Momentum Dependence of Betatron Functions near an IR

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The momentum dependence of the lattice functions β, α at the interaction point will make the position and values of the low-β point momentum dependent. From the equations in a field-free region we may estimate this variation. The expression is

$$\beta\ (s,\ \Delta) = \beta(0,\ \Delta) - 2s\ \alpha(0,\ \Delta) + \frac{s^2}{\beta(0,\ \Delta)}\ (1 + \alpha(0,\ \Delta)^2)$$

where $\Delta = \delta\ p/p$, and the reference position of s=0 is at the center of the IR.

$\beta(s,\ \Delta)$ has a minimum value at

$$s^*(\Delta) = \frac{\beta(0,\ \Delta)\ \alpha(0,\ \Delta)}{1 + \alpha(0,\ \Delta)^2}$$

of $\beta^*(\Delta) = \beta(0,\ \Delta) - s^*(\Delta)\ \alpha\ (0,\ \Delta)$

Betatron tuning places the central momentum value of β^* at 0.5 m (1.0 m) and the value of α (0,0) at 0.0. $|\alpha(0,\ \Delta)|$ at a typical IR region is less than 0.05 over $|\Delta| \lesssim 10^{-3}$, so that the location of the low beta point is shifted by only $\tilde{\pm}$ 2.5 cm.

The variation of β^* (Δ) is dominated by the dependence of $\beta(0,\ \Delta)$, which can vary by $\pm$ 10% over $\Delta = \pm\ 10^{-3}$.