Nonlinearities Due to Systematic Multipoles

David Neuffer

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1. Dipoles

It is desirable that particle motion be substantially linear over the region in phase space explored by the beam. This property may be expressed as a limit on the amplitude and momentum dependent tune shift of the form:

$$\Delta\nu \lesssim \pm .005,$$

where $\Delta\nu$ is the tune shift relative to the central trajectory and the number .005 is extrapolated from experience with previous near-linear machines[1]. The tune shift may be calculated in lowest order using the relation:

$$\Delta\nu = \frac{1}{4\pi}\int_0^{2\pi R} \beta(s)\, \delta\left(\frac{B'}{B\rho}\right) ds \tag{1}$$

where $\beta(s)$ is the betatron function of linear motion, and $\delta(B'/B\rho)$ is the perturbation of focussing fields due to multipoles coupled with amplitude and momentum off-sets. The dipoles have multipole components, given by b_n,

so $$\delta\left(\frac{B'}{B\rho}\right) = \frac{nB_o}{B\rho}\, b_n\, x^{n-1} \qquad \text{at } y = 0 \tag{2}$$

The large size of the SSC makes it particularly sensitive to systematic multipole effects. The tune shift due to a a systematic multipole $< b_n >$ is

$$\Delta\nu = \frac{n\bar{\beta}_\perp < b_n >}{2} \quad < x^{n-1} > \tag{3}$$

This expression may be evaluated using

$$< x > \sim \bar{\eta} \frac{\delta p}{p} \text{ and } < x^2 > = \varepsilon\bar{\beta}.$$

Dipole symmetry permits even systematic multipoles (b_2, b_4, ...), and the tune shifts are proportional to odd powers of $\eta\ \delta p/p$ times even powers of x ($< x^{2m} >$). For example, the decupole term is:

$$\Delta\nu_4 = 2\bar{\beta} \perp < b_4 > [3 < x^2 > \bar{\eta} \frac{\delta p}{p} + \bar{\eta}^3 (\frac{\delta p}{p})^3] \tag{4}$$

For an SSC with $\bar{\beta}$ = 200 m., $\bar{\eta}$ = 2 m.) this becomes:

$$\Delta\nu_4 = A_4 (.0024 < x^2 > \Delta + .00032\ \Delta^3) \tag{5}$$

with $< x^2 >$ in $(mm)^2$, $\Delta = \delta p/p$ in units of 10^{-4}, which are typical SSC RMS beam size parameters, and $A_4 = b_4$ in units of 10^{-4} cm^{-4}. Thus, with $A_4 \lesssim 1$, tune shifts are only of order 10^{-3} over the SSC beam amplitude and momentum spreads.

However, accelerator experience indicates a somewhat broader linear aperture in amplitude and momentum is desirable. Observers[1] have suggested linear aperture criteria of the form $\Delta\nu \lesssim \pm .005$ over amplitudes $A_x \lesssim 0.5$ cm at $\Delta = 10$, or $A_x \lesssim 0.7$ cm at $\Delta = 0$. With these criteria the dominant term is the first term in equation (5), obtaining

$$\Delta\nu \cong A_4 (.3),$$

$$\text{or from} \quad \Delta < \pm .005; \qquad A_4 \lesssim .017.$$

Following this procedure, estimates of maximum allowable systematic b_{2n} may be obtained for all n from the linear aperture criteria; these are

displayed in Table (1). The estimates of Table 1 assume no correction magnets are included to increase the linear aperture.

The results can be compared with the expected systematic multipole components of the SSC dipoles. For $n \gtrsim 8$ the expected values are much less than the estimated limits. For $n = 4$ (decupole) significant reduction and/or correction is desired.

Table 1

Maximum permitted $< b_{2n} >$ in dipoles under the aperture criterion: $[\Delta\nu < \pm .005$ for $< A_x > \lesssim 0.5$ cm, $\Delta p/p \lesssim 10^{-3}]$ for SSC lattices with $\beta = 200$, $\eta = 2$ m.

Multipole Coefficient	Maximum	$< b_n >$
b_4	$.017 \times 10^{-4}$	cm^{-4}
b_6	$.07 \times 10^{-4}$	cm^{-6}
b_8	$.2 \times 10^{-4}$	cm^{-8}
b_{10}	$.5 \times 10^{-4}$	cm^{-10}
b_{12}	1.6×10^{-4}	cm^{-12}

2. Systematic Multipoles in the Cell Quadrupoles

Symmetry forbids $< b_n > \neq 0$ for n odd in the dipoles; however, such components are allowed in quadrupoles. The tune shift calculation may be performed for these multipoles in the dipoles, obtaining

$$\Delta\nu_N \cong \frac{(N-1)}{4\pi} \frac{B'\ell_Q}{B_\rho} n_{cell} \beta_{max} g_{N-1} < x^{n-2} > \tag{6}$$

where the magnetic field in the dipole is represented by

$$B_y = B' \times (1 + g_3 x^2 + g_5 x^4 + \ldots)$$

n_{cell} is the total number of cells, ℓ_Q is the quad length, β_{max} is the maximum betatron value and $< x^{N-2} >$ is evaluated at the focussing quad. Assuming a requirement that $\Delta\nu_N \lesssim .005$ at orbits with $x_{max} = 0.7$ cm, we obtain the limits in table (2). Roughly, $< g_n > \lesssim 10^{-4}$ $cm^{-(n-1)}$ is desired.

A limit on random multipoles can be found by considering a statistically randomized version of (6). The limit is $\sim \sqrt{n_{cells}}$ larger.

Table 2

Limits on systematic multipole content in the arc quadrupoles under the constraint ($\Delta\nu$ (A_x = .007 m) < .005)

n	multipole	$\lvert g_n$ (limit)$\rvert$
3	(octupole)	$.53 \times 10^{-4}$
5	(12-pole)	$.87 \times 10^{-4}$
7	(16-pole)	1.5×10^{-4}
9	(20-pole)	1.74×10^{-4}

g_n is the magnetic field (at x = 1 cm) of the multipole field divided by the quadrupole field.

3. Systematic Multipoles in IR Quads

Multipole content in the low-Beta quads (where $\beta_\perp$ reaches maximum values of 8000 m when $\beta^* = 0.5$ m) can also cause extremely large tune shifts. The difficulty is somewhat ameliorated by the fact that a low-beta configuration is not developed until the beam reaches full energy (20 TeV), at which particle amplitudes are adiabatically damped. However the high-β value increases coordinate values, making the beam roughly five times larger in the IR quads than in the cells:

$$\sigma_{beam} \cong \sqrt{\frac{10^{-6}\ 8000}{20{,}000}} \cong .00063 \text{ m}$$

As a minimum quality criterion, we would like linear motion up to amplitudes of 8σ or .005 m in the IR quads. The resulting constraints on the uncorrected IR quad multipole content are estimated and displayed in Table 3.

Table 3

Limits on systematic multipole content in the Low-β quads under the constraint:

$$[\Delta\nu \lesssim .005 \text{ at } \langle x_{max}\rangle_{Hi-\beta} = .5 \text{ cm}]$$

n	$\langle g_n\rangle_{max}$
3	$.33 \times 10^{-4}$
5	1.1×10^{-4}
7	3.6×10^{-4}
9	13.0×10^{-4}

{.5 cm in IR quad corresponds to ~ .1 cm in arcs.}

These limits on multipole component in the IR quads should also be approximately followed at low energy, where β is smaller in the IR quads but the beam emittance is larger. Similar constraints must be followed for random multipole components, with a statistical increase in the limit by $\sim \sqrt{m}$ where m is the number of independent multipoles. Since there are only 6 IR's, m will only be ~ 12 - 24, so the random components may only be ~ 3-5 times larger.

References

1. D. Edwards, SSC-23 (1985).
2. E.D. Courant and H.L. Snyder, Annals of Physics 3, 1 (1957).

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