Estimate of the Bremsstrahlung Lifetime of Collider Beams

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December 1985

Bremsstrahlung occurs in collisions with momentum transfer less than $Q \sim 2Mc$, corresponding to scattering through angles that are less than $2/\gamma$. The major effect is the loss of longitudinal momentum $\Delta p = \hbar\omega$. The loss of beam particles will be governed by the rate of creation of particles with momentum change greater than Δp_{bucket}, where Δp_{bucket} is the momentum spread permitted by the phase stability.

The bremsstrahlung cross section in the high energy limit is

$$\frac{d\sigma}{d\omega} = \frac{16}{3}\frac{\alpha r_o^2}{\omega}\left(1 - \frac{\omega}{p} + \frac{3\omega^2}{4p^2}\right)\left[\ln\left(\frac{2p(p-\omega)}{M\omega}\right) - \frac{1}{2}\right]$$

where $r_o = e^2/Mc^2$ is the classical <u>proton</u> radius. The number of particles lost per unit time per IR is the integrated cross section times the luminosity:

$$\frac{dN}{dt} = \frac{16\alpha r_o^2}{3}\mathcal{L}\int_{\Delta p}^{p}\frac{d\omega}{\omega}\left(1 - \frac{\omega}{p} + \frac{3\omega^2}{4p^2}\right)\left[\ln\left(\frac{2p(p-\omega)}{M\omega}\right) - \frac{1}{2}\right]$$

The integral can be approximated for small $\Delta p/p = \Delta\gamma/\gamma$ as

$$\frac{dN}{dt} \simeq \frac{16}{3}\alpha r_o^2\mathcal{L}\left\{\ln(2\gamma\sqrt{\gamma/\Delta\gamma})\left[\ln(\gamma/\Delta\gamma) - 1\right] + \frac{3}{8}\ln(2\gamma) - \frac{1}{16} - \frac{\pi^2}{6}\right\}$$

Neglected terms are of order

$$\frac{\Delta\gamma}{\gamma} \text{ or } \frac{\Delta\gamma}{\gamma}\ln(\Delta\gamma) .$$

For protons, the numerical coefficient has the value,

$$\frac{16}{3}\alpha r_o^2 = 0.917 \times 10^{-33}\text{cm}^2$$

The bremsstrahlung lifetime τ_{brems} is given by

$$\frac{1}{\tau_{brems}} = \frac{n_{IR}}{N}\frac{dN}{dt}$$

where n_{IR} is the number of collision points with luminosity $\mathcal{L}$ around the ring and N is the total number of protons per beam.

For the nominal SSC parameters, $\mathcal{L} = 10^{33}\text{cm}^{-2}\text{s}^{-1}$, $E \simeq 20$ TeV ($\gamma = 2 \times 10^4$), $\Delta\gamma/\gamma = 10^{-4}$, we find

$$\frac{dN}{dt} = 116\ \text{s}^{-1}\ .$$

For $n_{IR} = 6$, $N = 1.3 \times 10^{14}$, the lifetime is $\tau_{brems} \simeq 6 \times 10^3$ years.

For an electron-position collider such as CESR, with nominal parameters, $\gamma = 10^4$, $\Delta\gamma/\gamma = 10^{-3}$, $\mathcal{L} = 3 \times 10^{31}\text{cm}^{-2}\text{s}^{-1}$, $n_{IR} = 2$, $N = 1.5 \times 10^{11}$, the result for the lifetime is $\tau_{brems}(e) \simeq 10^4\text{s} \simeq 2.8$ hours.