SURFACE IMPEDANCE OF A TWO-LAYER BEAM TUBE AND RESISTIVE-WALL CONTRIBUTION TO MACHINE IMPEDANCES

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The longitudinal and transverse impedances presented to beam particles by the finite conductivity of the walls of the beam tube are important quantities for beam stability considerations[1]. Numerical estimates made in the RDS were based on a copper-walled beam tube with a low temperature relative resistance ratio (RRR) of 30, and a copper thickness large compared to the skin depth. The present note examines the impedances in rather complete generality for a two-layered beam tube, the inner layer of thickness t_1 and conductivity σ_1, the outer layer of thickness t_2 and conductivity σ_2. No significant assumptions (about thickness relative to either skin depth, for example) are made. The treatment is valid for frequencies less than 100 GHz for materials with conductivity as good as or better than stainless steel.

The resistive-wall contributions to the longitudinal and transverse impedances for the beam are related to the <u>surface impedance</u> Z_s of the beam tube wall according to

$$Z_{\parallel} = \frac{R}{b} Z_s, \quad Z_{\perp} = \frac{2R}{b^2} \frac{Z_{\parallel}}{n} \tag{1}$$

where R is the effective radius of the accelerator (the circumference of the equilibrium closed orbit is $2\pi R$), b is the inner radius of the beam pipe, and n is the harmonic number ($\omega = nc/R$).

1. Simple description of the RDS

For a beam tube that is many skin depths thick, the elementary treatment of surface impedance[2] yields

$$Z_s = \frac{1-i}{\sigma\delta} = \frac{\omega\delta}{2c} Z_0(1-i) \tag{2}$$

where $\delta = \sqrt{2/\mu_0\omega\sigma}$ is the skin depth, $Z_0 = \sqrt{\mu_0/\epsilon_0} = 377$ ohms is the impedance of free space, and $c = 1/\sqrt{\mu_0\epsilon_0}$ is the speed of light. [Note that, while SI units are used, the phase of Z_s is given in the physicist's convention. Electrical engineers, please substitute $i \rightarrow -j$!]. With $\omega = nc/R$, one obtains the longitudinal impedance, divided by n, at frequency $\nu = n\nu_0 = nc/2\pi R$:

$$\frac{Z_{\parallel}}{n} = \frac{Z_0\delta}{2b}(1-i) \tag{3}$$

This is the corrected form of RDS Eq. (4.3.2). The numerical values of RDS Table 4.3-1 are based on Eqs. (3) and (1), with the skin depth computed with a 4K, B = 6T resistivity of $\rho = \sigma^{-1} = 5\times10^{-10}\ \Omega - m$.

2. Surface impedance for a two-layer beam pipe

With the assumption that the total thickness $t = t_1 + t_2$ is small compared to the radius b, the geometry may be approximated by two adjacent slabs. The standard treatment[2] can be generalized directly. The displacement current can be ignored for frequencies whose free space wavelengths satisfy the inequality, $\lambda \gg 2\pi\rho/Z_0$, where ρ is the a-c resistivity. (For $\lambda > 1$ mm, the condition holds for all but the poorest conductors). Within the two layers, the magnetic field is generally much larger than the electric, by a factor of the order of $4\pi\sigma\delta/c$ (Gaussian units) or $\sigma\delta Z_0$ (SI units).

Outside the beam pipe, the fields are required to be those of radiation electromagnetic fields in free space, with equal magnitudes of E and H (Gaussian units). The impedance mismatch at the outer surface of the pipe forces the magnetic field in the outer layer to vanish at that surface, regardless of the dimensions relative to the skin depth. The corrections are of order the reciprocal of $\sigma\delta Z_0$.[3] With this approximation, there are no fields outside the pipe and the boundary value of the tangential electric field at the inner surface of the beam pipe is determined in terms of the tangential magnetic field there in a straightforward manner.

It is convenient to write the surface impedance as Eq. (2) times a correction factor ζ :

$$Z_s = \frac{(1-i)}{\sigma_1\delta_1}\zeta = \frac{\omega\delta_1}{2c} Z_0(1-i)\zeta \tag{4}$$

The magnitude and phase of ζ give the departures from the simple model of the RDS for $Z_{\parallel}$ and $Z_{\perp}$. The result for ζ is

$$\zeta = \left[\frac{1 + \sqrt{\frac{\sigma_2}{\sigma_1}} \left(\frac{1-z_1}{1+z_1}\right)\left(\frac{1-z_2}{1+z_2}\right)}{\frac{1-z_1}{1+z_1} + \sqrt{\frac{\sigma_2}{\sigma_1}} \left(\frac{1-z_2}{1+z_2}\right)} \right] \tag{5}$$

where

$$z_i = \exp\left[-2(1-i)t_i/\delta_i\right] . \tag{6}$$

Various familiar results emerge by taking appropriate limits:

(a) $\underline{t_1 \gg \delta_1}$ $z_1 \to 0$ and $\zeta = 1$, as expected, regardless of t_2 or σ_2.

(b) $\underline{t_2 \gg \delta_2}$ $\quad z_2 \to 0$ and ζ becomes the result given by K.Y. Ng.[4]

(c) $\underline{t_2 = 0}$ $\quad \zeta = \dfrac{1+z_1}{1-z_1}$.

Here a familiar limit occurs for $t_1 \ll \delta_1$.

$$\zeta \to \frac{\delta_1}{(1-i)t_1}, \quad Z_s \to \frac{1}{\sigma_1 t_1},$$

as occurs for a tube of thickness t_1 and conductivity σ_1 with current uniformly distributed throughout its cross section.

(d) $\underline{t_1 = 0}$ $\quad \zeta = \sqrt{\dfrac{\sigma_1}{\sigma_2}}\left(\dfrac{1+z_2}{1-z_1}\right)$.

This is the same as (c) with $1 \to 2$, the factor $\sqrt{\sigma_1/\sigma_2}$ being necessary to convert $1/\sigma_1\delta_1 \to 1/\sigma_2\delta_2$ in Eq. (4).

3. Numerical results

For reference, we give some numerical valves for skin depths at low temperatures and at the fundamental frequency of rotation for a SSC of 83 km circumference (ν_0 = 3612 Hz).

Values of δ in mm at $\nu = \nu_0$:

Material	Resistivity (Ω -m)	δ(mm)
Copper (RRR = 30)	5×10^{-10}	0.187
Aluminum (hard)	4×10^{-9}	0.530
Aluminum (RRR = 400)	8×10^{-11}	0.075
Stainless steel	5×10^{-7}	5.92

To scale to other values of circumference, resistivity, or frequency note that $\delta \propto (R\rho/n)^{1/2}$. It is clear that at frequencies associated with the bunch spacing and size the standard result ($\zeta = 1$) holds. Only for $n = O(1)$ is there a question of corrections.

The correction factor ζ is shown in Fig. 1 for a single-layer beam pipe. The magnitude and phase are plotted as a function of t/δ. It is significant that $|\zeta|$ departs less that 25% from unity for $t/\delta > 0.6$ and its phase is negligible for $t/\delta > 1.2$. The beam pipe does not need to be thick compared to the skin depth for the simple result (2) to apply.

Figure 2 displays the correction factor in magnitude and phase as a function of t_1/δ_1 (properties of the inner layer) for several values of σ_2/σ_1 and t_2/δ_1. For small σ_2/σ_1, ζ is independent of t_2/δ_1, except for extremely small values of t_1/δ_1. For $\sigma_2/\sigma_1 = 10^{-1}$, there is some dependence on t_2/δ_1, but it is relatively unimportant, at least for $|\zeta|$. The same lesson is learned from Fig. 2 as from Fig. 1. For $t_1/\delta_1 > 0.6$, the magnitude of Z_s is given to within 25% by the simple result (2), and the phase is closely the same for $t_1/\delta_1 > 1.2$, whatever the nature and thickness of the outer layer.

4. Some Examples

The ratio $\sigma_2/\sigma_1 = 10^{-3}$ is characteristic of a stainless steel tube with an inner layer of copper with RRR = 30. The skin depth of the copper at the fundamental frequency of rotation is $\delta_1 \simeq 0.19$ mm. For mechanical reasons the beam tube will need to have a total thickness of order 1.0 mm. Thus $(t_1 + t_2)/\delta_1 \simeq 5$.

If the low frequency ($n \simeq 1$) resistive-wall contribution to the impedance can be permitted to be a factor of two larger than the RDS value, Fig. 2 shows that an inner layer thickness of $t_1 \simeq 0.4\delta_1 = 75$ μm (approximately 3 mils)

is acceptable. If a factor of 3 larger is allowed, t_1 = 50 μm (2 mils) is permitted. Of course, if n = 0.25 is relevant for the dominant instability, these t_1 values will increase by a factor of two.

An aluminum beam pipe is an attractive possibility. A single-layer tube of "hard" aluminum and thickness 1.1 mm has $t/\delta \simeq 2$ for n = 1. Figure 1 shows that $\zeta \simeq 1$ and the simple result (2) holds. The skin depth of "hard" aluminum is $\sqrt{8} \simeq 2.8$ times that of the RDS copper. Thus, if a factor of 3 greater resistive-wall impedances than the RDS could be tolerated, a "hard" aluminum tube could be employed without any inner liner.

Another possibility is a two-layer aluminum beam tube. A sample tube from Japan has approximately the characteristics of the two kinds of aluminum whose skin depths were calculated above. The high purity inner layer (RRR ≃ 400) has a skin depth 0.4 times the RDS copper. Figure 2 shows that, if resistive-wall impedances twice as large as the RDS are allowed, ie $|\zeta| \simeq 5$, we need only $t_1/\delta_1 \simeq 0.1$, or t_1 = 7.5 μm (0.3 mils). The fabrication of the tubing is a mechanical process and so gives t_1 values considerably larger (~100 μm). Clearly, such tubing gives impedances 2.5 times smaller than RDS values.

5. Conclusions

The simple expressions for the resistive-wall contribution to the machine impedances are generalized to a two-layer beam tube. The numerical results show that the naive expressions, based on the good inner conductor being many skin depths thick, are a good approximation for $t_1/\delta_1 \geq 1.0$. The magnitude and phase of the actual multiplicative correction factor ζ are displayed in Fig. 2.

If the SSC resistive-wall impedances could be a factor of two or three larger than those assumed in the RDS, it appears that a simple "hard" aluminum beam tube (with a low-temperature resistivity of $\rho \lesssim 4 \times 10^{-9}\ \Omega$-m) would be adequate. Alternatively, a stainless steel tube with a inner layer of copper (with $\rho = 5 \times 10^{-10}\ \Omega$-m) of 2 or 3 mils thickness (or 5 or 6 mils, if $n \approx 0.25$ is relevant) would serve. A hard aluminum tube with an inner layer of high purity aluminum, such as has been recently fabricated in Japan, would exceed the RDS specifications. From the point of view of mechanical stresses during a quench, however, such a high RRR material is undesirable.

References

1. See Section 4.3 of the Report of the SSC Reference Designs Study (May 1984), p. 95 ff.

2. See, for example, J.D. Jackson, Classical Electrodynamics, 2nd ed., John Wiley and Sons, N.Y. (1975), p. 335-339.

3. At $\nu = 100$ GHz, $\sigma\delta Z_0 \approx 800$ for stainless steel at 4K and 2.5×10^4 for copper with RRR ≈ 30.

4. K.Y. Ng, in "Accelerator Physics Issues for a Superconducting Super Collider", Ann Arbor, Michigan, 12-17 December 1983, University Michigan report UM HE 84-1, p. 139.

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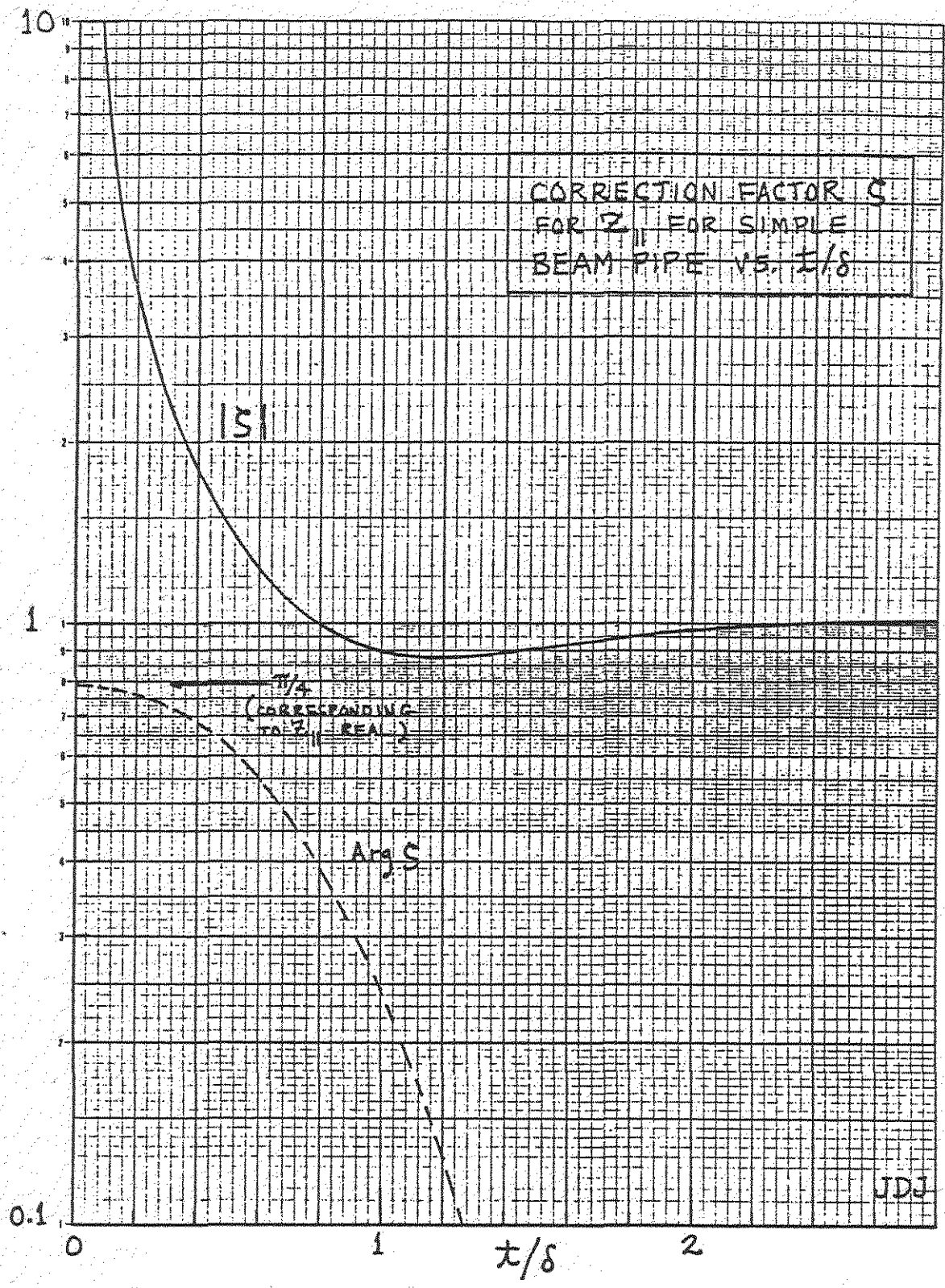
CORRECTION FACTOR S
FOR Z∥ FOR SIMPLE
BEAM PIPE VS. t/δ
|S|
π/4
(CORRESPONDING
TO Z∥ REAL)
Arg S
JDJ
10
1
0.1
0
1
2
t/δ

Figure 1

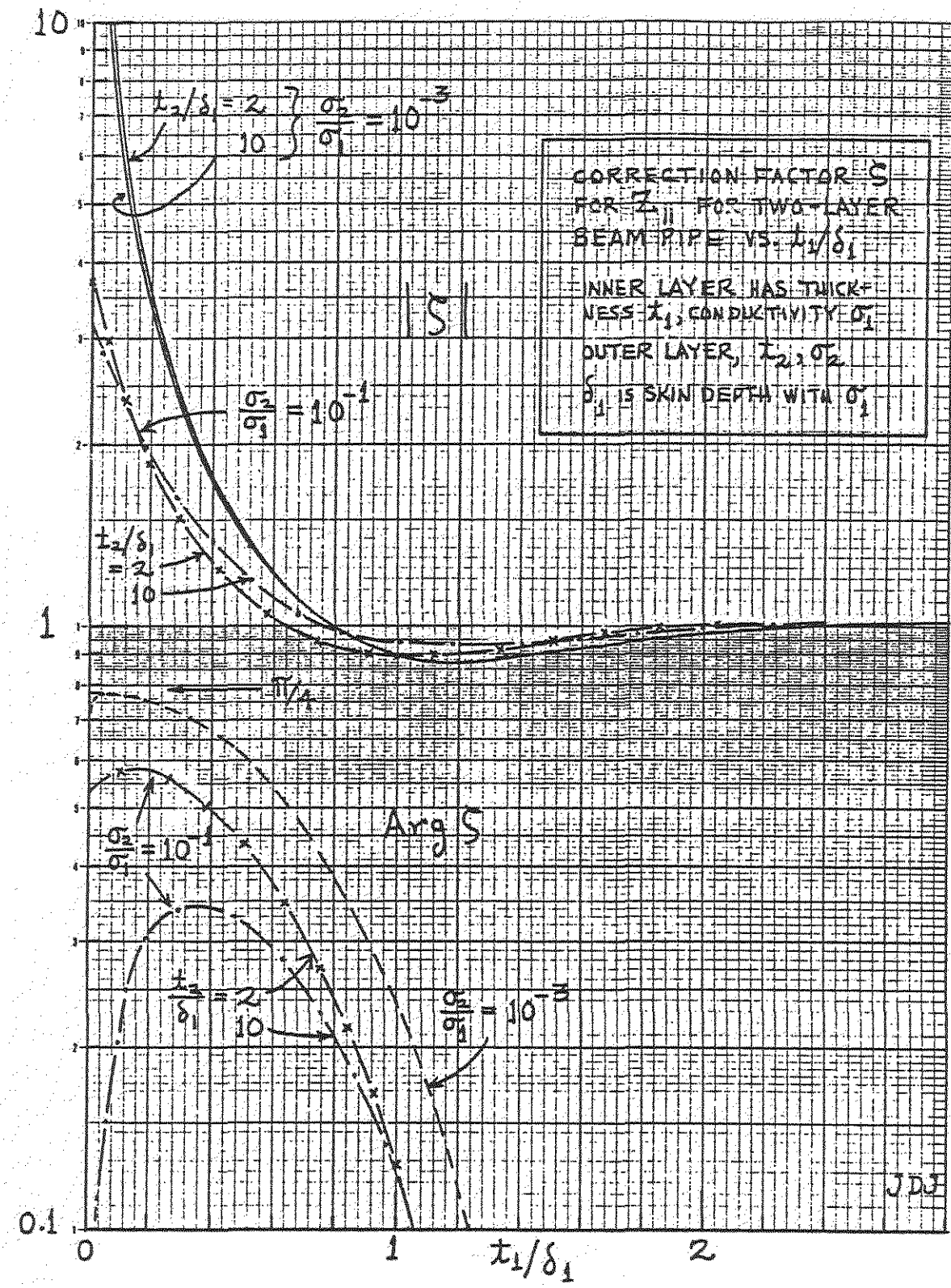
CORRECTION FACTOR S FOR $Z_{\parallel}$ FOR TWO-LAYER BEAM PIPE VS. t_1/δ_1
INNER LAYER HAS THICKNESS t_1, CONDUCTIVITY σ_1
OUTER LAYER, t_2, σ_2
δ_1 IS SKIN DEPTH WITH σ_1
$t_2/\delta_1 = 2, 10$
$\frac{\sigma_2}{\sigma_1} = 10^{-3}$
$|S|$
$\frac{\sigma_2}{\sigma_1} = 10^{-1}$
$t_2/\delta_1 = 2, 10$
$\pi/4$
Arg S
$\frac{\sigma_2}{\sigma_1} = 10^{-1}$
$\frac{t_2}{\delta_1} = 2, 10$
$\frac{\sigma_2}{\sigma_1} = 10^{-3}$
JDJ
10
1
0.1
0
1
2
t_1/δ_1

Figure 2