1}	5.14	42.0	2.43×10^{-2}	10.6	212
λ_2	1.64×10^{-2}	3.29×10^{-1}	1120	3.74×10^{-1}	18.4	368
λ_3	1.46×10^{-1}	2.92	48.9	4.21×10^{-2}	7.12	142
λ_4	4.87×10^{-2}	9.74×10^{-1}	253	1.26×10^{-1}	12.3	247

The dimensionless combinations of frequencies and τ are displayed in Table IV.

Table IV

	$\omega_c\tau$ (185 GHz)	$\omega_c\tau$ (9.24 GHz)	$\omega\tau$ (3.16 GHz)	$\omega\tau$ (1.42 GHz)	$\omega\tau$ (2500 GHz)
$\tau 1$	6.30×10^{-1}	3.15×10^{-2}	1.08×10^{-2}	4.84×10^{-3}	8.52×10^{-9}
$\tau 2$	9.68	4.84×10^{-1}	1.65×10^{-1}	7.43×10^{-2}	1.31×10^{-7}
$\tau 3$	1.09	5.45×10^{-2}	1.86×10^{-2}	8.37×10^{-3}	1.47×10^{-8}
$\tau 4$	3.26	1.63×10^{-1}	5.58×10^{-2}	2.5×10^{-2}	4.41×10^{-8}

The ratio of f to f_c for f_c = 185 GHz is for 3.16 GHz, 1.71×10^{-2}; 1.42 GHz, 7.68×10^{-3}; 2.5 kHz, 1.35×10^{-8} and for f_c = 9.24 GHz is for 3.16 GHz, 3.42×10^{-1}; 1.42 GHz, 1.54×10^{-1}; 2.5 kHz, 2.71×10^{-7}. The τ_1 through τ_4 correspond to λ_1 through λ_4 in Table II.

Magnetoresistance in a d.c. Electric Field

For d.c. electric fields or low frequencies copper in static magnetic fields exhibits magnetoresistance. In weak magnetic fields defined by $(\omega_c\tau)^2 << 1$ the resistance R(H) at magnetic field H is related to the resistance R(0) at zero magnetic field by[4,8,9,17,26,27]

$$\frac{R(H) - R(0)}{R(0)} \approx (\omega_c\tau)^2 << 1 \quad . \tag{16}$$

For strong magnetic fields, $(\omega_c\tau)^2 >> 1$, the magnetoresistance differs for different crystals. Kittel[9] has described three cases for the transverse magnetoresistance. With crystals of type 1 with closed Fermi surfaces the resistance saturates for high magnetic fields for all orientations; the saturated value is perhaps several times the zero magnetic field value. For crystals of type 2 with equal number of holes and electrons the resistance increases for all strengths of magnetic field. For crystals of type 3 with Fermi surfaces with open orbits the resistance saturates in some directions but not in others. Copper forms crystals of type 3. The dependence of the magnetoresistance with orientation of a pure copper crystal is shown in Figure 1 (Figure 4.29 on p. 109 in Rosenberg's book which in turn is from a paper[28]). As the sharp changes in resistance with angle are seen only in single crystals, the angular dependence with respect to crystal direction of the d.c. magnetoresistance should not be important for copper coated pipes.

The transverse magnetoresistance ($\vec{B}_{appl} \perp \vec{E}$) is more commonly studied than longitudinal magnetoresistance ($\vec{B}_{appl} \parallel \vec{E}$) because it is usually much greater. $\vec{E}$ is the electric field here. Since the theories are complicated and did not always explain the phenomenon in the past, Kohler's[29] rule has proven useful. This is a simple rule relating resistivity change to the magnetic field and the reference resistivity based on dimensional arguments.[30] The change in resistivity $\Delta\rho = \rho(H,T) - \rho(0,T)$ divided by the resistivity $\rho(0,T)$ at zero magnetic field is a function of $H/\rho(0,T)$ where $\vec{H}$ is the magnetic

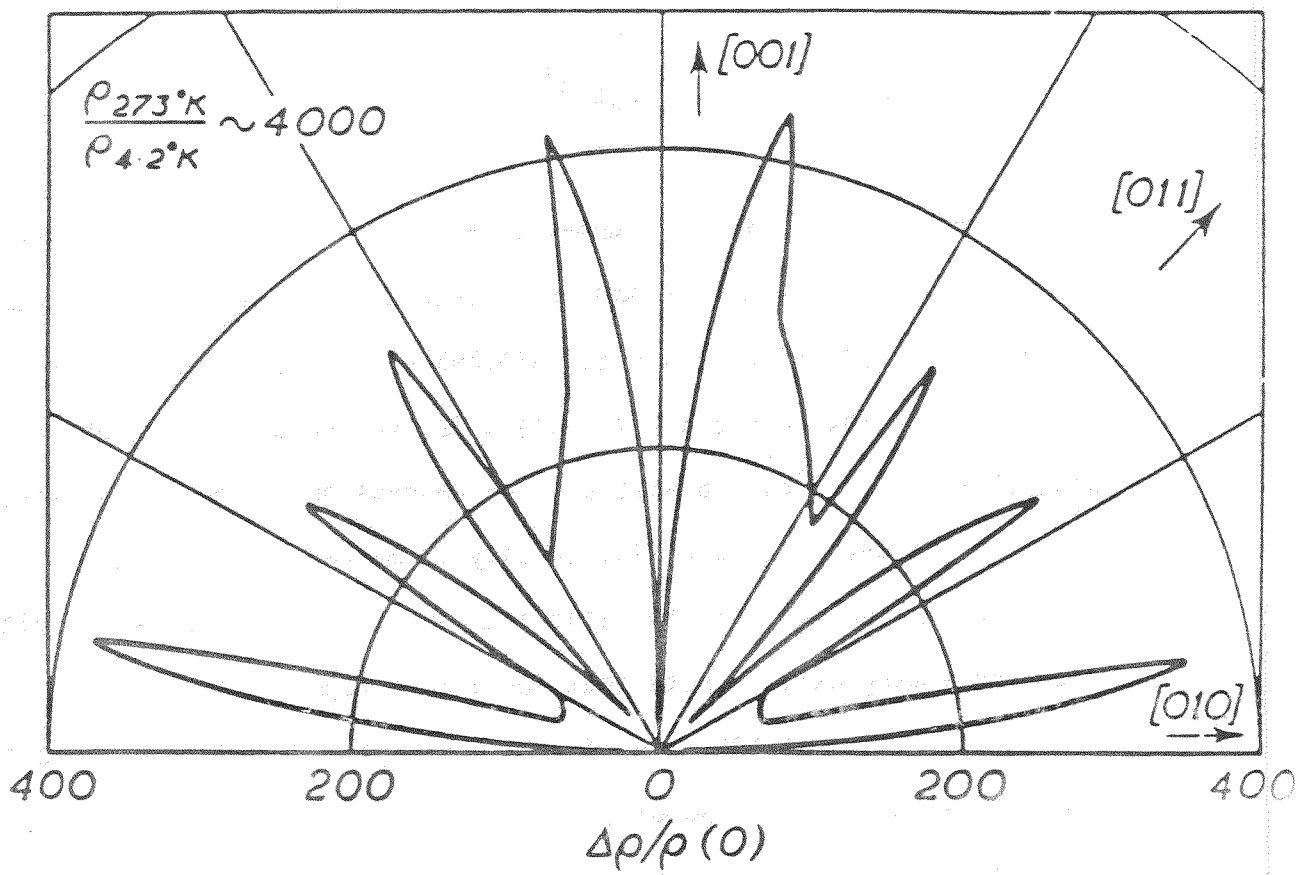


FIG. 4.29. A polar diagram showing the variation of the magneto-resistance of a h
purity copper single crystal as a transverse field is rotated about the specimen
$T = 4{\cdot}2°$ K, $H = 18$ k oersted, specimen axis approximately [100] (Klauder and Kun
1960).

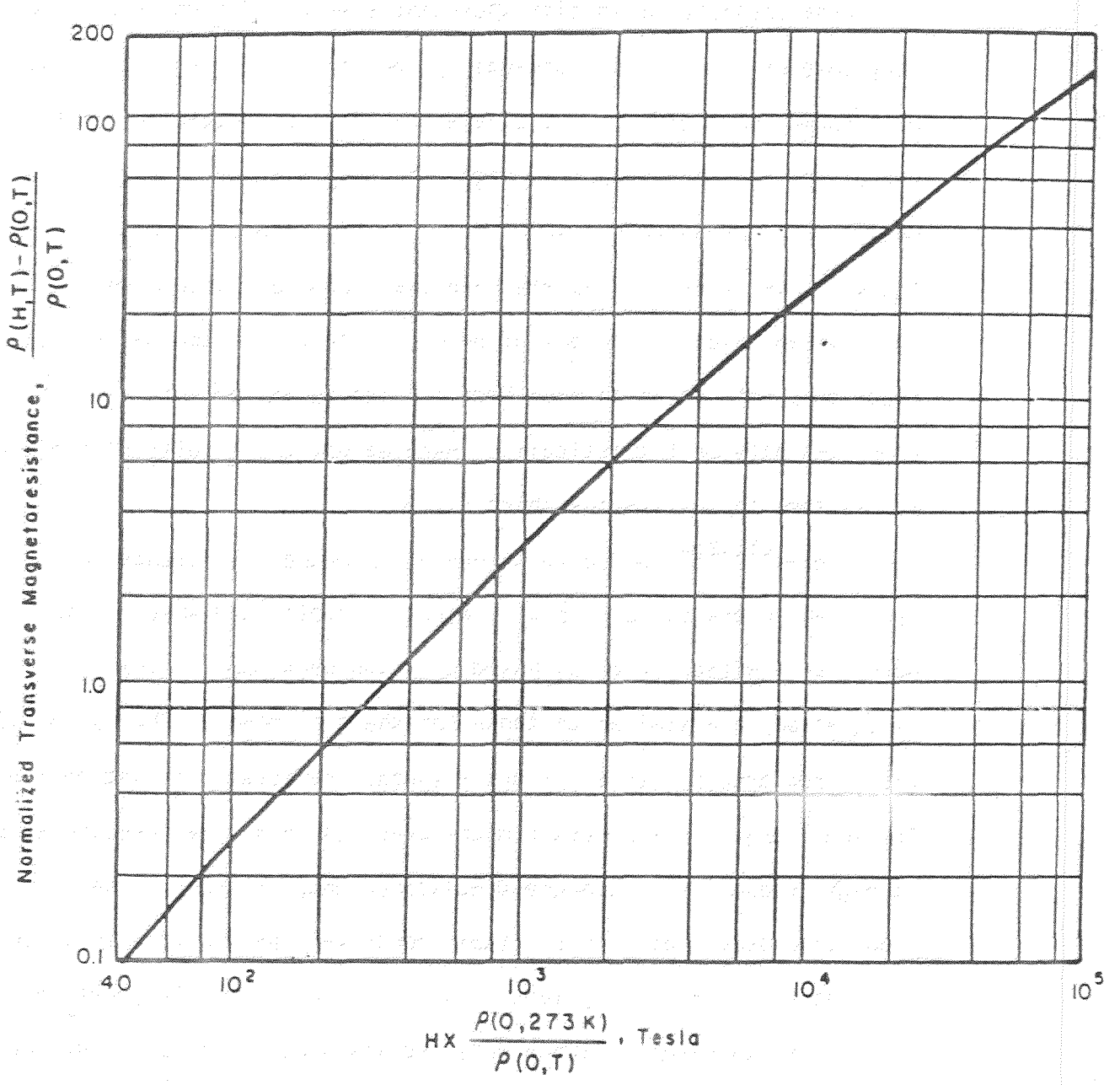
Figure 1

field. Experimental data for transverse and longitudinal magnetoresistance are displayed by a reduced Kohler plot. The reduced Kohler rule is

$$\frac{\Delta\rho}{\rho(0,T)} = f\left(\frac{H\ \rho(0,273^\circ K)}{\rho(0,T)}\right) \tag{17}$$

for f some function of its argument. Ralph R. Shutt[31] has furnished the reduced Kohler plot for the transverse magnetoresistance of copper in Figure 2. If we let B = 6.6T, $\rho(0,295^\circ K)/\rho(0,T) = 100, 300$ we find from Figure 2 that $\rho(0,273^\circ K)/\rho(H,T) = 32, 42$, respectively, where $\rho(0,295^\circ K)/\rho(0,273^\circ K) = 6.45/5.8$. The depression of the resistivity ratio by the peak magnetic flux density is marked and additional increases in the resistivity ratio increase the effective resistivity ratio only marginally. Note that for a zero magnetic field resistivity ratio of 100, the effective resistivity ratio is close to the SSC design value of about 30.

Theoretically, the magnetoresistance can be explained by departures from the free electron model that has proven helpful in describing the anomalous skin effect. In fact Kittel shows for a single carrier, isotropic effective mass and constant relaxation time in a simple geometry that no transverse magnetoresistance is predicted. Since the magnetoresistance is usually seen experimentally, departures from this model include two carrier types, variable relaxation time and open orbits.



TRANSVERSE MAGNETORESISTANCE (ELECTRICAL) OF COPPER
(REDUCED KOHLER PLOT)

Figure 2

Magnetoresistance in Time-Dependent Electric Fields

Magnetoresistance in time-dependent electric fields is an even more complex phenomenon. Several systems have been studied: metals with normal skin effect in a.c. electric fields, metals in which cyclotron resonance occurs in high-frequency electric fields and metals with the anomalous skin effect in high-frequency electric fields. The first two systems have been reported in some detail; the last system which is the one of primary interest in the SSC design study does not appear to have been treated to any extent experimentally or theoretically. Here we use the results of the first two systems to analyze the third system.

Donovan[25,32,33] proposed a theory for magnetoresistance in metals which are subject to a.c. electric fields and which exhibit the normal skin effect ($\lambda << \delta$) based on a two-band model. The calculations are similar to those for the d.c. model. The factor of $1/\tau$ is replaced by $(1+j\omega\tau)/\tau$ and a surface resistance is introduced. The model predicts a constant magnetoresistance at low frequencies but at high frequencies the magnetoresistance drops to zero. The transition occurs at $\omega\tau \simeq 1$. Experiments were performed with bismuth, since it has an unusually long mean free path at room temperature. At 300 MHz the magnetoresistance retained its d.c. value, at 3 GHz the magnetoresistance was one-half its d.c. value[34] and at infrared frequencies the magnetoresistance dropped to zero.[35] Since even for the d.c. magnetoresistance the two-band model does not give quantitative results,[24] this model is probably qualitative. We also

may doubt that a normal metal theory applies when $\omega\tau \simeq 1$. If we require $\lambda << \delta$, $\omega\tau \simeq 1$ with $\lambda = v_F\tau$, then

$$\sqrt{\frac{\sigma}{\lambda}} >> \left(\frac{\sigma}{\sigma_o}\right)\sigma_o\sqrt{\frac{v_F\mu_o}{2}} \quad . \tag{18}$$

If we use $\sigma_o = 5.8\times10^7$ siemens, $v_F = 1.45\times10^6$ m/sec, we find the conditions $0.20 >> \sigma/\sigma_o = 100$ and $0.34 >> \sigma/\sigma_o = 300$ for the most likely σ/λ to satisfy (18) from Table III. Since neither condition is satisfied, at all, we conclude that the normal metal condition does not apply for most copper samples when $\omega\tau \simeq 1$.

The magnetoresistance of copper in high-frequency fields such that the extreme anomalous skin effect regime holds was not directly treated to any extent. However, the models for cyclotron resonance and some of the measurements are relevant since the models include the effect of a magnetic field on a surface impedance in the extreme anomalous regime even when the electric field frequency is too low for the resonance condition. Cyclotron resonance in metals and semiconductors has been studied experimentally and theoretically.[17,18,26,27,36-42] The resonance affords a useful method of finding the effective masses of electrons and holes in the valence and conduction bands. The microwave measurements are performed at liquid helium temperature since the mean free collision time is sufficiently long such that the particle can complete at least one cyclotron orbit, $\omega_c\tau \gtrsim 1$. Actually cyclotron resonance has been detected for $\omega_c\tau$ somewhat less than one.

The surface impedance for the static magnetic field parallel to the surface of a metal in the extreme anomalous skin effect regime is[9,39,41]

$$Z_\infty(H) = Z_\infty(0) \tanh^{1/3}\left[\frac{\pi(1+j\omega\tau)}{\omega_c\tau}\right] . \tag{19}$$

Here $Z_\infty(0)$ is the surface impedance in a zero magnetic field in Eq. (11). Quantum corrections have been given[42] but should be negligible. The expression (19) is valid for longitudinal ($\vec{B} \parallel \vec{E}$) or transverse ($\vec{B} \perp \vec{E}$) cyclotron resonance. The surface impedance in Eq. (19) is the first term in an asymptotic series found from an integral for which $\lambda >> \delta$ and $\lambda >> \omega_c\tau\delta$ ($r_c >> \delta$) hold. From Table III we see for f a few gegahertz that $\lambda >> \delta$ always holds. On the other hand r_c is greater than δ if not overwhelmingly so for B = 6.6 T whereas it is always much greater than δ for B = 0.33 T. However, the condition for cyclotron resonance that ω/ω_c be an integer or ω_c is a subnarmonic of the excitation frequency does not hold at B = 6.6 T since $\omega/\omega_c << 1$ for f a few gegahertz. At the start-up magnetic flux density, B = 0.33 T, $\omega/\omega_c = 0.342$. For B = 6.6 T the correction factors $\tanh^{1/3}$ to the surface impedance in Eq. (19) which have been calculated for the four cases in Table IV are: τ_1, 1.00; τ_2, 0.679; τ_3, 0.998; τ_4 0.907. The corresponding complex numbers have been calculated for the start-up magnetic field but are so close to 1 that we neglect ω/ω_c. We find that the surface impedance, in marked contrast to the d.c. dependence, decreases with

increasing magnetic field. Experimental measurements on copper at 4.2°K at f = 24 GHz for B up to about 1.7 T confirm this dependence. The results are shown in Figure 3 taken from a paper by Fawcett.[39] The plot of $R_s(H)/R_s(0)$ shows the decrease with increasing H and is particularly suitable for our interests because it was on the ragged edge of the cyclotron resonance regime.

The original integral that leads to Eq. (19) can also be evaluated at low frequencies. The condition that $\lambda \gg \omega_c \tau \delta$ retrieves the normal skin effect result. If the transition condition is then that $\lambda \simeq \omega_c \tau \delta$ ($r_c \simeq \delta$), we can solve that for the frequency f. The values for f are: f = 28.0 MHz for B = 6.6 T and f = 69.9 kHz for B = 0.33 T for $\sigma/\sigma_0 = 100$; for $\sigma/\sigma_0 = 300$ f is one-third these values. Since both these frequencies are well above the revolution frequency $f_0 = 2.5$ kHz, we conclude that the d.c. magnetoresistance is valid for the surface impedance excited by coupled-bunch instabilities.

We have so far ignored the determination of the surface impedance for the magnetic field perpendicular to the surface of the metal. Early theories[37,38] concluded that the surface impedance was independent of the magnetic field in the extreme anomalous skin effect limit. However, later expressions for the surface impedance in the extreme anomalous skin effect regime were given which included nonlocal effects. The conditions for the inclusion of nonlocal effects is

$$1 + (\alpha\beta)^2 \ll \beta^2 \tag{20}$$

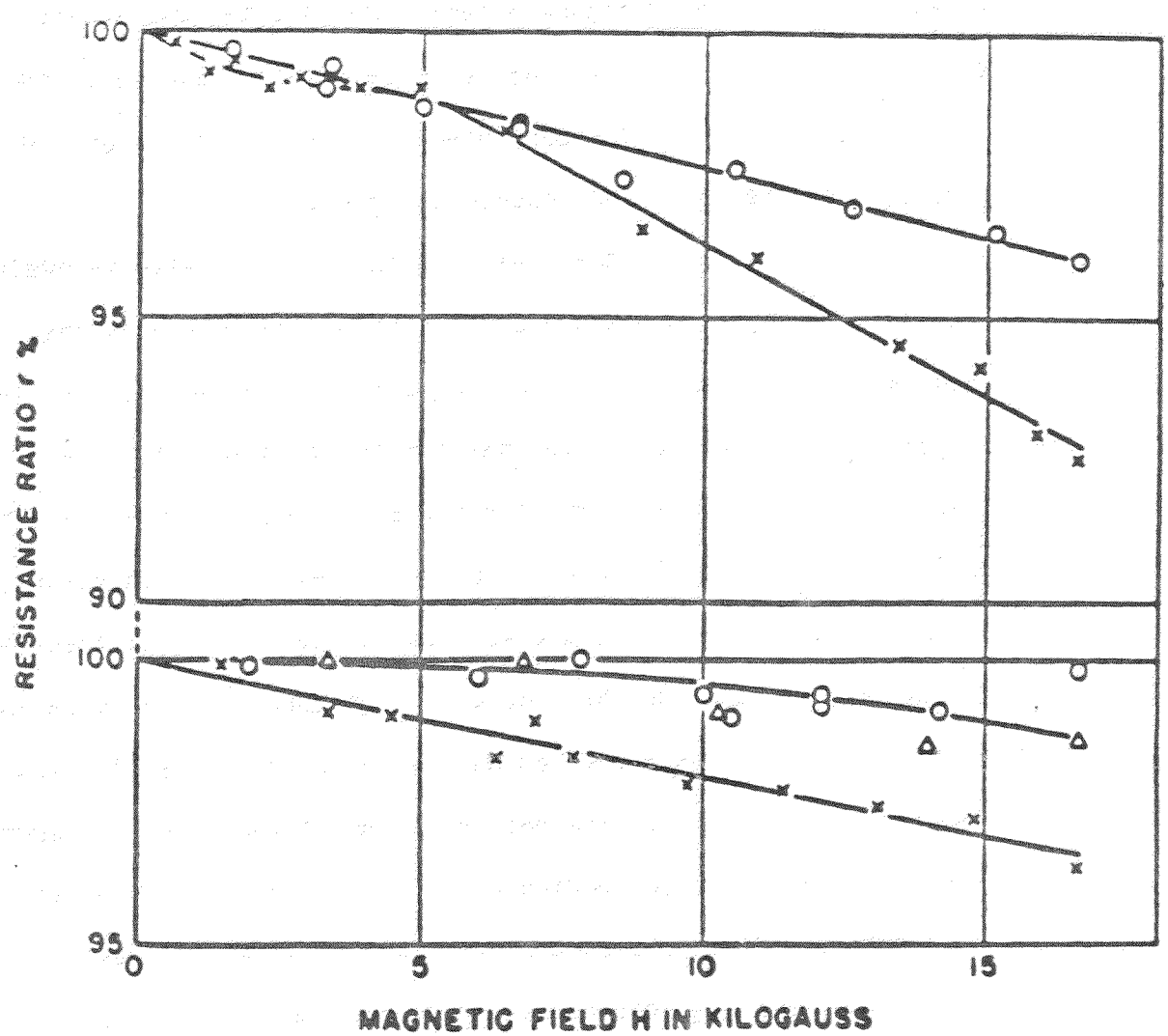


FIG. 2. Cyclotron resonance absorption in copper at 4.2°K for plane polarized radiation near 24 kMc/sec and magnetic field parallel to the metal surface. Specimen orientation—see text; upper curves—$J \| X$, lower curves—$J \| Y$; ○—$H \| J$, ×—$H \perp J$; $\omega\tau = 1.05$.

FIGURE 3

where $\alpha = \omega\delta_L/v_F\ ((\omega+\omega_c)/\omega)^{3/2}$, $\beta = \lambda/\delta_L\ (\omega/(\omega+\omega_c))^{1/2}$ and α is not the same α as in Eqs. (7) and (12). The London skin depth is

$$\delta_L = \frac{c}{\omega_p} = \delta\sqrt{\frac{\omega\tau}{2}}\ ,\quad \omega_p^2 = \frac{ne^2}{m_e\varepsilon} \tag{21}$$

If f = 1.42 GHz, B = 6.6 T, $\delta = 8.55\times10^{-6}$ m, we find the largest α and smallest β. Even for this case we are in the nonlocal regime as $\alpha = 0.262$ and $\beta = 26.4$. From Table I in the article by Miller and Haering we find an approximate expression for the surface impedance under the conditions $Z_s(H) = Z_\infty$ in Eq. (10) for this case. Since the zero magnetic field surface impedance is a good approximation for the least favorable case, we conclude that the magnetic field does not change the surface impedance if B is perpendicular to the metal surface in the extreme anomalous limit.

Thin Films

Thin films[4] are layers of materials where the layer thickness t is much less than the carrier mean free path. From the ratios of λ/t in Table III one sees that the proposed copper layers are not thin under that definition. However, even thick metal films may have higher resistivity than the bulk metal. Yet for Au and Ag[4] the evaporation process can produce films with a resistivity much higher than very high-purity bulk material. For d.c. electric fields a size effect dependence of longitudinal d.c. magnetoresistance has been shown. These calculations were done numberically for wires[44,45] and

films. The results are shown in Figure 4 from reference 4, p. 393. An analytical formula valid for $\beta = t/r_c >> 1$ has been given where this β is not the same as defined in Eq. (20). The quantity $\gamma = t/\lambda$. From Table III $\gamma > 2.67$ for our mean free paths which is off the graph for films. However, from the analytical formula valid for $\beta >> 1$

$$\frac{\rho_F}{\rho_B} = 1 + \frac{3\pi}{8\beta} \tag{22}$$

for ρ_F the resistivity of the film and ρ_B the resistivity of the bulk material, we find $\rho_F/\rho_B = 1.007$ for B = 6.6 T and 1.14 for B = 0.33 T.

Therefore at peak static magnetic fields we neglect any size effect of the copper layer but include it for the start-up magnetic field in the surface impedance at f = 2.5 kHz.

Conclusions

The surface impedance of 8 mil copper layer on the SSC beam pipe at 4.2°K has been investigated. Excitation frequencies excited by coupled-bunch instabilities and single-bunch instabilities at the revolution frequency of 2.5 kHz and several gigahertz have been considered where the applied magnetic flux densities range from the start-up value of 0.33 T to the operating value of 6.6 T.

For high frequencies of several gigahertz the surface impedance is roughly represented by the surface impedance for zero magnetic field in the extreme anomalous regime. The resistive wall impedance $Z_{\parallel}/n$ including magnetic corrections is then given by

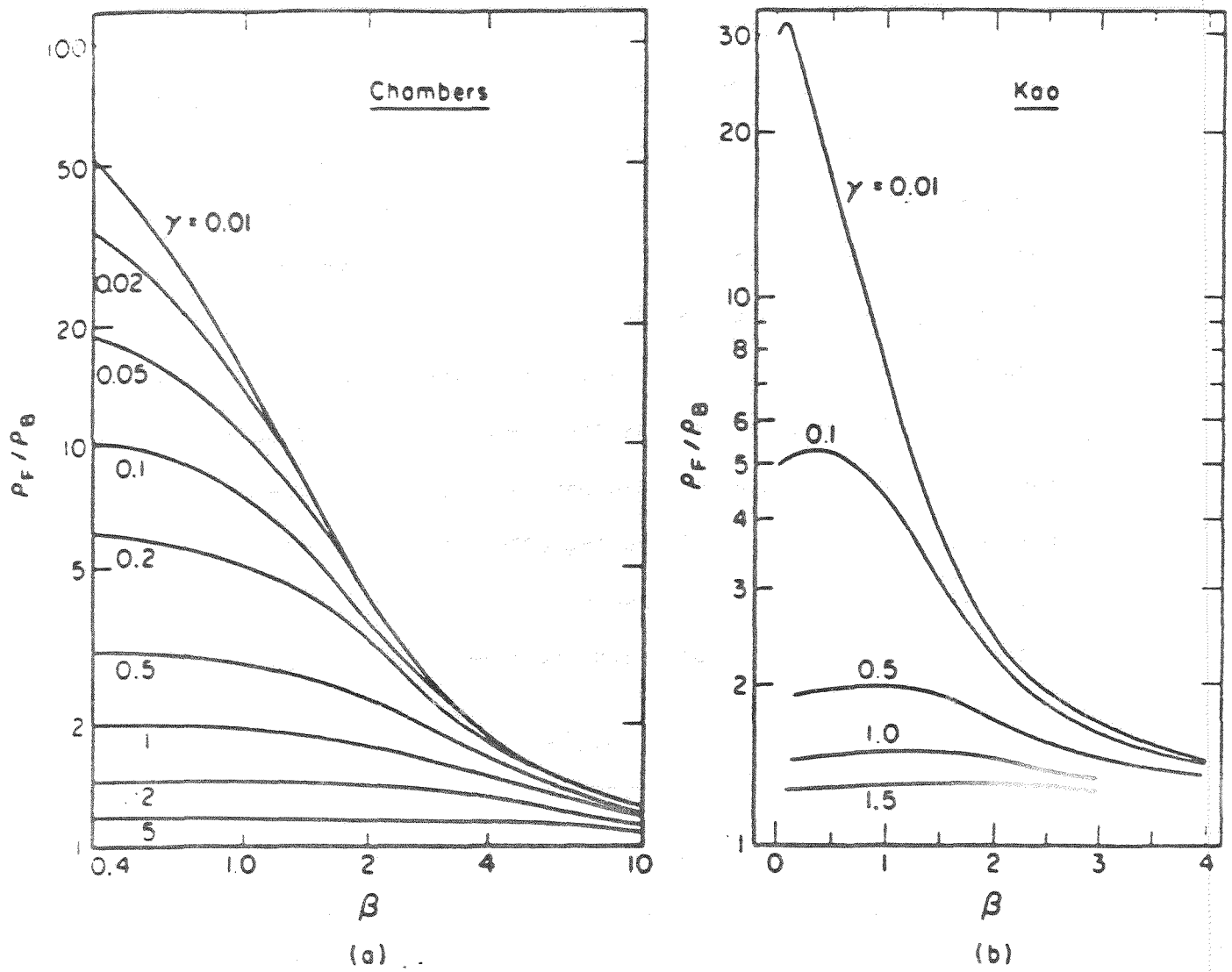


FIG. 18. Theoretical variation of the longitudinal magnetoresistance of (*a*) a wire based on Chambers' [197] calculations, (*b*) a film based on Kao's [199] calculations, as a function of $\beta = t/r$ (r is the orbital radius and t the specimen thickness). Both calculations assume $p = 0$ and *zero* bulk magnetoresistance.

FIGURE 4

$$\frac{Z_{\parallel}}{n} = \frac{c}{\omega b} Z_{\infty} = \frac{c}{\omega b} \frac{1}{\Sigma_{\infty}} (1+j\sqrt{3}) \tanh^{1/3}\left(\frac{\pi}{\omega_c \tau} (1+j\omega\tau)\right) \qquad (23)$$

where Σ_{∞} is the measured conductance in the microwave cavities measurements at 4.2°K in copper since we are in the extreme anomalous limit. The values of the impedances are tabulated in Table V. The impedances are given with (B = 6.6 T) and without (B = 0) the magnetic correction term $\tanh^{1/3} (\pi/\omega_c\tau\ (1+j\omega\tau))$. The impedances for B=0 apply to the case of the magnetic field perpendicular to the metal surface and for B = 0.33 T for the magnetic field parallel to the surface; the impedances for B = 6.6 T apply to the case for the magnetic field parallel to the surface. For the magnetic field parallel to the surface ω/ω_c is small compared to 1 for B = 6.6 T and neglected; for B = 0.33 T ω/ω_c is the order of 1 but $\pi/\omega_c\tau$ is so large compared to 1 that the correction factor is 1. The values of λ are the same as those used in Table III.

Table V

	$Z_{\parallel}/n$ (B = 0) (mΩ)	$Z_{\parallel}/n$ (B = 6.6 T) (mΩ)	$Z_{\perp}$ (B = 0) (MΩ/m)	$Z_{\perp}$ (B = 6.6 T) (MΩ/m)
λ_1	$2.63\ (1+j\sqrt{3})$	$2.63\ (i+j\sqrt{3})$	$0.335\ (i+j\sqrt{3})$	$0.335\ (i+j\sqrt{3})$
λ_2	$4.62\ (1+j\sqrt{3})$	$3.12\ (i+j\sqrt{3})$	$0.588\ (i+j\sqrt{3})$	$0.400\ (i+j\sqrt{3})$
λ_3	$4.16\ (i+j\sqrt{3})$	$4.15\ (i+j\sqrt{3})$	$0.530\ (i+j\sqrt{3})$	$0.528\ (i+j\sqrt{3})$
λ_4	$4.16\ (i+j\sqrt{3})$	$3.77\ (i+j\sqrt{3})$	$0.530\ (i+j\sqrt{3})$	$0.480\ (i+j\sqrt{3})$

For the revolution frequency f_o = 2.5 kHz the surface impedance for the normal skin effect given by Eq. (1) is appropriate. The zero magnetic field resistivity ratio must be corrected from the Kohler

plot to give the equivalent resistivity ratio in a magnetic field and for the start-up magnetic field a size effect correction made. Since the Kohler plot is for a reference temperature of 273°, we find the zero magnetic field resistivity ratios become 90 and 270 respectively. The effective resistivity ratios from the reduced Kohler plot in Figure 2 are 32 and 41.5 respectively. For B = 6.6 T the corresponding $Z_{\parallel}/n$ are: 2.78 (1+j)Ω and 2.44 (i+j)Ω. For the transverse impedance $Z_{\perp}$ the corresponding values are: 354 (1+j) MΩ/m and 311 (1+j) MΩ/m. The value for $Z_{\perp}$ for the lower resistivity ratio is nearly the same as the SSC value in Eq. (6). If the d.c. resistivity ratio is tripled, $Z_{\perp}$ decreases slightly. For the start-up B = 0.33 T the lower resistivity ratio gives a point off the graph. For RR = 270 we find with the size correction that $Z_{\parallel}/n$ = 1.18 (1+j)Ω and $Z_{\perp}$ = 150 (1+j)MΩ/m. The impedances are the narrow band averages.

To calculate the broadband average of the impedance we use

$$\overline{Z}_{\perp} = \frac{\sigma_{\ell}}{c\sqrt{\pi}} \int_{-\infty}^{\infty} d\omega \, Z_{\perp}(\omega) e^{-\frac{\omega^2 \sigma_{\ell}^2}{c^2}} , \quad \sigma_{\ell} = 7 \times 10^{-2} \text{ m} \quad . \tag{24}$$

from Eq. (4.3.1) in the SSC design study. From Eqs. (1) and (3) we see that $Z_{\perp}(\omega) \alpha \omega^{-1/2}$ for the surface impedance in the normal skin effect regime. From Eqs. (11) and (23) roughly we find that $Z_{\perp} \alpha \omega^{-1/3}$ in the anomalous skin effect regime. If $Z_{\perp} \alpha \omega^{-\overline{n}}$, then the integral evaluated in Eq. (24) gives

$$\overline{Z}_\perp = \left(\frac{\sigma_\ell}{c}\right)^{\overline{n}} \frac{\Gamma\left(\frac{1-\overline{n}}{2}\right)}{\sqrt{\pi}} Z_\perp(\omega)\, \omega^{\overline{n}} \tag{25}$$

by integral 662 in CRC tables.[47] For $\overline{n} = 1/2$ we find

$$\frac{\overline{Z}_\parallel}{n} = 0.011\ (1+j)\Omega\ , \quad \overline{Z}_\perp = 1.41\ (1+j)\ \frac{M\Omega}{m} \tag{26}$$

which are close to the design values of 0.01Ω and 1.5 $M\Omega/m$ respectively.

For $\overline{n} = 1/3$

$$\frac{\overline{Z}_\parallel}{n} = 8.181(1+j\sqrt{3})\ m\Omega\ , \quad \overline{Z}_\perp = 1.04\ (1+j\sqrt{3})\ \frac{M\Omega}{m}\ . \tag{27}$$

No magnetic correction is included in Eq. (27); that could reduce the impedances further. Actually the integral should be over the exact $Z_\perp(\omega)$. For low frequencies $Z_\perp \propto \omega^{-1/2}$ and for high frequencies $Z_\perp \propto \omega^{-1/3}$ in the normal and anomalous skin effect regimes respectively. The high transition frequencies (for $\frac{\sigma}{\sigma_0} = 100$ as found on page 23) between these two regimes are 28.0 MHz (6.6 T) and 69.9 kHz (0.33 T) which are small compared to the frequency $c/2\pi\sigma_1 = 682$ MHz at which the Gaussian in the integral in Eq. (24) is e^{-1}. Consequently, the integral over the normal regime can be approximated by replacing the Gaussian in Eq. (24) by 1;

that part of the integral is small compared to the total integral. We conclude that the effective values of cold copper impedances (broad band averages) are better represented by Eq. (27) than by Eq. (26) with the proviso that corrections at the peak magnetic field may need to be added. The correction factors given near the bottom of page 22 for our cases give at most a 30% reduction in the impedance depending on the strength of the magnetic field and magnetic field orientation with respect to the normal to the copper surface.

In conclusion the cold copper impedances associated with the coupled-bunch modes, the narrow band average impedances, are well represented by resistive wall impedances of a metal with normal skin effect. The resistivity ratio used to calculate the cold copper resistivity is an effective one found from the zero magnetic field resistivity ratio and the reduced Kohler plot for magneto resistance. The calculated values for this study are given on page 29. The cold copper impedances associated with single-bunch modes, the broad band average impedances, are well represented by the Gaussian-weighted average over the resistive wall impedances for a metal with anomalous skin effect. Reductions in the zero magnetic field values of the broad band average impedances up to 30% are found if the magnetic field corrections are included. The calculated values of these impedances in this study are given on page 30.

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