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Ely Shrauner
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STATISTICAL ANALYSIS OF QUENCH TEST DATA ON MAGNETS and CABLE STOCK

INTRODUCTION

The SSC requires so many large magnets that their testing and training must be accomplished as efficiently as possible. Determinations of the characteristics of magnet training and their implications for reliability are necessary for decisions that trade among machine operating level, reliability, and both time and dollar costs. Examples of extreme cases are the questions: Should every magnet be thoroughly tested and trained in a magnet test facility, or installed with minimal testing and training, and which is least costly? How much compromise of operating level is tolerable in order to gain reliability?

The objective of this note is to analyse the existing data on 6 model magnets made at LBL and 6 made at Brookhaven, and the data on the superconducting cable stock used in their fabrication so that this information can be brought to bear ultimately on these kinds of decisions. We will first compare each magnet against itself to exhibit the degree, as well as the fact, of their training. Second, we will observe the lack of useful correlations between the present data on the superconducting strands and cables and the data on the completed magnets. Third, we will use the data on the combined set of magnets to extrapolate predictions about production testing and training of a large number of magnets of which the models represent a normal sample.

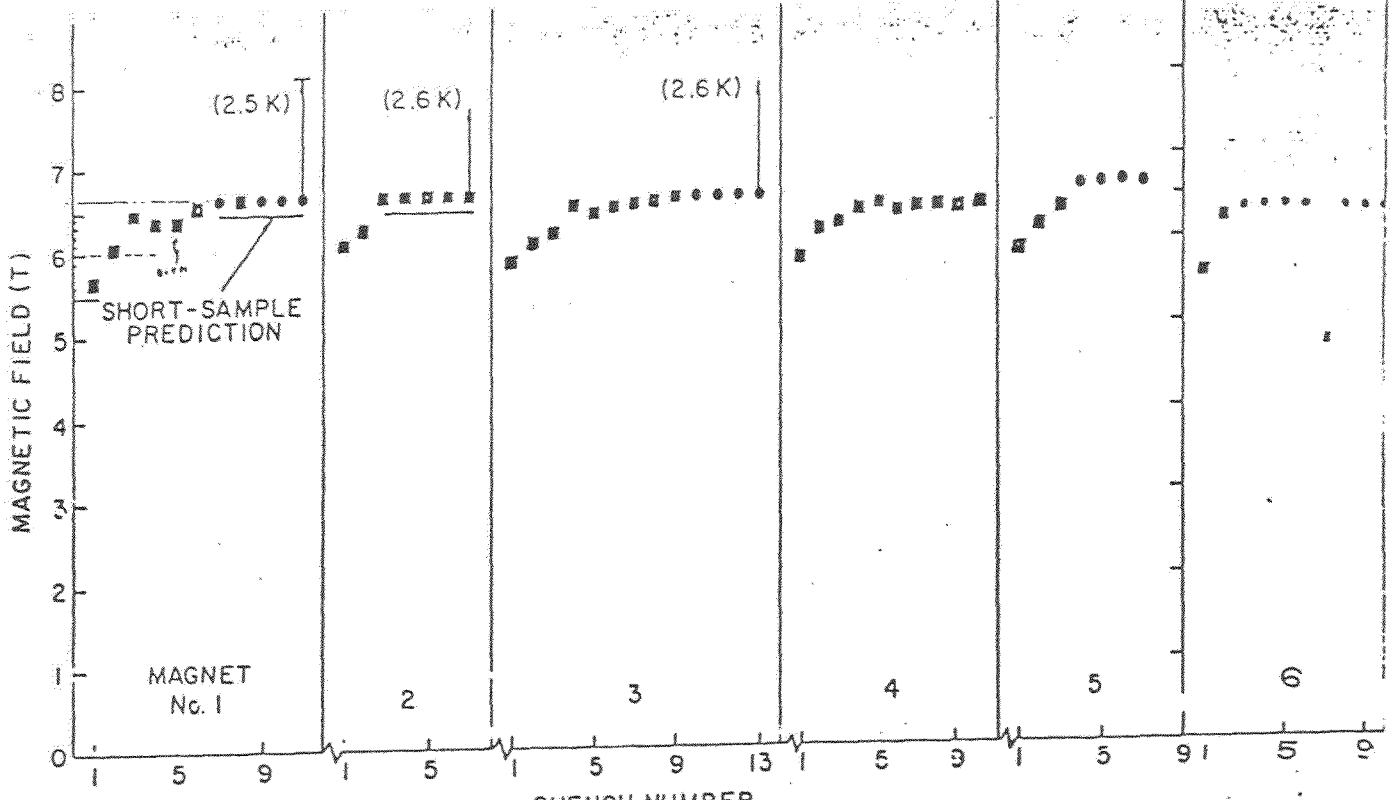
We find that training does occur in the amount of about 10 per cent change in field strength from its value at the first test quench to its asymptotic value after several training quenches, all at the same temperature, 4.5° K. We find that data from critical-current measurements on the strands and cables at 4.2° K are not of adequately predictive correlation with test data on the completed magnets to be of value in reducing the need for testing the completed magnets. Averages of the test-quench training curves of the model magnets indicate that after 2 or 3 training quenches the magnets could be installed with expected probabilities of further quenches at the level of a few per cent. This assumes scaling extrapolations to maximum operating temperature 4.35° K and minimum strand current density of $J_c = 2750$ A/mm^2 in applied field of 5 Tesla at 4.2° K.

Figures. 1a, 1b, 2a, 2b, and 3 :

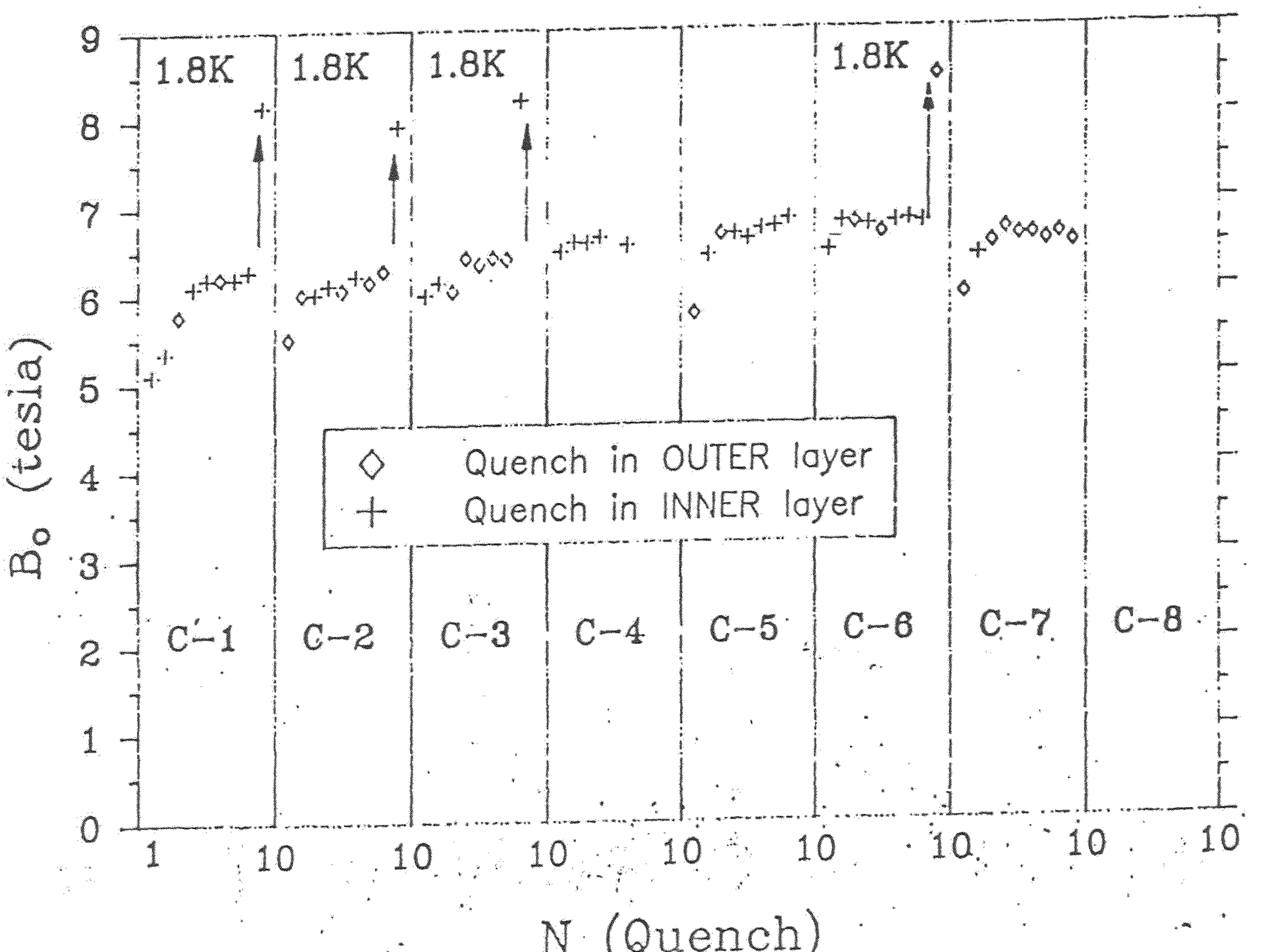
TRAINING

The quench test training curves for the 6 model magnets fabricated at BNL are shown in Figure 1a, and for the 6 magnets made at LBL in Figure 1b. For each magnet an asymptote is fitted, graphically or numerically, from which an asymptotic value of the field strength after many quenches $B_n^{(i)}$ is determined. To investigate trainability the field strength at the n-th quench of the i-th magnet $B_n^{(i)}$ is expressed as a fraction of its asymptotic field value $B_{AS}^{(i)}$ in

$$\beta_n^{(i)} = B_n^{(i)}/B_{AS}^{(i)} \quad .$$



TRAINING LBL–SSC DIPOLE MODELS FIGURE 1



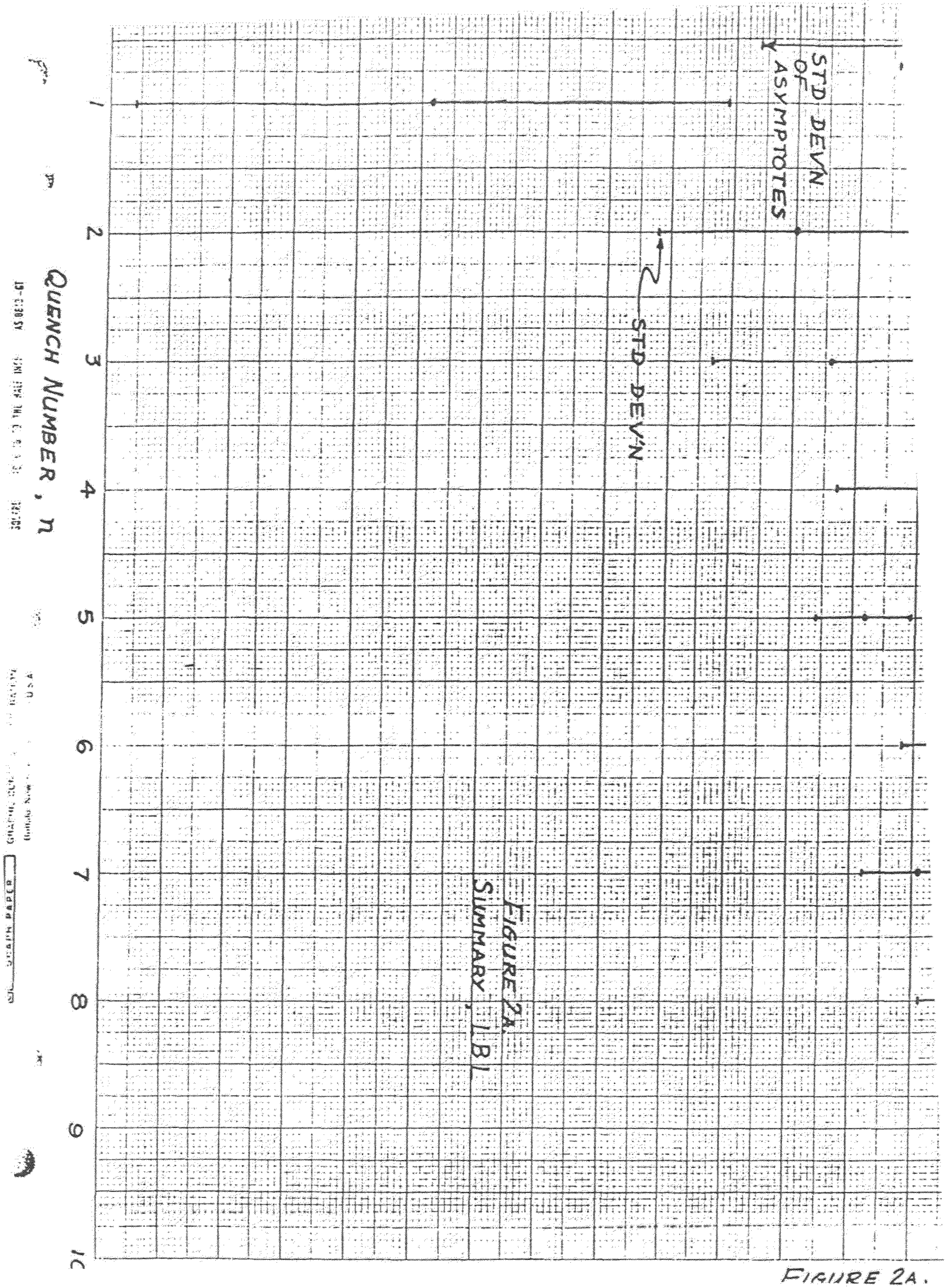

STD DEV'N OF ASYMPTOTES
STD DEV'N
QUENCH NUMBER, n
1
2
3
4
5
6
7
8
9
10
FIGURE 2A
SUMMARY, I.B.I

FIGURE 2A.

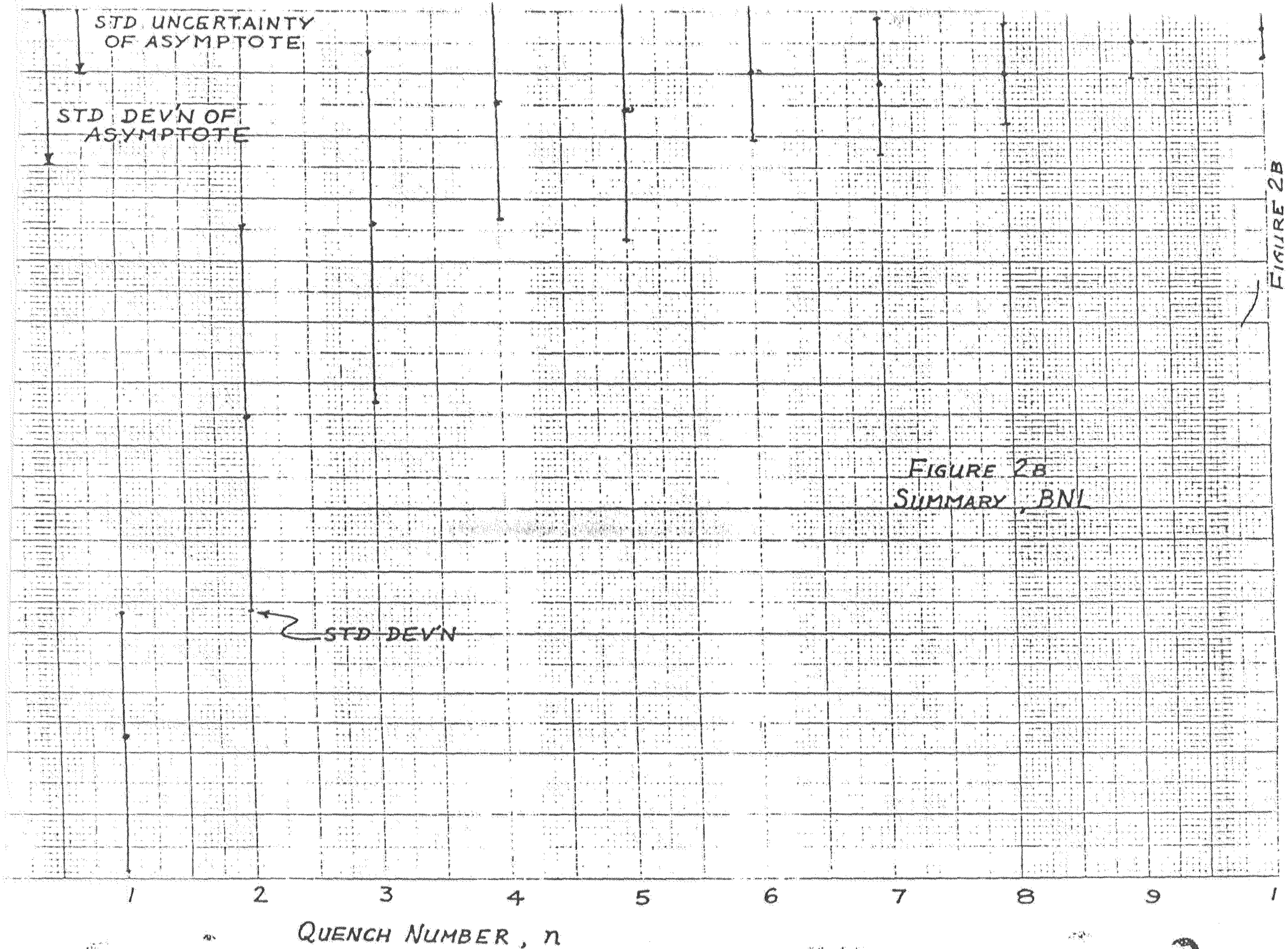


FIGURE 2B
SUMMARY, BNL

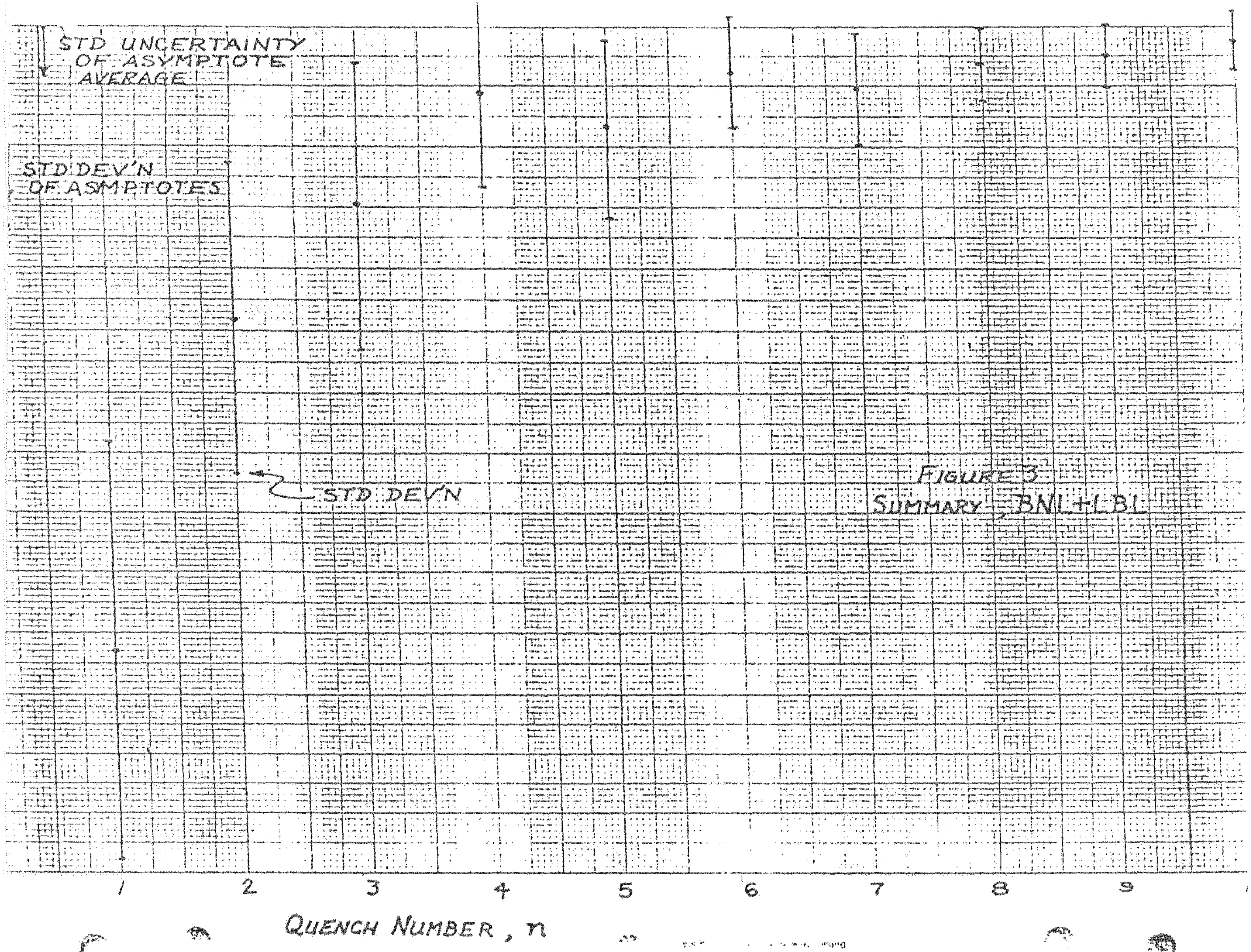


FIGURE 3
SUMMARY, BNL+LBL

For the 6-magnet LBL sample i runs 2 to 7, and for the 6-magnet BNL sample i runs 1 to 6. The LBL magnet number 1 was ignored on recommendation of C. Taylor.

The average β_n of the 6 $\beta_n^{(i)}$ at a given quench number n for the 6-magnet LBL sample is shown in Figure 2a. The uncertainty bars shown are the standard deviation among the 6 $\beta_n^{(i)}$'s about the mean β_n at each n. The standard deviation among the 6 asymptotic fields about their mean $<B_{AS}^{LBL}>$ is also shown as a fraction of this mean in Figure 2a. Figure 2b shows the corresponding data for the 6-magnet BNL sample. Figure 3 shows the corresponding data for the combined 12-magnet LBL + BNL sample.

Standard deviations are shown rather than standard errors of the means because we are interested in the likelihood of the next magnet of which a given sample is representative falling within our cut rather than determination of the precision of a given mean value. The shrinkage of these standard deviations as we advance toward higher quench-number for each of the samples is clear evidence that on average each magnet is approaching an asymptote. $B_n^{(i)}$ approaches $B_{AS}^{(i)}$ as n grows. Keeping in mind that $\beta_n^{(i)}$ compares the progress the i-th magnet has made in n quenches in approaching its own training asymptote, the first 5 or so points of the mean training curves clearly show that training does occur (on average). This is summarized in Table I.

TABLE I

LBL

The probability of $\beta_1 < 96$ % is 85 %
" " " $\beta_2 < 95$ % " 14 %
" " " $\beta_3 < 96$ % " 14 %
" " " $\beta_4 < 97$ % " 15 %

BNL

The probability of $\beta_1 < 90$ % is 83 %
" " " $\beta_2 < 95$ % " 83 %
" " " $\beta_3 > 94$ % " 84 %
" " " $\beta_4 > 97$ % " 83 %

The asymptotic quench field strengths averaged over the 6-magnet samples and their fractional standard deviations are:

LBL: $<B_{AS}> = 6.595$ Tesla, $\sigma/<B_{AS}> = 2.50$ %
BNL: $<B_{AS}> = 6.567$ Tesla, $\sigma/<B_{AS}> = 3.51$ % .

The nominal temperature in both cases is 4.5 degrees Kelvin. Since β_n 's basically compare a magnet with itself, there is no need to question this here. Later we will take possible temperature differences into account.

CORRELATIONS OF STRAND/CABLE DATA WITH MAGNET TEST DATA

In addition to the quench test training curves for the magnets we also have data on the winding stock used in the magnets. Each magnet has 4 coil windings: upper and lower halves of the inner and outer layers of the cosΘ windings. Each of these 4 windings could come from different cable spools, as in magnets number 3,4,5, and 6 of the BNL sample. On the 1-meter-model magnets of the LBL sample the upper and lower halves of inner or outer windings were all taken from the same spools, as was also the case for magnets number 1 and 2 of the BNL sample. For each spool the data given are: J_c , the strand critical current density at 4.2° K in a 5 Tesla field applied transversely; $I_{c\perp}$, the critical current of the fabricated cable at 4.2° K in a 5 Tesla field applied perpindicular to the flat width of the cable; and $I_{c\parallel}$, the critical current at 4.2° K in a 5 Tesla field applied parallel to the flat width and perpindicular to the edge of the cable. For the BNL magnets $I_{c\parallel}$ is not given. These data are given in Table II. The Q's in Table II indicate that the asymptotic quenches of a given magnet originate predominantly in its inner or outer winding layers.

Table II.

Causal correlation of the strand and cable quench data with the location of the origin of asymptotic quenches of a given magnet in its inner or outer winding layers is absent or very weak. For a given magnet its asymptotic quenches originate predominantly in its inner or outer winding layer. For our combined 12-magnet sample asymptotic quenches predominantly originate in the inner layers of 5 magnets and in the outer layers of 7 magnets.

This reflects the fact that the maximum quench currents for the cables used in inner and outer winding layers have been optimally adjusted for the different maximum field strengths in the cross section regions of the two layers at $I_{c\perp}^{(in)} = 10.50$ kA, $I_{c\perp}^{(out)} = 8.40$ kA, $I_{c\parallel}^{(in)} = 11.17$ kA, and $I_{c\parallel}^{(out)} = 8.50$ kA. The ratio is 1.32 for both $\perp$ and $\parallel$ cases. Uncertainties are about 4 per cent for all the I_c 's .

On top of this balancing it might be expected that the likelihood of asymptotic quenches for a given magnet originating predominantly in its inner or outer winding layer would be determined by whether the given layer was wound with cable stock from a spool whose short-sample quench data were above or below average by about a standard deviation. This does not occur.

In both the LBL and BNL magnet sets there are subsets wound with cable stock of identical strand/cable data and in which asymptotic quenches predominantly originate equally in inner and outer coil layers of different magnets. This kind of cancelation of correlation occurs in 2 pairs in the BNL sample, for which all 6 magnets have negligible variances of inner winding stock and about 5% standard deviation in outer winding stock. The other two BNL magnets suffer asymptotic quenches predominantly in their outer layers inspite of one of them having weaker-than-average cable in one half inner coil. The LBL sample also has a pair of nearly identically supplied magnets in which one quenches mainly in the inner and one in the outer layer. There is a triplet of nearly identically supplied LBL magnets two of which have asymptotic quenches mostly in their inner layers and one in its outer layer. In this triplet the strand/cable data for the outer layer is stronger than average by a standard deviation.

Locations of the origins of quenches in asymptotically trained magnets of these samples are not noticably correlated with variations from the means of the quench currents measured in the strands and cables of which they are fabricated.

The first two training quenches of all 6 magnets of the BNL sample originate in the inner winding layers. In the LBL sample first quenches occurred in the inner layer of 3 magnets and in the outer layers of the other 3, with no apparent correlation with strand/cable quench test data.

Correlation of asymptotic quench field B with variances of strand current densities $J_c^{(i)}$(in) and $J_c^{(i)}$(out) for a given magnet (i) are quite close in the LBL sample. Correlations of the signs of the variances of $B_{AS}^{(i)}$ with those of $J_c^{(i)}$(in) , $J_c^{(i)}$(out) and $J_c^{(i)}(in) \wedge J_c^{(i)}(out)$ are of the order of 80 per cent. For the whole 6-magnet LBL sample the standard deviations of B_{AS} (3.5%) , J_c (in) (4.7%)

		Inner Layer			Outer Layer		
LBL	B_{AS} (T)	J_c (kA/mm²)	$I_{c\perp}$ (kA)	$I_{c\parallel}$ (kA)	J_c (kA/mm²)	$I_{c\perp}$ (kA)	$I_{c\parallel}$ (kA)
~~i = 1~~	~~6.24~~	~~(Q) 2,280~~	~~9,100~~	~~10,900~~	~~2,273~~	~~6,985~~	~~7,250~~
2	6.24	2,238	9,450	10,525	(Q) 2,435	7,825	8,510
3	6.40	2,500	10,600	11,980	(Q) "	"	"
4	6.64	(Q) 2,545	10,400	11,450	"	"	"
5	6.83	(Q) "	"	"	2,719	8,130	8,380
6	6.80	(Q) 2,509	10,450	10,800	2,719	8,280	8,540
7	6.66	"	"	"	(Q) "	"	"
av. =	6.595	2.474	10.292	11.167	2,577	7,027	8,498
σ =	.231	.117	.419	.548	.155	.228	.0598
σ/av. =	.035	.047	.041	.049	.060	.028	.0070

i = 1 data not included in averages

BNL

i = 1	6.65	2,500 "	10,600 "	2,435 (Q) "	7.480 "
2	6.[illegible]5	" (Q) "	" "	" "	" "
3	6.68	2,509 2,500	11,120 10,510	" (Q) 2.719	" 8,130
4	6.44	2,500 (Q) 2,500	10,600 10,500	" 2,435	" 7,480
5	6.[illegible]	2,500 2,509	" 11,120	2.719 (Q) "	8,130 8,280
6	6.29	" 2,500	" 10,600	" (Q) 2,435	" 7,480
av =	6.567	2,502	10,705	2,553	7.776
σ =	.251	.0041	.254	.146	.369
σ/av =	.025	.0016	.024	.057	.047

and the ratios B_{AS}/J_C (in) (3.0%) and $B_{AS}/\sqrt{J_C \text{ (in)}}$ (1.2%) ; and likewise of B_{AS} , J_C (out) (6.0%) and their ratios B_{AS}/J_C (out) (3.7%) and $B_{AS}/\sqrt{J_C \text{ (out)}}$ (2.2%) , indicate closer correlations of $B_{AS}^{(i)}$ with $J_C^{(i)}$(in) and $J_C^{(i)}$(out)than if they were statistically independent. Warning should noted that if the (standard) errors of the current measurements on individual strands are greater than about 4 percent and are included, then most of the above statistical correlations would be washed out.

For the BNL sample there is no net correlation in the signs of the variances of $B_{AS}^{(i)}$ and $J_C^{(i)}$ (out) . There is no significant variance at all among the $J_C^{(i)}$(in) = 2.502 kA/mm^2 for the BNL sample. The statistical standard deviations of B_{AS} (2.5%), J_C(out) (5.7%), and of B_{AS}/J_C(out) (6.2%) and $B_{AS}/\sqrt{J_C(\text{out})}$ (3.8%) suggest nothing beyond statistical independence.

We have short-sample data for all cables in both BNL and LBL magnets on the maximum quench current, $I_{C\perp}$, measured in an external magnetic field of 5 Tesla perpindicular to the width plane of the flat cables at 4.2 degrees K. Neither $I_{C\perp}^{(i)}$(in) for the inner coil layers nor $I_{C\perp}^{(i)}$(out) for the outer layers correlates with the asymptotic quench field $B_{AS}^{(i)}$ of the trained magnets as well as do the strand current densities $J_C^{(i)}$(in) and $J_C^{(i)}$(out) discussed above. This is somewhat surprising because $I_{C\perp}$ should already incorporate and reflect the cumulative traumatic effects of cable frabrication. Measurement of $I_{C\perp}$ is apparently less precise than for J_C . And apparently it is this lack of precision that prevents observable correlations of variances of $I_{C\perp}^{(i)}$ with those of $B_{AS}^{(i)}$ or with locations of the origins of quenches in the magnets.

For the cable stock used in the LBL magnets we also have data on the quench currents $I_{C\parallel}^{(i)}$ measured at 4.2° K in 5 Tesla magnetic field applied parallel to the flat cable width. Because this is the field orientation that corresponds most closely to that of the cables in the magnets, it is disappointing that correlations of $I_{C\parallel}^{(i)}$ with magnet measurements are not at all evident. This correlation is presumably washed out by the uncertainties in each measured $I_{C\parallel}^{(i)}$.

Any signifcant usefulness of the quench test data on the strands and cables for predicting behavior of the asymptotically trained magnets has eluded discovery in this analysis.

PROBABILITIES OF FURTHER QUENCHING AFTER PARTIAL TRAINING

Statistical analysis can be applied more successfully toward estimating the vulnerability of magnets installed after some chosen number of training quenches against their suffering further quenches and its dependence on operating levels of the SSC. The probability that a magnet represented by our sample will suffer a further quench after after n-1 training quenches is related almost directly to the mean value and standard deviation of field strength at the n-th quench of the (mean) training curve for our sample. We assume that our sample is a normal statistical sample that is representative of the installed magnets.

In order to use data from both LBL and BNL 6-magnet samples we combine them into a single 12-magnet sample. To do this we first make the LBL and BNL equivalent. It has been suggested by C. Taylor that there is likely a difference of the actual temperatures used for the nominally 4.5° K training quenches at LBL and BNL that is of the order of 0.1 K° . We compensate for these and/or any other effects contributing to the difference of the means of the asymptotic quench fields for LBL ($<B_{AS}^{LBL}>$ = 6.595 T.) and BNL ($<B_{AS}^{BNL}>$ = 6.567 T.) by raising all the BNL values of $B_n^{(i)}$ by the ratio of the means of the asymptotic fields. The 12 corrected values for given n are called $\overline{B}_n^{(i)}$, with the 6 observed at LBL requiring no correction, $\overline{B}_n^{(i)LBL} = B_n^{(i)LBL}$; and the 6 $B_n^{(i)BNL}$, as observed at BNL entering in the corrected form

$$\overline{B}_n^{(i)BNL} = B_n^{(i)BNL} <B_n^{LBL}>/<B_n^{BNL}> = B_n^{(i)BNL}(6.595/6.567) \; .$$

Thus, $<\overline{B}_{AS}^{BNL}> = <B_{AS}^{LBL}> = <\overline{B}_{AS}>$. These data are plotted in Figure 4 as the ratios

$$\gamma_n = <\overline{B}_n>/<\overline{B}_{AS}> \quad ,$$

where all averages indicated by <...> are now over the full 12-magnet sample. It should be noted that the difference of the means $<B_{AS}^{LBL}>$ minus $<B_{AS}^{BNL}>$ is small; only about one-tenth of the standard deviation of B_{AS}^{LBL} .

The bars through graph points in Figure 4 are standard deviations of 12 $\overline{B}_n$'s divided by $<\overline{B}_{AS}>$. This is because we are interested in comparing a given SSC magnet represented by our sample after n-1 training quenches against average asymptotic performance standards predicted by our sample.

Figure 4 suggests that if magnets are installed after 3 training quenches and the SSC is operated at 4.5° K and an energy corresponding to 95.2 % of $<\overline{B}_{AS}>$, i.e., 6.278 Tesla, then P = 16 % of the magnets will be expected to suffer further quenchs. For other operating levels B we get the percentages:

P = 16 %	at B = .952 $<\overline{B}_{AS}>$	= 6.28 Tesla	,
P = 10 %	" " .943	" 6.22 "	,
P = 2 %	" " .919	" 6.06 "	,
P = 1 %	" " .911	" 6.01 "	.

The present sample has $<\overline{B}_{AS}> = 6.595$ Tesla with a nominal working temperature of 4.5° K and mean strand current density $J_C = 2500$ A/mm^2 (at 4.2° K) . If we assume a supply of cable stock with $J_c \geqq 2750$ A/mm^2 (at 4.2° K) can be relied on, then we might also expect asymptotic quench fields to be scaled up to

$$<B_{AS}> = (2750/2500)^{1/2} <\overline{B}_{AS}> = 9.92 \text{ Tesla} \quad .$$

Assuming the same statistics hold, we would then expect that after 3 training quenches and operating at the same working temperature (4.5° K), the probabilities P of further quenches in the installed magnets at operating levels B are:

P = 16 %	at B = .952(6.92)	= 6.59 Tesla	,
P = 10 %	" " .943	" 6.53 "	,
P = 2 %	" " .919	" 6.36 "	,
P = 1 %	" " .911	" 6.30 "	.

Assuming everything above plus a maximum operating temperature of 4.35° K and .85 Tesla change of effective dipole field strength per degree change of working temperature, the correction from the above 4.5° K to 4.35° K gives probabilities P of further quenches after 3 training quenches at operating levels B as:

P = 16 %	at B = 6.72 Tesla		,
P = 10 %	" "	6.66 "	,
P = 2 %	" "	6.49 "	,
P = 1 %	" "	6.43 "	.

Perhaps 3 training quenches are more than is required. With all the above assumptions excepting only that the magnets are installed and observed after 2 training quenches at 4.35° K , i.e., that all the data on the present 12-magnet sample scales as described to a final operating temperature of 4.35° K and an effective strand current density of $J_C = 2750$ A/mm (at 4.2 K) , then the probabilities P of further quenches at operating levels B are:

$P = 16\ \%$ at $B = 6.60$ Tesla ,
$P = 8.6\%$ " " 6.50 " ,
$P = 2\ \%$ " " 6.31 " ,
$P = 1\ \%$ " " 6.24 " .

Under these extrapolated conditions of $T = 4.35°$ K and $J_c = 2750\ A/mm^2$ (at 4.2° K) and operating-field levels B , the probabilities P (i.e., the fractions of installed magnets,) of suffering further quenches after n training quenches are:

for $n = 2$, 3
$P = 3.7\ \%$, $2.0\ \%$ at $B = 6.4$ Tesla ,
$P = 8.6\ \%$, $5.0\ \%$ at $B = 6.5$ Tesla ,
$P = 16\ \%$, $10\ \%$ at $B = 6.6$ Tesla .

Thus, in a quarter-sector of 180 magnets we would expect about 18 quenches in the tunnel before we could operate at 6.6 Tesla and 4.35° K if the magnets had each undergone 3 training quenchs at 4.35° K before installation. There remains the question of how many of these 18 quenches should involve identifying and/or replacing the offending magnet.

The training curves for the present samples of model magnets do not indicate much profitability from considering more than 3 training quenches. However, there are seeming irregularities in the data on the 4-th and 5-th training quenches in the present test samples. These irregularities would hopefully normalize in better, more nearly-normal samples. Indeed, we have normalized them in our use of the data in Figure 4 by shifting the means γ_4 and γ_5 to the best-fit curve shown there. All the γ_n except γ_4 and γ_5 fall almost exactly on the curve $\gamma_x = 1 - \exp(-2.273\ x^{0.4204})$ at $x = n$. The standard deviations of γ_4 and γ_5 about these shifted means were used so as to obtain the most accurate and conservative extropolations.

There is also the question of whether or not the residual quenches to be expected might be reduced by performing the training quenches at lower temperature in a magnet test facility than the working temperature in the ring. Only tests explicitly directed to this question can answer it.