High Energy Facilities

Accelerator Development Branch

BROOKHAVEN NATIONAL LABORATORY

Associated Universities, Inc.

Upton, New York 11973

SSC Technical Note No. 42
SSC-N-116

AZIMUTHAL COIL PRESTRESS AND MOTION IN SUPERCONDUCTING COSθ MAGNETS, IN PARTICULAR SSC DIPOLES WITH COLLARED COILS, AND PRESTRESS LOSSES DURING COOLDOWN.

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Summary

Formulas are derived for calculations of coil motions when a "cosθ" dipole is azimuthally prestressed and powered. It is assumed that a distribution of azimuthal Lorentz force components has been calculated from field distributions and currents. Azimuthal stress distributions and minimum prestress to minimize coil motion are calculated, as well as coil motions if applied prestress is less than minimum. Angular location of maximum coil motion for different prestress conditions is given. Calculations are also discussed for different elastic moduli of spacers used at and near the magnet poles. For a linear stress-strain relationship in the operating range, and for force distributions which depend more or less on azimuthal angular location only, coil motions are proportional to the square of the coil radius, and minimum prestress is linearly dependent on radius. Coil motion is dependent on the inverse of the elastic modulus, and minimum prestress is independent. Radial pressure exerted by the coil is independent of the radius. Since integrated coil forces are proportional to coil radius, friction effects, therefore, would begin to become important at higher fields for smaller magnets. Since motion decreases with the square of decreasing radius, heat energy produced by friction is much smaller for smaller magnets. Generally, from the point of view of the present considerations, smaller magnets should encounter fewer difficulties than larger ones. For the stainless steel-collared SSC dipole, maximum coil motions vary from approximately 0.002" at the poles without prestress to 0.0003" to 0.0004", at smaller angles, with minimum or more prestress. More than minimum prestress has no direct effect on coil motion but will increase friction forces. Minimum calculated prestresses (at operating temperature) are about 3000 psi. For non-metallic pole spacers (with smaller elastic modulus), coil motions increase only by a few tenths of a thousandth of an inch which is a small fraction of the expected magnet construction accuracy.

Loss of prestress due to cooldown of coil-collar assemblies has been calculated, also taking into account increases of elastic moduli with decreasing

temperature, indicated by measurements of straight conductor stacks at BNL. If for actual collared magnet coils the average modulus also changes in this manner, loss of prestress during cooldown becomes greater than for constant modulus. With Nitronic 40 collars prestress losses of 800 to 1500 psi are calculated for pertinent cases. With aluminum collars prestress gains of 400 to 600 psi are found, and with direct constraint by the iron yoke prestress losses are roughly twice as large as with Nitronic 40. If the elastic modulus of the constraining structure is much larger than that of the coil, which is indeed the case for relevant materials, good approximate results can be calculated by means of a simple expression (eq. 49).

Finally, a procedure is given for considering prestress conditions when the stress-strain relation for coils is non-linear, which will be especially the case for low stress. Calculations will be performed at a later date.

Relevant and illuminating discussions with R. J. LeRoy were highly appreciated.

1. Calculations assuming pole spacers with large elastic modulus.

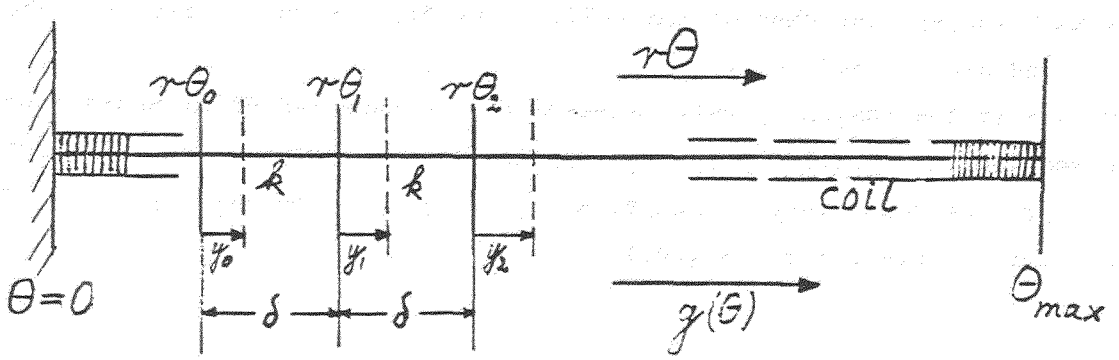
In order to limit conductor motion in superconducting magnets employing coils with a $\cos\theta$ current density distribution (θ measured from magnet midplane), it is necessary to prestress the coil azimuthally. This reduces field perturbations and the probability that the coil becomes normally conducting (quenches). Prestress will also tend to preserve the mechanical integrity of the coil, reducing abrasion of insulation, loosening of shims, breakage of bonds, etc. In ISABELLE Technical Note No. 280, "Estimates of Dynamic Energy Releases due to Elastic, Frictional, Bondbreaking, and Grinding Motions in ISA Dipoles, and a Comparison with Estimates of Quench Initiating Energies", many of the mentioned effects were considered in detail. Here we merely intend to derive general formulas for estimates of required prestress, or lack of it, and resulting coil motion, for the static case, neglecting friction, bonds, etc. Coil motion will be of particular interest.

The Lorentz force acting azimuthally on a conductor element is determined by the local radial field component multiplied by the local current density and by the size of the element. (The azimuthal field component is responsible for radial forces, as is the azimuthal prestress when combined with azimuthal force components, resulting in radially outward pressure. These radial forces also affect azimuthal conductor motion through friction (see the already mentioned Tech. Note 280).)

The magnetic field inside the inner coil diameter of a dipole ideally has the same direction at the poles as in the midplane. Therefore the azimuthal force would be maximum at $\theta = 90^{\circ}$ and zero at $\theta = 0$, if the field remained parallel to the central field, as it passes through the conductors (actually it does not remain parallel). The azimuthal component would follow a $\sin\theta$ distribution. Since the conductor density distribution, ideally, follows a $\cos\theta$ distribution, assuming that the current in the conductors is constant, the azimuthal force distribution would follow a $\sin\theta \cos\theta \sim \sin 2\theta$ distribution. In spite of this much oversimplified consideration, for actual single-layer coils the $\sin 2\theta$ approximation, multiplied by the right amplitude, turns out to be quite good. For two- or more-layer coils, of course, we cannot expect such an agreement for individual coil layers. For the inner layer of a two-layer coil the conductor distribution would usually still become zero near the poles, but for the outer coil, this happens at a much smaller θ, and therefore the force distribution deviates strongly from $\sin 2\theta$. Generally, we shall call the angular force distribution $g(\theta)$.

The azimuthal stress-strain relation for a coil rises slowly (coil is "soft", has low "elastic modulus" E) for low stresses, and much faster for the usual operating stresses, where E then is usually fairly constant at a value of 1 to 2 x 10^6 psi at room temperature and possibly higher at magnet operating temperature (see Section 3). In order to take into account the presence of wedges in a coil quadrant, we can use a somewhat modified modulus for an average value of E. Otherwise, one would have to consider each coil block separately, complicating calculations and resulting formulas unnecessarily.

We present a very simple model for a magnet coil quadrant:



As a result of force distribution $g(\theta)$, we obtain deflections y_o at angle θ_o, y_1 at θ_1, etc. Length elements are $\delta = r\Delta\theta_n$. r is the average radius for each coil. If the azimuthal Lorentz force per coil turn (f_n) is given, $g(\theta)$ is given by

$$g(\theta) \approx \frac{f_n}{\delta}$$

Integrating $g(\theta)$ would then result in

$$\int_{\theta_1}^{\theta_2} g(\theta) r d\theta \approx \sum_{n_1}^{n_2} f_n \ .$$

A scaled-up magnet would contain more turns for fixed current. Therefore the given integral is indeed proportional to the coil radius r.

$\int_{\theta_1}^{\theta_2} g(\theta) d\theta$, as used below, is then obviously independent of r.

$k = hE$ is the spring constant per unit axial length of the coil for a unit azimuthal length. h is the radial width of the coil. The spring constant per element of length δ is thus k/δ. (The average elastic modulus E is calculated from

$$\frac{\theta_{max}}{E} = \sum_n \frac{\Delta\theta_n}{E_n}$$

where θ_{max} is the maximum coil angle ($\pi/2$, if pole spacer and shims are to be included or, otherwise, the angle where conductors end, if the pole spacer elastic modulus is much larger than that of the coil). The $\Delta\theta_n$ are the ranges of θ where the material has elastic modulus E_n.)

(y_1-y_0) is the length by which element δ is compressed or extended from its unstressed length. Similarly (y_2-y_1) applies to the next element. The forces necessary to produce these length changes are $(k/\delta)(y_1-y_0)$ and $(k/\delta)(y_2-y_1)$. The difference between these forces is $g(\theta)\delta$:

$$(k/\delta)\left[\left(y_2-y_1\right)-\left(y_1-y_0\right)\right]=-g(\theta)\delta \tag{1}$$

As can be seen from the sketch, $g(\theta) > 0$ as drawn, acts so that element $r(\theta_1-\theta_0)$ is under greater tension than element $r(\theta_2-\theta_1)$. Therefore the negative sign at the right side of eq. 1. Proceeding to differentials

$$\left[y_2 \approx y_1 + \frac{dy}{rd\theta}\, rd\theta,\ \text{etc.}\right]:$$

$$\frac{k}{r^2}\ \frac{d^2y}{d\theta^2} = -g(\theta) \tag{2}$$

We will be interested in compressive strains due to compressive stresses. Thus $g(\theta) < 0$ (referring to the above-given sketch). Also $y(\theta)$ would be < 0. As a simplification, we shall count both compressive stress and negative y as positive and thus use eq. 2 without further concern with sign reversals.

Boundary conditions for eq. 2 are:

$$\begin{aligned} \theta &= 0 \quad : \quad y = 0 \\ \theta &= \theta_{max} \quad : \quad \frac{dy}{d\theta} = 0 \end{aligned} \tag{3}$$

if coil motion is not constrained at $\theta = \theta_{max}$ and

$$\theta = \theta_{max} \quad : \quad y = y_s \tag{4}$$

if the coil is stressed prior to being powered ("prestressed"). If the prestress becomes insufficient, boundary conditions (3) will again apply after it is exceeded.

Assume that the coil, to begin with, is not prestressed, up to some distribution $g_a(\theta)$. Then integration of eq. 2 with boundary conditions (3) results in

$$\frac{k}{r^2}\, y_a(\theta) = \int_0^{\theta}\int_{\theta}^{\theta_{max}} g_a(\theta)\, d\theta\, d\theta \equiv I_a(\theta) \tag{5}$$

The maximum deflection y_{amax} is given by

$$\frac{k}{r^2}\ y_{amax} = I_a\left(\theta_{max}\right) \tag{6}$$

Now assume that we prestress the (unpowered) coil to a deflection $y_s = y_{amax}$ at $\theta = \theta_{max}$. Then, when the coil is powered, it will reach force distribution $g_a(\theta)$ without motion at $\theta = \theta_{max}$. Before powering, the prestressed coil is deflected according to

$$y_a(\theta) = \frac{\theta}{\theta_{max}} y_{amax} = \frac{\theta}{\theta_{max}} y_s \tag{7}$$

since we have assumed that the stress-strain relation for the coil is linear in the operating range. The prestress required to provide $y_0(\theta)$ is

$$\sigma_a = \frac{y_s}{r\theta_{max}} E = \frac{r}{h\theta_{max}} I_a\left(\theta_{max}\right) \tag{8}$$

Note that this prestress is proportional to the average coil radius r. (The double integral $I_a(\theta)$, as presented here, again, is independent of the radius.) A small aperture $\cos\theta$ magnet requires less prestress for a given magnetic field, simply because the integrated azimuthal force is smaller than for a larger magnet. Note also that σ_a is independent of elastic modulus E.

Next, assume that we power the coil to still higher fields so that force distribution $g(\theta) > g_a(\theta)$ results, thus exceeding the constraint provided by prestress σ_a. Corresponding to eq. 5, we now would obtain

$$\frac{k}{r^2} y_b(\theta) = \int_0^{\theta} \int_{\theta}^{\theta_{max}} g(\theta)\, d\theta d\theta = I(\theta) \tag{9}$$

if σ_a (or $g_a(\theta)$) had been zero. If $\sigma_a \neq 0$, we must subtract eq. 7 from eq. 9, since we are only interested in the net motion between prestressed unpowered and powered states. Combining eqs. 6, 7 and 9:

$$\frac{k}{r^2} y(\theta) = \frac{k}{r^2}\left[y_b(\theta) - y_a(\theta)\right] = I(\theta) - \frac{\theta}{\theta_{max}} I_a\left(\theta_{max}\right) \tag{10}$$

Summarizing:

(1) If no prestress at all is provided:

$$g_a(\theta) = 0$$

$$y(\theta) = \frac{r^2}{k} \int_0^{\theta} \int_{\theta}^{\theta_{max}} g(\theta)\, d\theta d\theta \tag{11}$$

(2) If minimum prestress σ or more (to be proved below) is provided to prevent motion at $\theta = \theta_{max}$:

$$g_a(\theta) = g(\theta)$$

$$y(\theta) = \frac{r^2}{k} \left[\int_0^{\theta} \int_{\theta}^{\theta_{max}} g(\theta)\, d\theta d\theta - \frac{\theta}{\theta_{max}} \int_0^{\theta_{max}} \int_{\theta}^{\theta_{max}} g(\theta)\, d\theta d\theta \right] \tag{12}$$

$$\sigma \geq \frac{r}{\theta_{max} h} \int_0^{\theta_{max}} \int_{\theta}^{\theta_{max}} g(\theta)\, d\theta d\theta$$

(3) If less than minimum prestress is provided,

$$g_a(\theta) < g(\theta)$$

$$y(\theta) = \frac{r^2}{k} \left[\int_0^{\theta} \int_{\theta}^{\theta_{max}} g(\theta)\, d\theta d\theta - \frac{\theta}{\theta_{max}} \int_0^{\theta_{max}} \int_{\theta}^{\theta_{max}} g_a(\theta)\, d\theta d\theta \right] \tag{13}$$

$$\sigma_a = \frac{r}{\theta_{max} h} \int_0^{\theta_{max}} \int_{\theta}^{\theta_{max}} g_a(\theta)\, d\theta d\theta$$

For application of these formulas it may also be convenient to write

$$y(\theta) = \frac{r^2}{k} \left[\int_0^{\theta} \int_{\theta}^{\theta_{max}} g(\theta)\, d\theta d\theta - \frac{\sigma_a h}{r} \theta \right] \tag{14}$$

which is valid up to $\sigma_a = \sigma$.

If σ exceeds the minimum value given in eqs. 12, we shall use the boundary condition given by eq. 4. The solution of eq. 2 is then, for the total motion y_t,

$$\frac{k}{r^2} y_t(\theta) = I(\theta) - \frac{\theta}{\theta_{max}} \left(I\left(\theta_{max}\right) + \frac{k}{r^2} y_s \right)$$

The prestress alone (without powering) must have resulted in the function $y_s\theta/\theta_{max}$ which must again be subtracted in order to obtain the net motion, resulting again in eq. 12. Therefore, prestress exceeding the minimum value has no further effect beyond the result obtained with the minimum. This is not surprising because of the assumed linearity of eq. 2 (constant E). Obviously some prestress margin may be desirable but we shall see that there is not very much need for it. Of course, we must use the conditions for the coil when it is at operating temperature; effects due to temperature changes and also due to stress relaxation in time must have been taken into account.

While we have assumed linearity for these derivations, intentions are also to investigate effects of the actual non-linear stress-strain relation. This would be of particular interest for prestress near minimum, where quite low stresses would result near the poles (see Section 4 for some derivations). A report on this subject may appear at a later date.

Without prestress, the maximum coil deflection y_{max} must occur at location $\theta = \theta_{max}$. With prestress y_{max} will move to smaller θ. With minimum prestress no coil deflection will occur at θ_{max}. (We are still assuming that the pole spacers, extending from $\theta = \theta_{max}$ to $\theta = 180^{\circ} - \theta_{max}$, are made of a material with elastic modulus much larger than that of the coil.) To find the location $\theta = \theta_m$ where y_{max} occurs, differentiate eq. 13 and set $dy/d\theta = 0$. Result:

$$I'\left(\theta_m\right) = \frac{\nu}{\theta} I\left(\theta_{max}\right) \tag{15}$$

where $\nu = g_a(\theta)/g(\theta) = \sigma_a/\sigma_{max}$, assuming that $g_a(\theta)$ is proportional to $g(\theta)$, and

$$I'\left(\theta_m\right) = \int_{\theta_m}^{\theta_{max}} g(\theta)d\theta .$$

Equation 15 can simply be solved by using the figures 1 and 2 given below. y_{max} as a function of ν is then

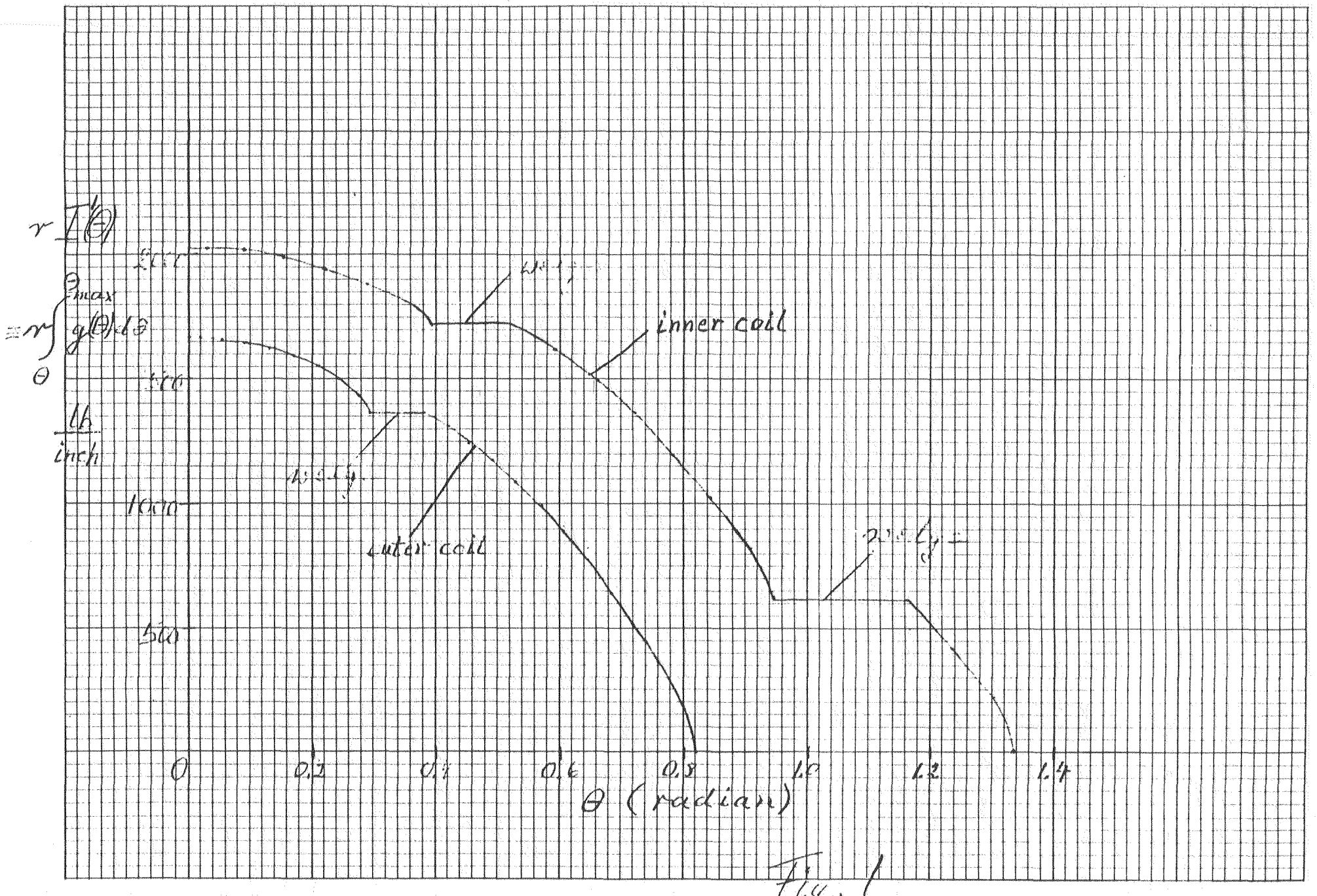
$$y_{max}\left(\theta_m\right) = \frac{r^2}{k}\left[I\left(\theta_m\right) - \frac{\nu\theta_m}{\theta_{max}} I\left(\theta_{max}\right) \right] \tag{16}$$

We present results for the SSC dipole magnet. Field distributions at the coils had been calculated by G. H. Morgan and forces f_n on the conductors presented by R. J. LeRoy. We have applied eqs. 13. Based on the available force distributions, figures 1 and 2 give the expressions

$$r\int_{\theta}^{\theta_{max}} g(\theta)d\theta \equiv rI'(\theta), \text{ and } rI(\theta)$$

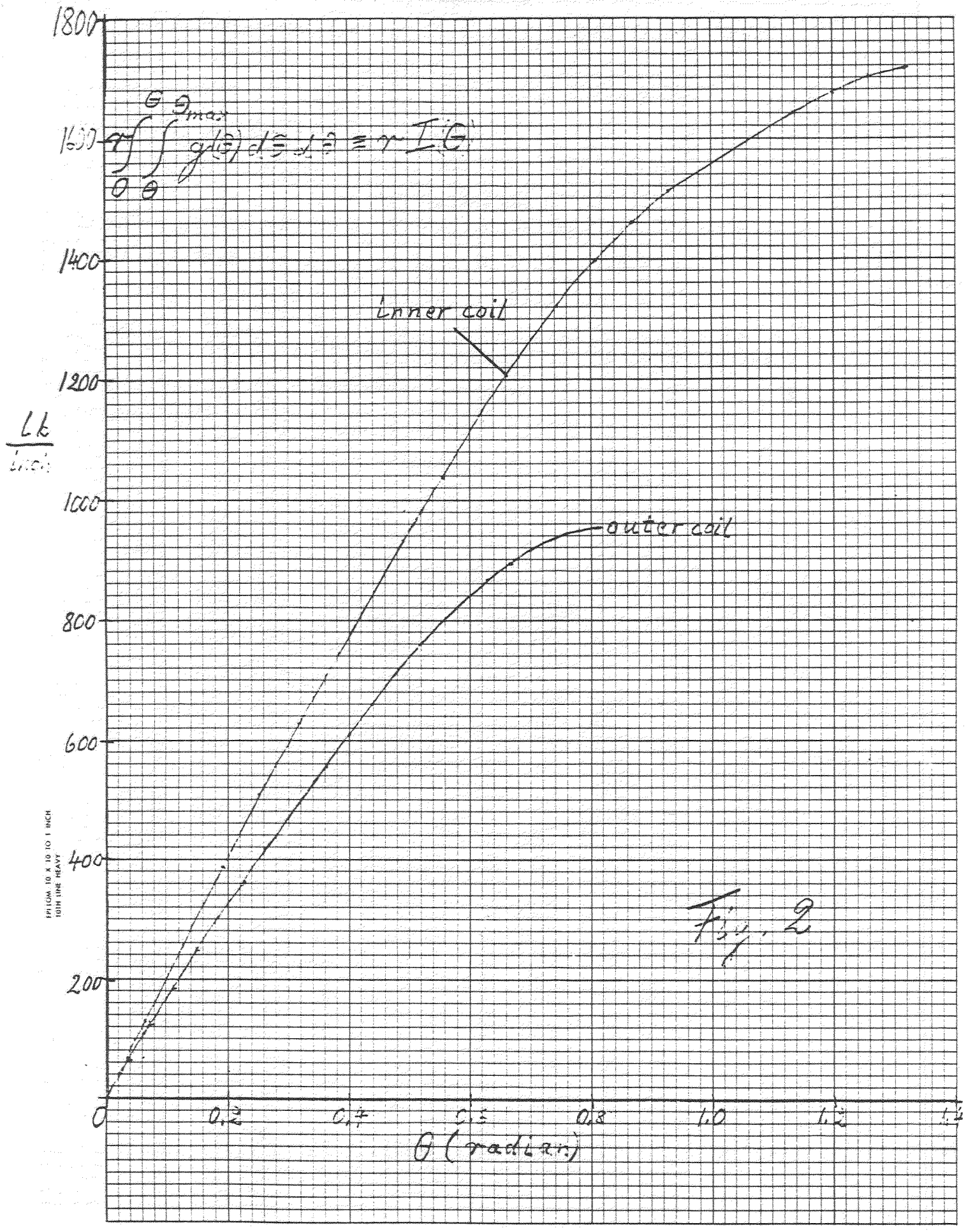
required for the calculations, and fig. 3 gives $y_{max}(\theta_m)$ and θ_m as functions of ν. For these calculations a value of $E = 2 \times 10^6$ psi has been used for the elastic modulus of the coils. y_{max} is $\sim 1/E$ and $\sim r^2$, thus increases fast with coil radius.

From fig. 3 one sees that for the inner coil θ_m moves from $\theta = \theta_{max} = 76^\circ$ without prestress ($\nu = 0$) to $\theta = 42^\circ$ with minimum (or more) prestress. For the outer coil θ_m moves from 47° to 28°. y_{max} moves from 0.00210" to 0.00044" for the inner, and from 0.00165" to 0.00026" for the outer coil. It is obvious that operation at somewhat less than minimum prestress should not be particularly detrimental, but it should not be forgotten that coil motion can proceed in a "stick-slip" frictional mode which can easily produce sufficient heat to quench a coil as has been analyzed in detail in the Technical Note No. 280 referred to above. From the expression for the azimuthal stress distribution given below one can simply calculate the radial pressure which at radius r would be given by $p_r \approx \sigma_t h/r$ if σ_t is the stress due to combining prestress and coil force. The radial pressure at the outer coil surface is somewhat less by, approximately, the ratio r/r_{oi} if r_{oi} is the outer radius of the coil. At minimum prestress, the maximum radial pressure (at $\theta = 0$ and outer coil radius) for the inner coil is 1690 psi and, similarly, for the outer coil alone it is 1040 psi. For inner and outer coils



$r\,I'(\theta)$
$\equiv r\int_{\theta}^{\theta_{max}} g(\theta)\,d\theta$
$\frac{lb}{inch}$
2000
1500
1000
500
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
θ (radian)
inner coil
outer coil

Fig. 1



$r\int_0^{\theta}\int_{\theta}^{\theta_{max}} g(\theta)\,d\theta\,d\theta \equiv r\,I(\theta)$
1800
1600
1400
1200
1000
800
600
400
200
0
Lb/inch
inner coil
outer coil
0.2
0.4
0.6
0.8
1.0
1.2
1.4
θ (radian)

Fig. 2

θ_m = angle where maximum azimuthal deflection occurs.

y_{max} = maximum azimuthal deflection, for inner and outer SSC dipole coils. y_{max} is proportional $1/$(el. modulus E). $E = 2\times10^6$ psi was used here ("cold" modulus).

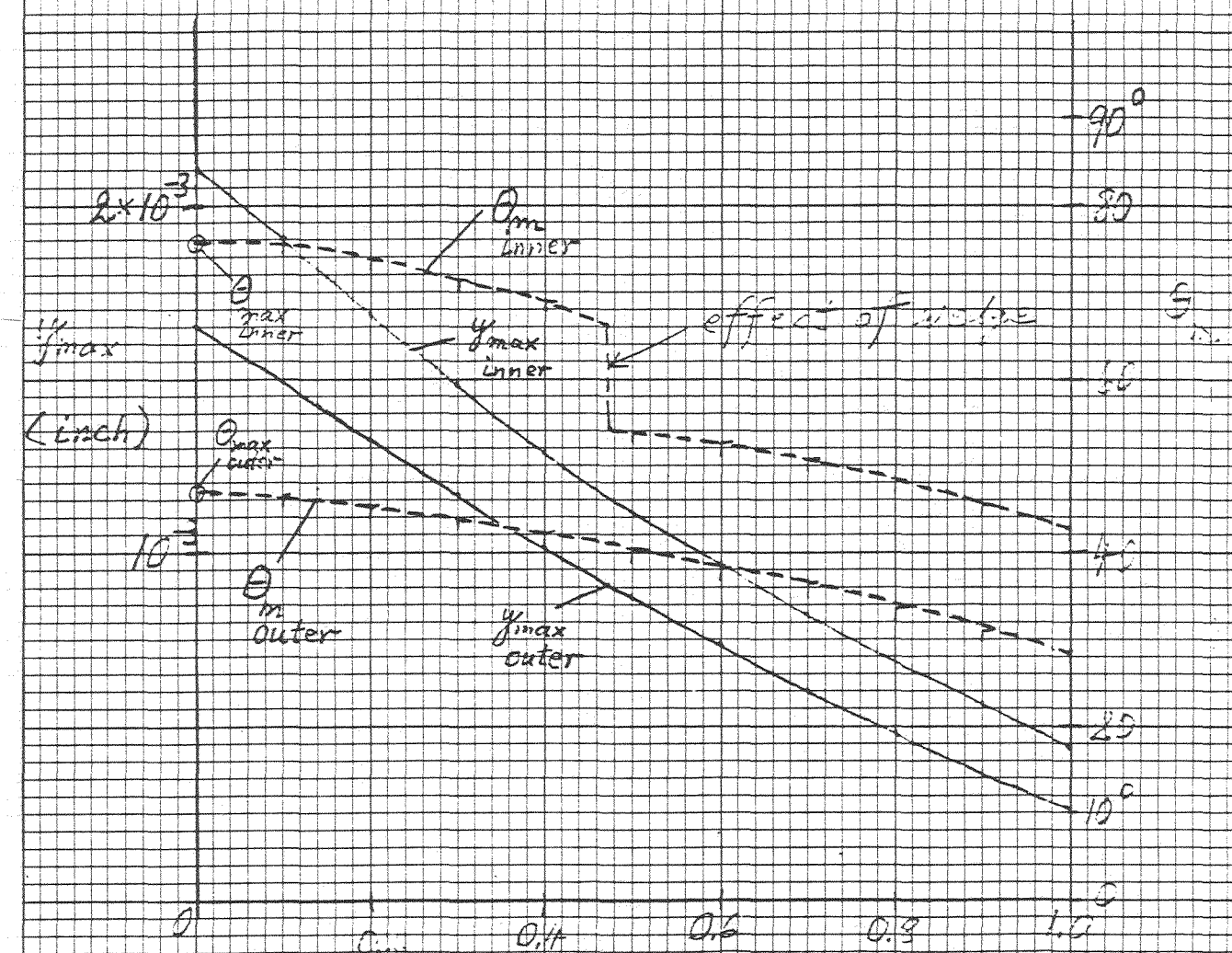
ν = fraction of minimum prestress applied. (Minimum prestress just "prevents" motion at poles* when coil is powered.)

Minimum prestress (independent of E): inner coil: 3250 psi ($\sim r_{av}$); outer coil: 2940 "

$y_{max} \sim r_{av}^2$

(* more generally: Min. prestr. prevents separation of coil from pole spacer)

r_{av} = average radius of inner or outer coil



$\nu = g_a(\theta)/g(\theta)$ Fig 3

combined one finds 2300 psi, having reduced the value for the inner coil further by the ratio r_{oi}/r_{oo}, r_{oo} being the outer radius of the outer coil. With a friction coefficient of $f = 0.2$, a frictional force per square inch of 460 psi is then obtained. To this one must add the radial component of the Lorentz force which is 2580 psi at radius r for the inner coil and 200 for the outer coil at its average radius. Combining pressures, $p_r = 3840$ for inner coil at $\theta = 0$, and radius r_{oi}, $p_r = 1210$ for outer coil at radius r_{oo}, and $p_r = 4100$ psi at r_{oo} for both coils combined. For $f = 0.2$, the azimuthal friction force per square inch of coil surface at $\theta = 0$ is then 820 psi. For the inner coil alone one obtains 770 psi at radius r_{oi}. The integrated azimuthal Lorentz force at $\theta = 0$ is 2020 lbs/inch (see fig. 1) which varies slowly near $\theta = 0$. Comparing this with the friction force shows that at full coil power the friction force would not restrain motion at $\theta = 0$. Near $\theta = \theta_{max}$, for minimum prestress only a small radial Lorentz force acts and the integrated azimuthal Lorentz force is also small. Resulting coil motion here is not clear without detailed analysis. At low power, the prestress provides enough radial pressure that friction should prevent motion. Obviously prestress above minimum would also restrain coil motion more at full power. On the other hand, if stick-slip motion should occur then, more heat would be produced.

Fortunately, for the SSC coils expected coil motions are quite small (fig. 3). (Since the azimuthal pressure for a coil is $\sim r$ for given $y(\theta)$, and radial pressure $p_r \sim 1/r$, p_r is independent of r. But coil motions are $\sim r^2$. Therefore much more heat can be produced in a larger coil due to friction than in a smaller one. Quenches could then start at lower fields.) Maximum inner coil motion amounts only to a fraction of one-thousandth of an inch as long as at least nearly enough prestress is provided. Field perturbations should also be small for this reason. They would be expected to increase with decreasing coil radius for a given movement. But, since coil movements decrease $\sim r^2$, harmonics due to Lorentz force-caused coil motion should decrease with coil radius r.

The required minimum prestress σ to prevent coil motion at $\theta = \theta_{max}$ is given by eqs. 12. Using fig. 2, we obtain $\sigma = 3250$ psi for the inner coil and 2940 psi for the outer. We are also interested in the total stress distribution in a coil when it is powered. This is obtained from

$$\sigma_t(\theta) = E \frac{dy_t(\theta)}{r d\theta} ,$$

where $y_t(\theta)$ is the total deflection as a function of θ of the coil, due to prestress and powering. We obtain

$$\sigma_t(\theta) = \frac{r}{h}\left[I'(\theta) - \frac{1}{\theta_{max}} I\left(\theta_{max}\right) \right] + \sigma_s \tag{17}$$

where σ_s is the prestress, now arbitrary but equal to or greater than minimum. Using figs. 1 and 2, we obtain for the inner coil

$\theta = 0$: $\sigma_t(0) = \sigma_s + 1800$ psi

$\theta = \theta_{max}$: $\sigma_t(\theta_{max}) = \sigma_s - 3250$ psi

and for the outer coil

$$\sigma_t(0) = \sigma_s + 1230 \text{ psi}$$
$$\sigma_t(\theta_{max}) = \sigma_s - 2940 \text{ psi}$$

Note that the stress increase at $\theta = 0$ is much smaller than the decrease at θ_{max}. However,

$$\sigma_t(0) - \sigma_t\left(\theta_{max}\right) = \frac{r}{h} I'(0) \tag{18}$$

where, according to fig. 1, $rI'(0)$ is the total integrated azimuthal force.

For interest, we present fig. 4 which gives maximum coil motion and θ_m for an azimuthal Lorentz force distribution $f\sin 2\theta$. Here $y_{max}(\nu=0)/y_{max}(\nu=1)$ is equal to π while for the inner SSC coil we had a ratio of 4.8 and for the outer 6.3; the actual force distributions differ considerably from the $\sin 2\theta$ distribution, and the actual given ratios are larger than π because the force distributions are skewed toward $\theta = \theta_{max}$.

2. Pole spacers with small elastic modulus.

So far we have assumed that the elastic modulus of the pole spacer material is much larger than that of the coils, so that the pole spacer will not "follow" the coil at $\theta = \theta_{max}$ when the latter is powered. This assumption is well justified (even for the outer coil which has a very large pole spacer) for the presently used stainless steel collars. However, if collars were not to be used for some reason, larger deviations from the presented results can be expected. Furthermore, instead of using average elastic moduli for the coils, one could take

f_1/f = fraction of minimum prestress provided.

$\psi_{max} = \frac{f}{4\pi} r^2 z_{max}$ = maximum motion in coil for a Lorentz force distribution $= f \sin 2\theta$.

θ_m = angle from midplane where ψ_{max} occurs.

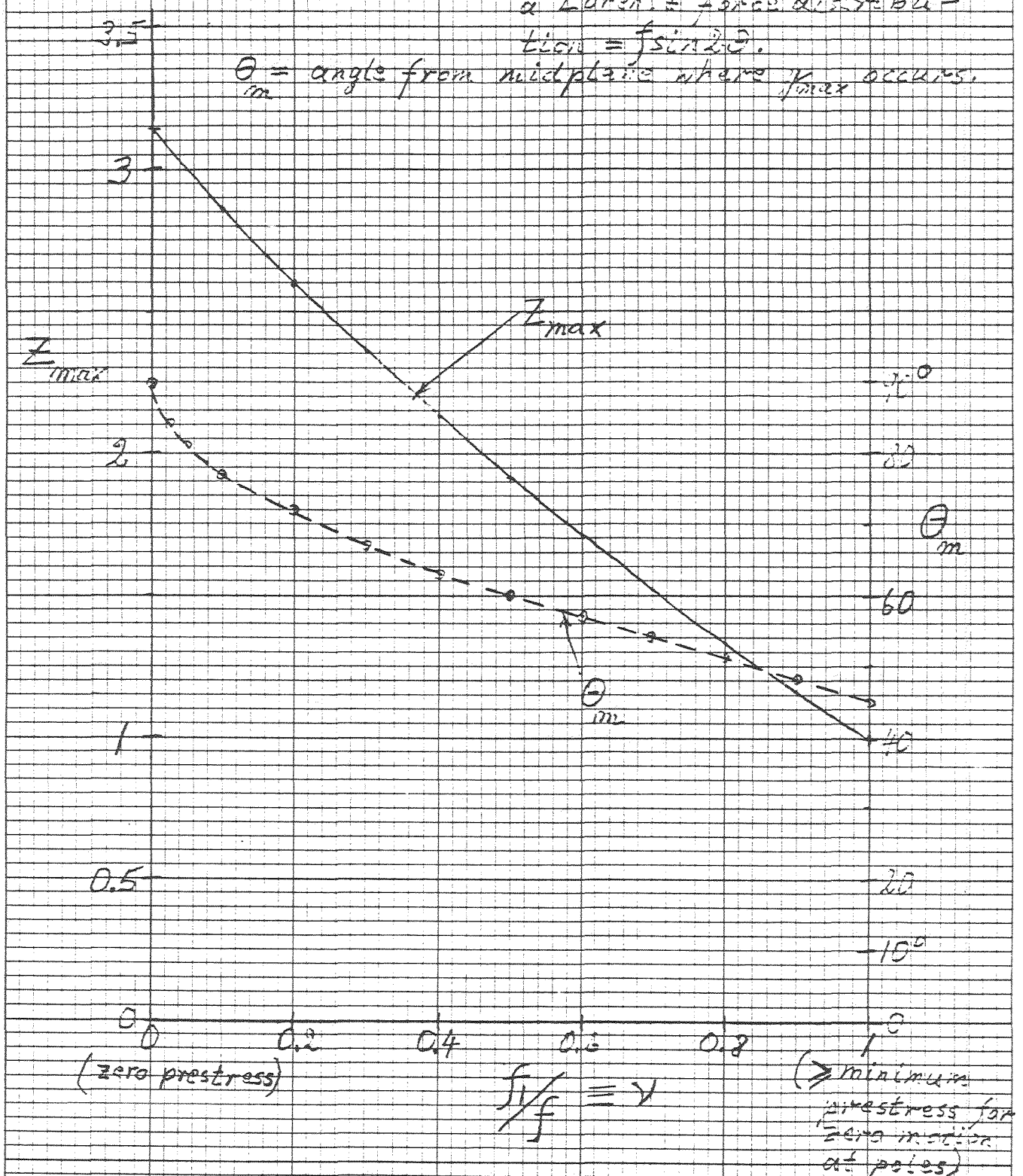


Fig. 4

the two inner coil wedges and the outer coil wedge into account individually. This would result in 6 equations of the type of eq. 2 to be solved with 12 boundary conditions for the inner coil (and 4 equations for the outer one) which can be done easily with a computer program but does not appear to be necessary, since the wedges are relatively small. We shall restrict ourselves to considering the pole spacers with elastic modulus E_2, or $k_2 = E_2 h$. Analogous to eq. 2:

$$\frac{k_1}{r^2} y_1'' = -g(\theta)$$

$$\frac{k_2}{r^2} y_2'' = 0 \tag{19}$$

since $g(\theta) = 0$ for $\theta > \theta_{max}$. Subscript (1) refers to the coil, (2) to the pole spacer. $y''_1 \equiv d^2y_1/d\theta^2$, etc. Boundary conditions:

$$\theta = 0: \qquad y_1 = 0$$

$$\theta = \theta_{max}: \qquad y_1 = y_2,\ k_1 y_1' = k_2 y_2'$$

$$\theta = \frac{\pi}{2}: \qquad y_2 = \Delta\ell$$

$\Delta\ell$ produces the prestress. It could be applied anywhere else, for instance by a shim at $\theta = \theta_{max}$. Giving some more detail on the proper procedure for the integration of the general function $g(\theta)$,

$$\frac{k_1}{r^2}\left[y_1'(\theta_{max}) - y_1'(\theta)\right] = -I'(\theta) + C_1, \tag{20}$$

where, again, $I'(\theta) = \int_{\theta}^{\theta_{max}} g(\theta)d\theta$,

$$\frac{k_2}{r^2} y_2'(\theta) = C_2 \tag{21}$$

Next

$$\frac{k_1}{r^2}\left[y_1(\theta) - y_1(0)\right] = I(\theta) - C_1\theta + \frac{k_1}{r^2}\theta y_1'(\theta_{max}) + D_1 \tag{22}$$

$$\frac{k_2}{r^2} y_2(\theta) = C_2\theta + D_2 \tag{23}$$

Since $y_1(0) = 0$ and $I(0) = 0$, $D_1 = 0$. Similarly, since $y_2\left(\frac{\pi}{2}\right) = \Delta\ell$

$$D_2 = \frac{k_2\Delta\ell}{r^2} - C_2 \frac{\pi}{2}$$

Applying the boundary conditions at $\theta = \theta_{max}$ results in

$$\frac{k_1}{r^2} y_1' (\theta_{max}) - C_1 = C_2 \tag{24}$$

since $I'(\theta_{max}) = 0$ (therefore also $C_1 = 0$ as seen from eq. 20).

$$C_2 = \left[\frac{\Delta\ell}{r^2} - \frac{1}{k_1} I(\theta_{max}) \right] \frac{k_2}{\left(\frac{\pi}{2} - \theta_{max}\left(1 - \frac{k_2}{k_1}\right)\right)} \tag{25}$$

$$y_1(\theta) = \frac{r^2}{k_1} \left[I(\theta) + C_2\theta\right] \tag{26}$$

$$y_2(\theta) = \Delta\ell - C_2 \left(\frac{\pi}{2} - \theta\right) \frac{r^2}{k_2} \tag{27}$$

$\Delta\ell_{min}$ (necessary "shim" size) to produce minimum prestress is obtained by setting $k_1\, y'_1(\theta_{max}) = 0$, resulting in the condition that $C_2 = 0$ or

$$\Delta\ell_{min} = \frac{r^2}{k_1} I(\theta_{max}) \tag{28}$$

Note that $\Delta\ell_{min}$ is independent of k_2. The minimum prestress follows by setting $g(\theta) = 0$ and obtaining

$$\sigma_{min} = \frac{k_2 y_2'}{rh} = C_2 \frac{r}{h} \tag{29}$$

remembering that the azimuthal force is $k_2 \frac{dy_2}{rd\theta}$.

Thus

$$\sigma_{min} = \frac{k_2}{k_1} \frac{r}{h} I\left(\theta_{max}\right) \Big/ \left(\frac{\pi}{2} - \theta_{max}\left(1 - \frac{k_2}{k_1}\right)\right) \tag{30}$$

If $k_2 \to \infty$, $\sigma_{min} \to \frac{r}{h\theta_{max}} I(\theta_{max})$. Thus, a factor

$$R\left(\frac{k_2}{k_1}, \theta_{max}\right) = \frac{\theta_{max}\,(k_2/k_1)}{\left(\frac{\pi}{2} - \theta_{max}\left(1 - \frac{k_2}{k_1}\right)\right)} \tag{31}$$

reduces the prestress found for $k_2 \gg k_1$, when k_2 is not $\gg k_1$.

Finally, the relative motion when the coil is powered is found by subtracting the initial deflection y_{10} due to prestressing from the total deflection due to prestressing and powering, namely y_1. To obtain y_{10}, we set $g(\theta) = 0$ in eqs. 25 and 26. The relative coil motion for $\sigma = \sigma_{min}$ at $\theta = \theta_{max}$ is then

$$y_1(\theta_{max}) - y_{10}(\theta_{max}) = \frac{r^2}{k_1} I(\theta_{max})(1-R) \tag{32}$$

The following Table gives R and $(y_1 - y_{10})$.

	INNER COIL		OUTER COIL	
k_2/k_1	R	$y_1 - y_{10}$ 10^{-4} inch	R	$y_1 - y_{10}$ 10^{-4} inch
1	0.84	1.4	0.52	3.2
2	0.91	0.8	0.68	2.1
3	0.94	0.5	0.76	1.6
5	0.96	0.3	0.84	1.1
10	0.98	0.2	0.92	0.5
15	0.99	0.1	0.94	0.4
∞	1	0	1	0

All of the values for $(y_1 - y_{10})$ are very small. For $k_2 \approx k_1$ they are approximately equal to the motion for $\nu = 1$ found for the outer coil for R = 1 (fig. 3). If non-metallic pole spacers were to be used, their elastic modulus

would probably be 2 to 3 times as large as that of the coil, therefore $k_2/k_1 = 2$ or 3; the calculated effect appears to be negligible. The minimum prestress (eq. 30) could be reduced somewhat for a plastic pole spacer.

3. Prestress loss or gain during cooldown.

Due to differences in thermal expansion with temperature, the coil prestress will change when a magnet is cooled down, depending on the materials involved. In addition, it has been observed that the elastic modulus (in the linear stress-strain region) of the coil material increases considerably at low temperatures (presumably due to the Kapton and fiberglass-epoxy insulation between coil turns and the plastic shim at $\theta = \theta_{max}$). Call $\beta_c(T)$ the thermal expansivity of the coil assembly as a function of temperature T. ($\beta_c \equiv \frac{1}{\ell}\frac{d\ell}{dT} \equiv \frac{d\lambda}{dT}$, if λ is the elongation.) $E_c(T)$ is to be the elastic modulus of the coil assembly. Similarly $\beta_k(T)$ and $E_k(T)$ shall refer to the material of the constraining structure, such as the collars. $E_c(T)$ and $E_k(T)$ are defined as the slopes of the stress-strain relations $\sigma_c(\epsilon_c)$ and $\sigma_k(\epsilon_k)$ at temperature T. During a temperature change accompanied by a stress change the elongations (or contractions) per inch of the materials shall be $\lambda_c(\sigma_c,T)$ and $\lambda_k(\sigma_k,T)$. Since the collars are to constrain the coils, one obtains for small changes dT, $d\sigma_c$, still neglecting changes in length with radius,

$$d\lambda_c = \left(\frac{\partial\lambda_c}{\partial\sigma_c}\right)_T d\sigma_c + \left(\frac{\partial\lambda_c}{\partial T}\right)_{\sigma_c} dT = d\lambda_k = \left(\frac{\partial\lambda_k}{\partial\sigma_k}\right)_T + \left(\frac{\partial\lambda_k}{\partial T}\right)_{\sigma_k} dT$$

By definition, $(\partial\lambda_c/\partial\sigma_c)_T \equiv (\partial\epsilon_c/\partial\sigma_c) = 1/E_c$ and, similarly $(\partial\lambda_k/\partial\sigma_k)_T = 1/E_k$. Also $(\partial\lambda_c/\partial T)_{\sigma_c} = \beta_c(T)$ and $(\partial\lambda_k/\partial T)_{\sigma_k} = \beta_k(T)$, as defined above. Therefore we obtain, calling $d\sigma/dT = \sigma_k'$,

$$\frac{\sigma_c'}{E_c(T)} + \beta_c(T) = \frac{\sigma_k'}{E_k(T)} + \beta_k(T) \tag{33}$$

Assuming, for the moment, that $E_k \gg E_c$,

$$\sigma'_c \approx \left[\beta_k(T) - \beta_c(T) \right] E_c(T)$$

or

$$\sigma_c(T) \approx \sigma_{co} - \int_{T_o}^{T} \left[\beta_c(T) - \beta_k(T) \right] E_c(T)\, dT \tag{34}$$

Thus one cannot use simple integrated thermal expansions for thermal stress calculations if elastic moduli are temperature dependent.

Elongations $\beta(T)$ appear to decrease relatively slowly with temperature near room temperature. As the temperature decreases the decrease of $\beta(T)$ becomes precipitous. At $T \approx 30^o$ K, β becomes negligibly small. Quite good fits can be obtained by means of exponential functions of the type

$$\beta(T) = b - ae^{\gamma(T_w - T)} \tag{35}$$

where $T_w = 293^o$ K, a, b, and γ are determined from two values along a measured function $\beta(T)$ for a material and the value of the integral

$$\int_{T_o}^{T_w} \beta(T)dT,$$

which is more often available for materials (or coil assemblies) than $\beta(T)$. For different materials we shall merely use the values of the integrals for direct scaling of a and b, keeping γ constant.

Elastic moduli of relevant materials increase with decreasing temperature, some quite linearly, others less so, often depending on heat treatment of a material. For our purpose a linear approximation will be sufficient, especially since for coil assemblies only end values are usually available, at room (E_w) and at cryogenic (E_o) temperatures:

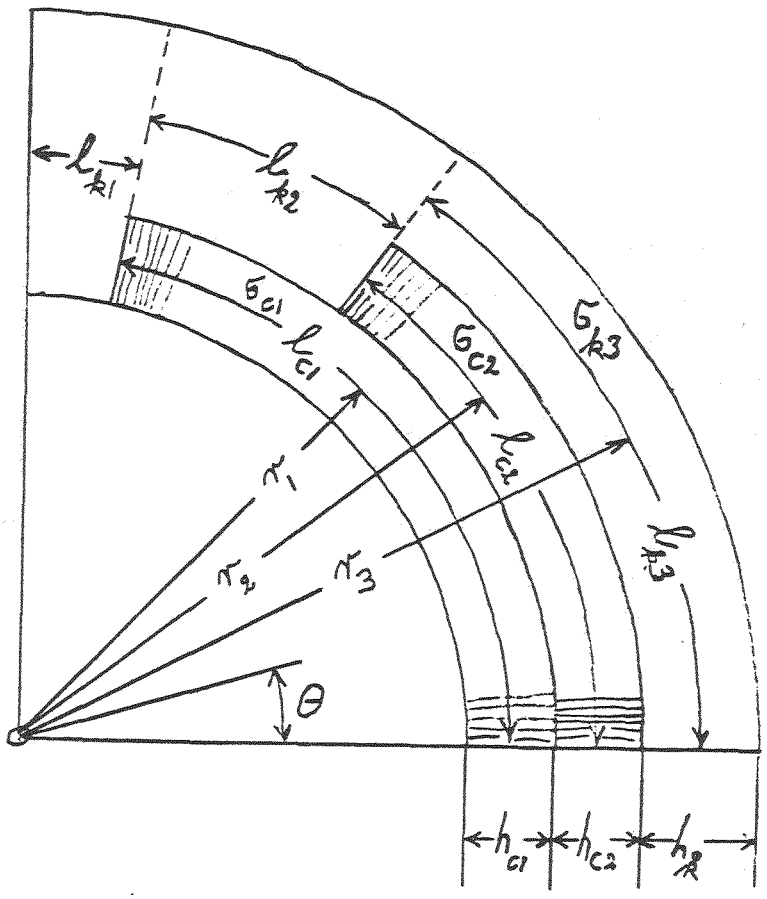
$$E(T) \approx E_o - (E_o - E_w) \frac{T}{T_w} \tag{36}$$

If, with the values for a_c, b_c, γ, E_{co} ($\equiv E_o$), E_c ($\equiv E_w$), to be given below, the integral, eq. 34, is evaluated, one can express a result for a coil constrained by stainless steel (still $E_k \gg E_c$) simply by

$$\sigma_c = A + (\sigma_{co} - \sigma_{min}) + B(E_{co} - E') + C(E_c - E') \tag{37}$$

(This expression has been generalized below. See eq. 49.) $A = -4100$, $\sigma_{min} = -3250$ (minimum required prestress for SSC inner coil), $B = -3.05 \times 10^{-4}$, $C = -5.45 \times 10^{-4}$, $E' = 10^6$. For instance, if $\sigma_{co} = -3250$ psi, $E_{co} = E_c = 10^6$ psi: $\sigma_c = -4100$ psi, a prestress loss of 850 psi during cooldown results. Making $E_c = 10^6$, but $E_{co} = 2 \times 10^6$, $\sigma_c = 4400$, an increase of the prestress loss to 1150 with doubled elastic modulus at low temperature. For $E_c = 1.5 \times 10^6$, $E_{co} = 2 \times 10^6$: $\sigma_c = -4680$.

We proceed to consider coils that are prestressed by means of collars made, for instance, of stainless steel or aluminum. The following sketch shows a quarter coil section.



All lengths are averaged over the radial width of the component under consideration. Thus

σ_{c1}: average azimuthal stress in inner coil of azimuthal length ℓ_{c1}, radial width h_{c1}, average radius r_1.

$\sigma_{c\ell}$: average stress in outer coil of length ℓ_{c2}, width h_{c2}, radius r_2.

Using average values here is appropriate because there is strong evidence that, for "keystoned" cables, these stress distributions are radially very non-uniform, and so far unexplored.

σ_{k1}: average stress in pole spacer region of collar, of average length ℓ_{k1}.

σ_{k2}: average stress in length ℓ_{k2} between end of outer coil and pole spacer region.

σ_{k3}: average stress in collar along outer coil, length ℓ_{k3}, with width h_k given as an average that takes into account that two different sections of ℓ_{k3} carry different loads. Roughly one-half of ℓ_{k3} of every other collar lamination can carry no hoop stress because of the way the laminations are pinned and keyed together. Considering this fact, we must use for h_k values that are only ~ 2/3 of the actual width h_{ka} of this collar section ($h_k = 2/3\ h_{ka}$). The resulting calculated average stress can then be multiplied by 2/3 to obtain the actual stress in the section where all collars carry equal hoop loads, and by 4/3 where only every other collar carries the load.

Next, for equilibrium we must set for the cold stresses (tension in collar sections, compression in coils):

$$\begin{aligned} -\sigma_{c_1} h_{c_1} - \sigma_{c_2} h_{c_2} &= \sigma_{k_3} h_k \\ -\sigma_{c_1} h_{c1} &= \sigma_{k2}\left(h_{c2} + h_k\right) \\ 0 &= \sigma_{k1}\left(h_{c_1} + h_{c_2} + h_k\right) \end{aligned} \tag{38}$$

Using a condition for equilibrium of radial pressures, of course, must - and does - result in the same set of equations. It follows that $\sigma_{k1} = 0$; there is no average (hoop) stress in the pole spacer region. Each half side of the coil could be prestressed by only a half collar. But then these assemblies would tend to

straighten in a vertical direction. Therefore the pole spacer region of the collar must supply the bending moment to prevent straightening. The prestressed assembly will thus deflect inward at the midplane and outward at the poles.

Next, we can write

$$\ell_{c1}\left(1 + \lambda_{c1}\right) = \ell_{k3} \frac{r_1}{r_3} \left(1 + \lambda_{k3}\right) + \ell_{k2} \frac{2r_1}{\left(r_2 + r_3\right)} \left(1 + \lambda_{k2}\right)$$

$$\ell_{c2}\left(1 + \lambda_{c2}\right) = \ell_{k3} \frac{r_2}{r_3} \left(1 + \lambda_{k3}\right) \tag{39}$$

where $\lambda c1$, $\lambda c2$, $\lambda k2$, $\lambda k3$ are the elongations per inch of the components due to stress and temperature changes.

For linear stress-strain relations we have

$$\epsilon_{c1} = \frac{\sigma_{c1}}{E_c} \quad , \quad \epsilon_{c2} = \frac{\sigma_{c2}}{E_c} \tag{40a}$$

$$\epsilon_{k2} = \frac{\sigma_{k2}}{E_k} \quad , \quad \epsilon_{k3} = \frac{\sigma_{k3}}{E_k} \tag{40b}$$

where E_c, E_k are the relevant (temperature dependent) elastic moduli. In equation 39 we have reduced the lengths ℓ_{k2} and ℓ_{k3} by the proper ratio of the pertinent radii. At constant temperature the three sets (38, 39, 40) of a total of 9 simple equations result in

$$\sigma_{c1} = - \frac{A}{B} E_c \tag{41}$$

$$\sigma_{c2} = - \left(\sigma_{c1} \frac{h_{c1}}{h_k} + C_1\right) \frac{1}{C_2} \tag{42}$$

(σ_{k2}, σ_{k3} follow then directly from eqs. 33) where

$$A = \Delta\ell_1 - \frac{r_1}{r_2}\frac{h_{c2}}{h_k}\frac{\Delta\ell_2}{C_2}$$

$$B = \frac{r_1}{r_2}\frac{h_{c1}}{h_k}\frac{\ell_{c2}}{C_2} + \frac{h_{c1}}{\left(h_{c2} + h_k\right)}\left(\ell_{c1} - \ell_{c2}\frac{r_1}{r_2}\right)\frac{E_c}{E_k} + \ell_{c1}$$

$$C_1 = \frac{\Delta\ell_2}{\ell_{c2}} E_k$$

$$C_2 = \frac{E_k}{E_c} + \frac{h_{c2}}{h_k}$$

$$\Delta\ell_1 = \ell_{c1} - \ell_{k3}\frac{r_1}{r_3} - \ell_{k2}\frac{2r_1}{\left(r_2 + r_3\right)} \tag{43}$$

$$\Delta\ell_2 = \ell_{c2} - \ell_{k3}\frac{r_2}{r_3}$$

Except for $\Delta\ell_1$ and $\Delta\ell_2$, we have made use of the fact that

$\ell_{c1} \approx \left(\ell_{k3}\frac{r_1}{r_3} + \ell_{k2}\frac{2r_1}{\left(r_2 + r_3\right)}\right) = \ell_k'$ and $\ell_{c2} \approx \ell_{k3}\frac{r_2}{r_3} = \ell_k''$. $\Delta\ell_1$ and $\Delta\ell_2$ are the shim thicknesses required to provide the prestresses if $\ell_{c1} = \ell_k'$, $\ell_{c2} = \ell_k''$, in other words, the interferences required to produce given prestresses σ_{c1} and σ_{c2} at a given temperature.

For given prestresses the $\Delta\ell_{1,2}$ can simply be expressed (using eqs. 41, 42, ff.) by

$$\Delta\ell_1 = -\frac{B}{E_c}\sigma_{c1} + \frac{r_1 h_{c2}}{r_2 h_k}\frac{\Delta\ell_2}{C_2}$$

$$\Delta\ell_2 = -\frac{\ell_{c2}C_2}{E_k}\left(\frac{h_{c1}}{h_k}\frac{\sigma_{c1}}{C_2} + \sigma_{c2}\right) \tag{44}$$

Assume now that the required prestresses σ_{c10} and σ_{c20} at magnet operating temperature are given (as, for instance, calculated in the first Section of this Note), and we wish to raise the temperature. We can then differentiate eqs. 39 and obtain, following the derivation leading to eq. 33,

$$\ell_{c1}\left[\frac{\sigma'_{c1}}{E_c(T)} + \beta_c(T)\right] = \ell_{k3}\frac{r_1}{r_3}\left[\frac{\sigma'_{k3}}{E_k(T)} + \beta_k(T)\right] + \ell_{k2}\frac{2r_1}{(r_2 + r_3)}$$

$$\times \left[\frac{\sigma'_{k2}}{E_k(T)} + \beta_k(T)\right] \qquad (45)$$

$$\ell_{c2}\left[\frac{\sigma'_{c1}}{E_c(T)} + \beta_c(T)\right] = \ell_{k3}\frac{r_2}{r_3}\left[\frac{\sigma'_{k3}}{E_k(T)} + \beta_k(T)\right]$$

Making use of eqs. 38 and of the above-stated fact that $\ell_{c1} \approx \ell_k'$ and $\ell_{c2} \approx \ell_k''$, we obtain

$$\sigma'_{c1} = \frac{Q + \zeta_2\sigma'_{c2}}{R}$$

$$\sigma'_{c2} = \frac{V}{W} \qquad (46)$$

$$Q = (\zeta_2 - \zeta_1)E_c(\beta_c - \beta_k)$$

$$R = \zeta_1 - (\zeta_2 - \zeta_1)gE_c/E_k$$

$$V = \beta_k - \zeta_2\beta_c - \frac{Q}{R}\frac{\mu_1}{E_k}$$

$$W = \frac{\zeta_2}{E_c} + \frac{\mu_2}{E_k} + \frac{\mu_1}{E_k}\frac{\zeta_2}{R}$$

Initial conditions for integrating eqs. 46 are

$$T = T_o = 30: \quad \sigma_{c1} = \sigma_{c20}$$
$$\sigma_{c2} = \sigma_{c20}$$

(Properly, we should use $T \approx 4$, the magnet operating temperature, but $\beta_c, \beta_k \approx 0$ for $T < 30^o$ K, and elastic moduli at low temperature are sufficiently uncertain that the exact low temperature used is unimportant.) We define further

$$\zeta_1 = \ell_{c1}r_2/\ell_{c2}r_1 = 1.6183$$
$$\zeta_2 = 1$$
$$\mu_1 = h_{c1}/h_k = 0.6413$$
$$\mu_2 = h_{c2}/h_k = 0.6701$$
$$g = \mu_1/(\mu_2 + 1) = 0.384$$

For Nitronic-40 stainless steel we have, for expression 35, obtained

$$a_k = 7.375 \times 10^{-7}$$
$$b_k = 1.5738 \times 10^{-5}$$
$$\gamma = 0.011637$$

For the coil assembly, scaling by ratios of integrated values of the $\beta(T)$'s, as mentioned above,

$$a_c = 9.575 \times 10^{-7}$$
$$b_c = 2.0432 \times 10^{-5}$$

After eqs. 46 have been integrated, we can obtain the stresses in the collar, epecially,

$$\sigma_{k3} = -\mu_1\sigma_{c1} - \mu_2\sigma_{c2} \tag{47}$$

in the link connecting top and bottom halves of the collars.

Assume now that $h_k E_k \gg h_{c1,2} E_c$. Then both eqs. 46 for σ_{c1} and σ_{c2} revert to the simple integration in eq. 34, the result of which we give:

$$\sigma_c = \sigma_{co} - \Delta b\left(T_w - T_o\right)\left[E_{co} - \left(E_{co} - E_c\right)\frac{\left(T_w + T_o\right)}{2T_w}\right]$$
$$- \frac{\Delta a}{\gamma}\left[E_{co}\left(1 - e^{\gamma\left(T_w - T_o\right)}\right) - \frac{\left(E_{co} - E_c\right)}{\gamma T_w}\left(1 + \gamma T_w - \left(1 + \gamma T_o\right)e^{\gamma\left(T_w - T_o\right)}\right)\right] \tag{48}$$

where

$$\Delta a = a_c - a_k$$
$$\Delta b = b_c - b_k$$
$$\gamma = 1.164 \times 10^{-2}$$
$$T_w = 293$$
$$T_o = 30$$

Eq. 48 results in a prestress change during cooldown of

$$\sigma_c - \sigma_{co} = 85.9\ \Delta a\left(13.2\ E_{co} + 7.15\ E_c\right) - 263\ \Delta b\left(0.449\ E_{co} + 0.551\ E_c\right) \tag{49}$$

a_k, b_k are found from

$$a_k = 2.59 \times 10^{-4}\ \alpha_k$$
$$b_k = 5.52 \times 10^{-3}\ \alpha_k$$

where

$$\alpha_k = \int_{30}^{293} \beta_k(T)\, dT$$

is the integrated thermal expansivity β_k. α_k, at least, is available for most materials that one may want to use for constraining the magnet coils.

For Nitronic-40 or aluminum collars, and for the iron yoke as a constraint, agreement between computer solutions for eqs. 46 and calculations by means of eq. 49 is obtained within 100 psi, the latter disagreement applying to aluminum collars of 15 mm radial width, aluminum having the lowest elastic modulus of the 3 materials considered.

The following Table gives the computer solutions. Additional, approximate results can easily be obtained by means of eq. 49.

	E_{co} psi 10^6 x	E_c psi 10^6 x	$-\sigma_{c10}$ psi 10^3 x	$-\sigma_{c1}$ psi 10^3	$-\sigma_{c20}$ psi 10^3	$-\sigma_{c2}$ psi 10^3	σ_{k3max} psi 10^3	$-(\sigma_{c1}-\sigma_{c10})$ psi 10^3
Nitronic-40 collars (15 mm)	1	1	3.25	4.06	2.94	3.74	10.2	- 0.81
	2	1	3.25	4.33	2.94	3.99	10.9	- 1.08
	1.5	1.5	3.25	4.44	2.94	4.10	11.2	- 1.19
	2.5	1.5	3.25	4.70	2.94	4.34	11.8	- 1.45
Aluminum collars (15 mm)	1	1	3.25	2.81	2.94	2.52	6.98	+ 0.44
	2	1	3.25	2.68	2.94	2.41	6.66	+ 0.57
	1.5	1.5	3.25	2.62	2.94	2.36	6.53	+ 0.63
Iron Yoke	1	1	3.25	4.93	2.94	4.62	-	- 1.68
	2	1	3.25	5.52	2.94	5.20	-	- 2.27
	1.5	1.5	3.25	6.05	2.94	5.73	-	- 2.80

coil data used to calculate this Table were

$h_{c1} = 0.379"$ $r_1 = 0.977"$ $\ell_{c1} = 1.289"$

$h_{c2} = 0.396"$ $r_2 = 1.364"$ $\ell_{c2} = 1.112"$

$h_k = 0.591"$

From the Table one sees that the for Nitronic-40 collars the prestress loss ($\sigma_{c1} - \sigma_{c10}$) increases if either E_{co} or E_c increases. Prestress losses are moderate, increasing from 0.81 kpsi for $E_{co} = E_c = 10^6$ to 1.45 kpsi for $E_{co} = 2.5 \times 10^6$ $E_c = 1.5 \times 10^6$. For aluminum, prestress gains (<1 kpsi) are found, due to its large expansivity. For direct constraint by the iron yoke prestress losses are about twice as large as for Nitronic-40 collars.

The maximum calculated hoop stress σ_{k3max} in the Nitronic-40 collar for $E_{co} = E_c = 1.5 \times 10^6$ is 11.2 kpsi while for aluminum with the same radial width 6.5 kpsi was found.

If we wish to calculate "warm" interferences $\Delta\ell_{1,2}$ between collars and coils, required to produce minimum "cold" prestress, we can use eq. 44, inserting the calculated values for σ_{c1} and σ_{c2}. Depending on the case considered, the $\Delta\ell_{1,2}$ amount only to a few thousandths of an inch.

Values for the elastic modulus E_c at room temperature for the coils have been measured at BNL to be $\approx 10^6$ psi or even less. For straight stacks of conductors considerably higher values (1.5 to 2 x 10^6) were found. This large discrepancy is probably due to the fact that the conductor cable is keystoned and thus much more compressed near the inner coil radii, especially the insulation material. At low temperatures a considerable increase of the elastic modulus, from E_c to E_{c0} has been measured here for straight stacks. An increase may also be expected for coils but has not yet been measured. Because of the high compression at the inner coil radii, the increase for coils may be much smaller than found for straight stacks.

4. Derivations for Non-Linear Stress-Strain Relation for Coils.

At the lower stresses, the coil stress as a function of strain is non-linear. Therefore eq. 2 must now be written, with $\sigma_c(\epsilon_c)$ for coil stress vs. $\epsilon_c \equiv \epsilon_c(\theta) = \frac{dy}{rd\theta}$ at angle θ

$$\frac{d}{rd\theta}\left[\sigma_c\left(\epsilon_c(\theta)\right)\right] = -\frac{g(\theta)}{h} \tag{50}$$

with boundary conditions for an arbitrarily prestressed case:

$\theta = 0$: $y = 0$

$\theta = \theta_{max}$: $y = y_s$

assuming that the powered coil remains stressed at $\theta = \theta_{max}$. Integration results in

$$\sigma\left(\epsilon_c(\theta_{max})\right) - \sigma_c\left(\epsilon_c(\theta)\right) = -\frac{r}{h}\int_{\theta}^{\theta_{max}} g(\theta)d\theta + K = -\frac{r}{h} I'(\theta) + K$$

or, calling

$$K - \sigma_c\left(\epsilon_c(\theta_{max})\right) = K_1 ,$$

$$\sigma_c\left(\epsilon_c\right) = \frac{r}{h} I'(\theta) - K_1 \tag{51}$$

Since $\sigma_c(\epsilon_c)$ is assumed to be given, one can find $\epsilon_c(\theta)$ with K_1 as a, so far unknown, parameter. Thus, eq. 51 can again be integrated:

$$y(\theta) - y(0) = r\int_0^{\theta} \epsilon_c(\theta)d\theta + D$$

where $D = 0$ because $y = 0$ at $\theta = 0$.
At $\theta = \theta_{max}$:

$$y\left(\theta_{max}\right) = y_s = r\int_0^{\theta_{max}} \epsilon_c(\theta)d\theta \tag{52}$$

With the help of a computer program, eq. 52 can be solved for K_1. So far, we will then have obtained the total motion in the coil. To obtain net motion, we have still to subtract motion before powering, due to prestressing:

$$y_0(\theta) = \frac{y_s\theta}{\theta_{max}} \tag{53}$$

resulting in

$$y(\theta) = r\int_0^{\theta} \epsilon_c(\theta)d\theta - y_s\frac{\theta}{\theta_{max}} \tag{54}$$

Calculations will be carried out at a later date. One can expect now that motion at $\theta = \theta_{max}$ is less likely than that for the linear case, some prestress remaining down to lower strains. Note that motions are now not independent of the prestress in excess of the minimum value, as they were for the linear case.

Prestress changes between warm and cold states for non-linear stress-

strain relations ($\sigma(\epsilon)$) can be treated by replacing constant elastic moduli by the slopes ($\partial\sigma/\partial\epsilon$) which then will be functions of σ as well as T.

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