SSC-N-107

UH-IBPD-003-1985

A STRING MAGNETOMETER FOR MAGNET FIELD MEASUREMENT

Sam Heifets
University of Houston
Texas Accelerator Center
2319 Timberloch Place
The Woodlands, Texas 77380

Introduction

High energy accelerators such as SSC have a diameter of the vacuum pipe of order of couple of inches and magnet field of order of 5T. At the same time multipole errors of the field have to be known with accuracy of order of 10e-05. This tough problem solved usually with a coil probe. The probe, placed in the beam pipe, is rotated and the signal, induced in the coil, gives the information of the field.

Such an approach is good for short dipoles and creates obvious problems, if it necessary to measure the magnet field along the dipole of 100m long and if measurements under helium temperature are needed.

In this paper we suggest to use a modification of a string magnetometer for such kind of measurements. We describe the basic design, give an estimation of the signal and discuss some adverse affects and choice of parameters.

The Basic Design

The magnetometer (fig.1) can be made as a compact device, about a foot long, which can be inserted in a beam pipe of any length and pulled through it. Some number of strings are stretched between two discs. The discs are fixed on an axes and have the oval shape of the pipe cross-section. They prevent the device from axial rotation, but allow the magnetometer to slide along the pipe, and provide the necessary stiffness of the system.

A.C. current applied to a string causes vibration of the string in the magnet field of a dipole, and that induced variation of the string tension. The tension variation can be measured with PZT transducer, implemented in the string, what gives the desired information on the field at the site of the string. The measurements repeated with all strings give the azimutal dependence of the field for a given longitudinal position, etc.

To increase the sensitivity of the device the frequency of AC current has to be in resonance with self frequency of a string. To reduce the cross-talk effect between strings the current with tunable frequency has to apply to each pair of strings in turn, and all strings can be initially set to different frequencies.

The accuracy of the measurement of higher order multipoles depends on the number of strings in the magnetometer and could be sufficiently high with a proper design.

Basic Formula

The deformation of the string stretched along y-axes is described by the vector $\vec{R}$,

$$\vec{R} = \vec{r}(s,y) - \vec{r}(s,0) = \vec{e}_x \cdot X(s) + \vec{e}_x \cdot Y(s) + \vec{e}_z \cdot Z(s)$$

The acceleration of the string element $\rho \cdot d\ell \cdot \ddot{R}$ is given by the sum of tension

$$d\vec{T} = ds \cdot \partial/\partial s(F \cdot \vec{T})$$

weigh

$$d\vec{P} = -\rho \cdot g \cdot dl \cdot \vec{e}_z$$

and the Ampere force (in CU units)

$$d\vec{F} = I \cdot dl \cdot \vec{T} X \vec{B}$$

Here s is the distance along the string

$$dl = ds \cdot \sqrt{(1+Y')^2 + X'^2 + Z'^2}$$

$\vec{T}$, $\vec{E}_z$ are unit vectors, T is the tangent unit vector

$$\vec{T} = (d\ell/ds)^{-1} \left[(1+Y')\vec{e}_y + X'\vec{e}_x + Z'e_z\right]$$

and $\rho = \rho_o \cdot S$ is the string density per unit length.

The exact equations can linearized. With an additional terms, describing damping with a rate ν, they take the form:

$$\rho\ddot{X} = \partial/\partial s\ (FX') + IB_z + \rho\nu\dot{X}$$

$$(1) \quad \rho\ddot{Y} = \partial/\partial s\ (F) - I(X' \cdot B_z - Z' \cdot B_x) + \rho\nu\dot{Y}$$

$$\rho\ddot{Z} = \partial/\partial/s\ (FZ') - \rho g - IB_x + \rho\nu\dot{Z}$$

The inductance B_z, B_x in the equations can be considered as constants at the location of the string in the equilibrium (X=Y=O). The tension F is given by the initial tension F_o and the elongation rate:

(2) $F = F_o - E \cdot S \cdot Y' - \chi \cdot E \cdot S \cdot T$

The last term describes the change of the tension with the temperature T, χ is a linear coefficient of thermal expansion, E is Young modulés of the string.

The temperature distribution is given by the equation:

(3) $\partial T/\partial t = (I^2/\rho_o C_v \sigma S^2) + D\partial^2 T/\partial s^2$

with $D = \lambda/C_v\rho_o$; σ is conductivity, λ is thermal conductivity and C_v is specific heat.

The first terms in f.(3) gives Ohmic heating of the string. For the copper string with cross-section 1mm and for 1 A current the rate of order of 5.e-03 ^oK/sec.

We neglect here the thermal radiation and convection. The thermal radiation is small if

$$\sigma_{SB} \cdot T^4 \cdot 2\pi\Gamma_o << \lambda S \cdot \partial^2 T/\partial s^2 \; ; \; \sigma_{SB} = 1 \cdot 35 \cdot 10^{-8} \; \text{cal/deg}^4 \cdot \text{sec} \cdot \text{m}^2$$

In our case it means $T<<10^3$ ^oK. Convection is small in cold measurements, and could be dominant for measurements under room temperature. In this case the eq. (3) gives the upper limit for temperature of the string.

Let us assume for the current

$$I = I_o \text{Cos } wt$$

In the eq.(1) for X, Z we can assume $F=F_o$. By this assumption we drop off a parametric resonance induced by rippling of the temperature:

$$T \sim I_o^2 \text{ Sin } 2wt/4w\rho_o C_v \sigma S^2$$

According to eq. (2), it gives for X the equation

$$\rho \ddot{X} = \partial/\partial s \left(F_o - \frac{I_o^2 \chi E \text{ Sin } 2wt}{4w\rho C_v \sigma}\right) X'$$

which describes a parametric resonance, but the width of the resonance is very narrow:

$$(\Delta w/w) = \chi E I_o^2/4w\rho_o C_v \sigma S F_o$$

or

$$(\Delta w/w) = \sim 1/wt_o \sim 10^{-5}$$

where t_o is the time of thermoexpansion of the order of initial elongation of the string by the force F_o.

In this approximation, the solution for the first harmonics of X,Z with zero boundary conditions $X(0)=X(1)=Z(o)=Z(1)=0$ and in the resonance $W = W_o$.

(4) $W_o = (\pi/\ell)\sqrt{F_o/\rho_o S}$

takes the form:

$$X = -\frac{4I_o B_z}{\pi\rho_o S\nu W_o} \text{Sin}(\pi s/\ell) \text{ Sin } w_o t$$

(5)

$$Z = \rho g/2F_o\ s(\ell - s) + \frac{4I_o B_x}{\pi\rho_o S\ w_o\nu} \text{ Sin } w_o t \cdot \text{Sin}(\pi s/\ell)$$

In the equation for longitudinal displacement Y the terms $\rho\ddot{Y}$ and $\rho\nu y$ can be neglected:

(6) $Y'' = -\chi T' - (I_o/ES)(B_z X' - B_x Z') \text{ Cos } wt$

The distribution of temperature is given by eq. (3) with boundary conditions $T(0,t) = T(1,t) = 0$

(7) $T(s,t) = (2I^2_o/\pi\rho_o C_v \sigma S^2) \text{ Sin }(\pi s/\ell) \left[\tau(1-\ell^{-t/\tau}) - \text{Sin}2wt/2w\right]$

where relaxation time τ is

(8) $\tau^{-1} = (\lambda/\rho_o C_v) \cdot (\pi/\ell)^2$

Eq. (6), (7) with zero boundary conditions give the value of Y' at $s = 0,1$:

$$Y'(o) = Y'(\ell) = \frac{4I^2_o \chi \cdot \tau(1-\exp(-t/\tau))}{\pi^2\rho_o C_v \sigma S^2} - \frac{4I^2_o \text{Sin}2w_o t}{\pi^2 E\rho_o \nu w_o S^2}\left[B^2 + \frac{\chi E\nu}{2C_v\sigma}\right]$$

$$+ \quad I_o \ell^2 \rho_o g B_x \text{Cos } w_o t/12F_o E$$

The variation of the tension in the string

$$\delta F(o) = -ESY'(o)$$

induces a voltage on the PZT transducer V,

(10) $V = g_{33}\delta F(o) \cdot h/s = -g_{33} EhY'(o)$

where g_{33} is a piezoelectric pressure constant and h is the thickness of the transducer.

Discussion

The figure (8), (9) describe the induced voltage on the PZT transducer. The first terms in (8) describes the slow variation of tension due to Ohmic heating of the string. As a result, the self frequency of the string creepling with time with the rate

$$\Delta w/w = \frac{4I_o^2 \ X \ E}{\pi^2 \rho_o S \cdot \sigma C_v F_o} \qquad t$$

With current I_o = 1A for a copper string with the cross-section S=1 mm^2 and parameters:

$$\rho_o = 0.89 \cdot 10^4 \ kg/m^3$$

$$C_v = 0.385 \cdot 10^3 \ J/kg \cdot {}^oK$$

$$\sigma^{-1} = 1.67 \cdot 10^{-8} \ \Omega \cdot m$$

$$X = 1.65.10^{-5} \ 1/{}^oK$$

$$E = 1.1.10^{11} \ N/m^2$$

$$F_o = 10kG = 100N$$

the tune shift ($\Delta w/w$) is about 13% per hour.

The second term in the eq.(8) gives a signal with frequency 2 W_o. It is caused by the magnet field and Ohmic heating. The ratio of the two effects with parameters listed above and the field B=3 T, damping rate ν= 0.1sec^{-1} is

$$XE / 2C_v \sigma B^2 \simeq 4.3 \cdot 10^{-7}$$

So, the effect of heating is negligible not only in compare with B^2 but also in comparison with variations of B^2. The signal

(11) $V = g_{33} h(2I_o/\pi S)^2 \cdot (B^2/\rho_o \nu w_o) \ \mathrm{Sin}\ 2w_o t$

depends only on the current density frequncy and the density of the string. The last term varies with frequency w_o:

$$V_1 = g_{33} h(\ell^2 I_o \rho_o g B_x / 12F_o) \ \mathrm{Cos}\ wt$$

The ratio V_1 to the variation of V of order of

$$(V_1/\delta V) = (\pi^2/96)(mg/F_o)(m\nu w_o/BI_o); \ m = \rho_o S \ell$$

For the same parameters as before and the string length 0.2m the ratio of order of 2.e-06.

So, the main signal is given by figure (11). According to the formula it is probably better to use an aluminum string instead of the copper one, but the effect of heating is larger in this case.

The length of the string is limited by the tolerable value of sagitta (see eq. (5)):

$$\delta Z = mgl/8F_o = \pi^2 g/8w_o^2$$

To have $\delta Z << 1$ mm, the frequency $f_o = W_o/2\pi$ has to be big enough:

(12) $f_o = (1/2\ell)\sqrt{F_o/\rho_o S} >> 17.6$ Hz

The maximum tension F_o is given by the elastic limit:

$$(F_o)_{max} = ES(\Delta\ell/\ell)_{max}; \quad (\Delta\ell/\ell)_{max} \simeq 10^{-3}$$

For a copper string it gives the limit

(13) $\ell << 3.14$m

With l=0.3m, f=176Hz and S=1mm^2, the force F_o is F_o = 10kG and scaled down with decrease of the cross-section. The decrease of the cross-section reduces also the total Ohmic heating, which is proportional to a cross-section for a given current density. The actual choice of the cross-section is limited only by the parameters of the PZT transducer.

With S=1 mm^2 and g_{33}=20.e-03 (V/m)(N/m) the amplitude of the signal is

$$V/h = 0.8 \cdot 10^5 \text{ V/m}$$

It means, that the variation of the field of order of $\delta B/B \sim 1.0$ e-05 induces the change of the signal of order of millivolts, if h $\simeq$ 1mm.

The cross-talk between current in two strings is, probably, the most important adverse effect. Let r be the distance between two currents. The ineraction gives a signal with the amplitude, which is in

$$\mu_o I_o / 16\pi r B_z$$

times less than the main signal (11).

With r=1 cm, I = 1A, B = 3T this ratio is 0.25e-05, or less than variation of the main signal. Additional reduction of this parazitic signal can be obtained rather easily, because it has frequencies W_o and $3w_o$, while the main signal is induced on frequency $2w_o$.

Appendix

Here we outline some problems, which have to be studied during the magnetometer design.

With a single string it is possible to do the following experiments.

1. To check the elastic limit for a string.
2. To determine the density of the string by measurements self-frequency, length and the applied tension.
3. To study the change of self-frequency of the string due to Ohmic heating.
4. To measure damping time of the string by the width of the resonance measurements.

All that can be done without PZT transducer with two coils, winded along the string and placed on the opposite sides of the string (see Fig. 2). The AC current in the string induced in the coils two signals with the difference

$$V = (2/\pi)N \cdot \mu_o I_o f_o \ell x_o \, (1/r_1 - 1/r_2)$$

where f_o is the current frequency, l, r_1, r_2 are the length, internal and outer radii of the coils, N is the total number of turns in both coils, x_o is the amplitude of oscillations of the string. With I = 1A, ℓ=10cm, x_o= 1mm, r_1=2mm, N=200 and f_o=150 Hz the signal about 1.2mV can be expected.

When PZT transducer is implemented in the string, the following can be done:
5. An observation and calibration of the dependence of the signal, induced on the PZT transducer, vrs. the amplitude of the string oscillations.
6. Study of the cross-talk effect with two strings.
7. Study of the parametric resonance, driven by the voltage on the transducer.

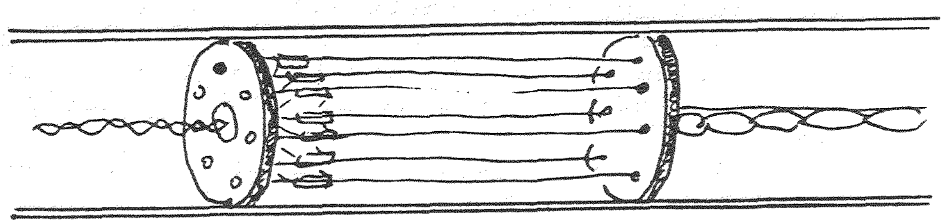

fig. 1

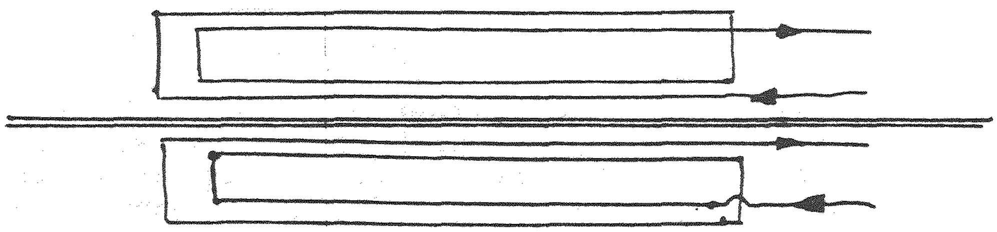

fig. 2