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Choice of the Accelerating Frequency in the SSC

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Abstract: A frequency of about 360 MHz is desirable for the RF acceleration system of the SSC from considerations of beam dynamics and practicality. The geometrical constraints of the latest ring geometry (82.944000 km circumference, clustered interaction regions) leads to a frequency of 374.74 MHz. However, if each of the eight insertions is slightly modified, several other frequencies closer to 360 MHz are possible.

The frequency of the RF accelerating system in the SSC is subject to the following constraints:

(1) For proper acceleration and storage the radio frequency f must be an exact harmonic h of the revolution frequency fo on the central orbit of circumference C_R (82.944000 km in SYNCH L JAN3 BIO, Garren).

$$f = hfo = \frac{h\beta c}{C_R} \tag{1A}$$

where β is the beam velocity in units of c, the velocity of light. Since β at 20 TeV is so close to unity ($1 - 1.100 \times 10^{-9}$), it is convenient and sufficient (for 8-place accuracy) to use the equivalent RF wavelength λ condition

$$C_R = h\lambda \tag{1B}$$

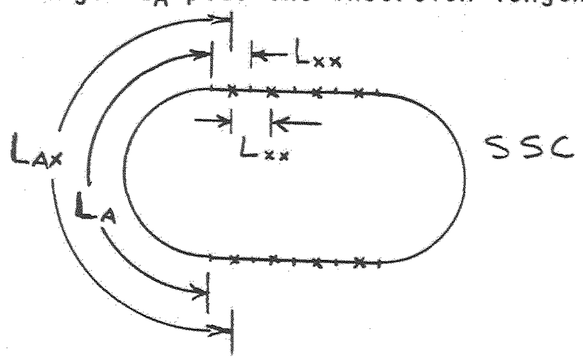
(2) In order to have collisions at all possible interaction points the inter-IP lengths L_{xx} and L_{Ax} must <u>both</u> be integral numbers of the half-inter-bunch spacing $S_B/2$

$$L_{xx} = n_x \frac{S_B}{2} \tag{2A}$$

and

$$L_{Ax} = n_{Ax} \frac{S_B}{2} = L_A + L_{xx} \tag{2B}$$

where we have noted that the around-the-arc-inter-IP spacing L_{AX} is equal to the regular-arc length L_A plus the insertion length L_{xx}.



For the lattice under consideration, $L_{AX} = 34{,}272$ m, $L_{XX} = 2{,}400$ m, and $L_A = 31{,}872$ meters. With the nominal inter-bunch spacing of 5 meters and a frequency near 360 MHz, the fraction of buckets filled with beam is 1/6. The conditions (2A) and (2B) can be expressed equivalently in terms of partial harmonics h_{XX} and h_a

$$L_{XX} = h_{XX}\,\lambda \tag{3A}$$

$$L_A = h_A\,\lambda \tag{3B}$$

where $h_{XX} = 3n_X$ and $h_A + h_{XX} = 3n_{AX}$.

(3) The third constraint is that the partial harmonics h_A and h_{XX} allow a reasonable number of different bunch spacings. If we express the possible bunch spacings as $n_f\,\lambda$, the condition on the filling integer n_f is that it be factor common to $2h_A$ and $2h_{XX}$. A similar but secondary constraint is that the possible n_f values include small multiples of 6 in order to be compatible with the "60 MHz" RF systems in the injector chain.

<u>Case 1. Fixed Geometry</u>

For the geometry specified earlier, the ratio of partial harmonics is

$$h_A/h_{XX} = 31{,}872.00/2400.00 = 13.28000 = 13\ 7/25.$$

From this numerology (including the factor of 3 requirement) we see that h_{XX} must be a multiple of 75--or, better yet, a multiple of 150, which satisfies the factor-of-6 feature. For the nominal 360 MHz, $h_{XX} \simeq 2400/.833 \simeq 2881$. The two nearest multiples of 150 are 2850 and 3000. For these two cases we obtain the following sets of parameters:

	(a)	(b)
h_{XX}	$2{,}850(= 2\cdot3\cdot5^2\cdot19)$	$3{,}000(= 2^3\cdot3\cdot5^3)$
$h_A(= 13.28\ h_{XX})$	$37{,}848(= 2^3\cdot3\cdot19\cdot83)$	$39{,}840(= 2^5\cdot3\cdot5\cdot83)$
common factors	$2^2\cdot3\cdot19$	$2^4\cdot3\cdot5$
possible n_f	2, 3, 4, 6, 12, 19, 38, 57 ,76, 114, 228.	2, 3, 4, 5, 6, ..., 240. (incl. 6x1, x2, x4, ..., x40)
possible S_B	1.7, 2.5, 3.4, 5.1, 10.1, ..., 192.	1.6, 2.4, 3.2, 4, 4.8, ..., 192. m
f	356.004	374.741 MHz
λ	0.842105	0.800000 m

The 374.7 MHz case is clearly preferable because of its greater compatibility with the "60 MHz" (i.e., f/6) RF systems in the injector chain. Having a "nice" wavelength (0.800 m) also is attractive.

Case 2. Variable Insertion Length

Let us consider what frequencies near 360 MHz are feasible if we let the insertion length L_{xx} vary slightly from 2,400 meters, but keep the arc length L_A fixed at 31,872 meters. The approximate partial harmonic numbers are

$$h_A = \frac{L_A}{\lambda} \approx \frac{31,872}{0.833} \approx 38,246$$

$$h_{xx} = \frac{L_{xx}}{\lambda} \approx \frac{2,400}{0.833} = 2,880 = 2^6 \cdot 3^2 \cdot 5$$

Since 2,880 has so many factors (including 6), let us adopt it as h_{xx}. Then for trial values of h_A, find the corresponding values of λ, f, L_{xx}, n_f, etc. Four examples are given below:

	a	b	c	d
h_A	38,244 ($2^2 \cdot 3 \cdot 3187$)	38,250 ($2 \cdot 3^2 \cdot 5^3 \cdot 17$)	38,280 ($2^3 \cdot 3 \cdot 5 \cdot 11 \cdot 29$)	38,400 ($2^9 \cdot 3 \cdot 5^2$)
λ	0.833 386	0.833 255	0.832 602	0.830 000 m
f	359.728	359.785	360.067	361.196 MHz
L_{xx}	2,400.150	2,399.774	2,397.893	2,390.400 m
ΔC_R	+1.200	-1.808	-16.856	-76.800 m
com.fac	$2^3 \cdot 3$	$2^2 \cdot 3^2 \cdot 5$	$2^4 \cdot 3 \cdot 5$	$2^7 \cdot 3 \cdot 5$
poss.n_f	2, 3, 4, 6, 8, 12, 24 [6x(1,2,4)]	2, 3, 4, 5, 6, ..., 180 [6x(1,2,3,5, ..., 30)]	2, 3, 4, 5, 6, ..., 240 [6x(1,2,4,5, ..., 40)]	2,3,4,5, ...,1920 [6x(1,2, ..., 320)]

These examples illustrate that many frequencies near 360 MHz are feasible and require only relatively small changes in the insertion or arc lengths.

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