A Comparison of Linear Aperture Between Lattices With 60° and 90° Per Cell

SSC-N-81
E.Forest/A.Chao
November 1985

One of the important parameters that specify the SSC lattice is the betatron phase advance per cell μ. To minimize the nonlinear effects due to systematic sextupoles such as the chromaticity sextupoles, two particular values for μ have been considered, namely 60° and 90°. This note is to summarize a few study results made to compare these two phase advance values as far as the linear aperture[1] is concerned.

The advantage of 90° lattice is that the ß-function and the dispersion function are smaller for a given cell length. (The maximum ß's in cell for 60° and 90° lattices are not too different but the average ß's are.) The smaller ß-function makes the particle motion less sensitive to the magnet field errors, thus providing more linear aperture. The smaller dispersion function makes off-momentum behavior more linear in the presence of magnet field errors even though the chromaticity sextupoles are stronger. In addition, 90° lattice may require a smaller needed aperture due to its smaller ß-function.

On the other hand, the 90° lattice requires stronger chromaticity sextupoles as already mentioned. More importantly, it needs stronger quadrupoles and is therefore more expensive than a 60° lattice for a given cell length. To reduce the cost of a 90° lattice, one way is to reduce the total number of quadrupoles by lengthening the cells. However, longer cells mean higher ß-function and dispersion function, thus substantially removing the advantage of 90° cells.

In this note, we give the study results on three lattices:

lattice	μ	half cell length	max ß in cell	max dispersion in cell
A	60°	100 m	345 m	3.9 m
B	90°	100	331	2.1
C	90°	120	403	2.8

These lattices are with 6 tesla bending field and with distributed interaction regions. The fractional part of the tunes are $\nu_x=0.28$ and $\nu_y=0.30$. Two families of interleaved chromaticity sextupoles are used in each lattice. The rms values of the random magnet multipole field errors are obtained from Ref.2.

For each lattice, particles with various initial amplitudes, designated by $\Delta x^*=\Delta y^*$ at the interaction point with $\beta^*=1$m, are tracked using mainly RACETRACK and sometimes also MARYLIE. The smears[1] in linear invariants, designated by $\Delta J_1/J_1$ and $\Delta J_2/J_2$, are computed to indicate the linearity of the lattices. Small smears indicate nearly linear motion.

1. Bare Lattices

In the absence of magnet field errors, the main source of nonlinearity comes from the chromaticity sextupoles.[3] Since the 90° cell lattices have stronger chromaticity sextupoles, particle motion is more nonlinear in them, as shown below for on-momentum particles:

Lattice	Δx*=Δy* (mm)	ΔJ1/J1	ΔJ2/J2
A	0.1	0.0022	0.0022
	0.15	0.0050	0.0050
	0.2	0.0087	0.0088
	0.3	0.020	0.020
	0.4	0.034	0.035
B	0.25	0.049	0.052
	0.3	0.068	0.074
	0.35	0.091	0.103
	0.4	0.12	0.14
	0.45	0.15	0.18
C	0.1	0.0058	0.0059
	0.15	0.013	0.013
	0.2	0.024	0.023
	0.3	0.054	0.052
	0.4	0.096	0.090

The smear in lattice A is very small even up to the highest amplitude studied. The smears in lattices B and C are larger because of the stronger sextupoles; but as we will see, they are still substantially smaller than the contribution from the magnet field errors.

2. Random Field Errors Without Sorting

As field errors are increased, starting with the bare lattices, the smears increase. The increases in smear are expected to be the largest for lattice A (ß-function is largest) and smallest in lattice B (ß-function is smallest). Indeed, when the random magnet field errors reach the level of the presently conceived values,[2] the 90° lattices have overcome their disadvantage as bare lattices and actually become more linear than the 60° lattice. The following table gives the average value of smear over 4 sets (3 sets for C) of random field errors:

Lattice		$\Delta x^* = \Delta y^*$ (mm)	$\Delta J1/J1$	$\Delta J2/J2$
A		0.1	0.10	0.14
		0.15	0.14	0.24
		0.2	0.25	0.34
	*	0.3	0.27	0.40
	*	0.4	0.46	0.66
B		0.1	0.07	0.08
		0.15	0.11	0.13
		0.2	0.17	0.20
		0.3	0.30	0.35
		0.4	0.49	0.50
C		0.1	0.08	0.07
		0.15	0.13	0.12
		0.2	0.18	0.17
		0.3	0.31	0.30
		0.4	0.50	0.44

Asteriks in the table mean the dynamic aperture is exceeded; the number of asteriks is the number of seeds when that happens.

The fact that one of the seeds in lattice A exceeds the dynamic aperture indicates the 60° lattice is the least linear. The smear of lattice A is larger than those of B and C. Lattices B and C are quite comparable. Their difference in ß-function (which favors lattice B) is balanced out by their difference in the bare lattice (which favors lattice C).

It should be emphasized here that no magnet sorting[1] has been applied to the field errors in obtaining the above table. As we will see later, as sorting is applied, the effective field errors are reduced; the 60° and 90° lattices then become quite comparable in linearity.

For off-momentum particles, the 90° lattices tend to be more linear because of the smaller dispersion functions. The following table gives the results for off-momentum cases with $\Delta E/E=10^{-3}$:

Lattice		$\Delta x^*=\Delta y^*$ (mm)	$\Delta J1/J1$	$\Delta J2/J2$
A	****	0.4	-	-
B		0.1	0.08	0.09
		0.15	0.12	0.14
		0.2	0.17	0.21
		0.3	0.30	0.35
		0.4	0.47	0.54

The following table gives the results for $\Delta E/E=-10^{-3}$:

Lattice		$\Delta x^*=\Delta y^*$ (mm)	$\Delta J1/J1$	$\Delta J2/J2$
A	*	0.4	1.49	0.61
B		0.1	0.08	0.08
		0.15	0.12	0.13
		0.2	0.18	0.18
		0.3	0.31	0.30
		0.4	0.45	0.44

We have included only the B results in the above off-momentum tables. For lattice A, only the results for $\Delta x^*=\Delta y^*=0.4$mm are given. All 4 seeds are unstable when $\Delta E/E=10^{-3}$ while one seed is unstable for $\Delta E/E=-10^{-3}$. Lattice A is clearly less linear than lattice B off momentum and without sorting.

3. With Sorting

The most relevant results are obtained when sorting is applied. In this study, we have used the preliminary sorting scheme[1] which is applied to only the sextupole (b_2) component of the magnet errors.

The same four seeds of random numbers as the previous section are used in this section. The following table is for on-momentum:

Lattice	$\Delta x^*=\Delta y^*$ (mm)	$\Delta J1/J1$	$\Delta J2/J2$
A	0.1	0.027	0.031
	0.15	0.05	0.05
	0.2	0.075	0.078
	0.3	0.15	0.15
	0.4	0.25	0.26
B	0.1	0.03	0.023
	0.15	0.055	0.044
	0.2	0.09	0.074
	0.3	0.17	0.15
	0.4	0.27	0.26
C	0.1	0.034	0.032
	0.15	0.063	0.055
	0.2	0.10	0.09
	0.3	0.20	0.18
	0.4	0.35	0.30

The lattices are much more linear than the cases without sorting. Lattices A and B give very similar on-momentum linear apertures while C is slightly less linear. All three cases are quite acceptable.

Results with sorting were also obtained for off-momentum particles. The following table is for $\Delta E/E=10^{-3}$:

Lattice	$\Delta x^*=\Delta y^*$ (mm)	$\Delta J1/J1$	$\Delta J2/J2$
A	0.1	0.053	0.11
	0.15	0.09	0.20
	0.2	0.13	0.36
	0.3	0.21	1.0
	0.4	0.31	0.8
B	0.1	0.033	0.026
	0.15	0.06	0.05
	0.2	0.09	0.08
	0.3	0.18	0.16
	0.4	0.30	0.28
C	0.1	0.040	0.044
	0.15	0.07	0.07
	0.2	0.10	0.11
	0.3	0.17	0.22
	0.4	0.29	0.43

The following table is for $\Delta E/E=-10^{-3}$:

Lattice	$\Delta x^*=\Delta y^*$ (mm)	$\Delta J1/J1$	$\Delta J2/J2$
A	0.1	0.08	0.06
	0.15	0.16	0.07
	0.2	0.29	0.15
	0.3	1.2	0.26
	0.4	1.5	0.39
B	0.1	0.04	0.03
	0.15	0.07	0.05
	0.2	0.10	0.08
	0.3	0.17	0.15
	0.4	0.26	0.25
C	0.1	0.04	0.044
	0.15	0.07	0.075
	0.2	0.11	0.11
	0.3	0.24	0.21
	0.4	0.45	0.54

For the C lattice, three sets of random numbers behave rather similarly to the B lattice but the fourth set gives a more nonlinear particle motion. Note that $\Delta E/E = \pm 10^{-3}$ is the conceived range of operation. The actual beam energy spread is much smaller than 10^{-3}.

One observes from the above off-momentum tables that (i) lattice B has the best linearity and (ii) lattice A has the least off-momentum linearity although the linear aperture reached by lattice A is about 0.2mm at the interaction point, which is considered acceptable.

4. Summary

Three cell lattices, A, B and C, are compared quantitatively for their linearities. We found that the three lattices each has its advantages and disadvantages.The differences among them (when random magnet field errors are included and sorting is applied) are small although noticeable. The best performance belongs to the 90° short cell lattice B, which is also the most expensive. For on-momentum particles, lattice A is as linear as lattice B. For off-momentum particles, to the extent that $\pm 10^{-3}$ energy window is needed, lattice B is more linear than lattice C which in turn is more linear than lattice A.

A qualitative comparison of these lattices can perhaps be summarized below:

lattice A: good on-momentum behavior
lattice B: good on-momentum and off-momentum behavior, but expensive
lattice C: good off-momentum behavior

The study shows that all three lattices provide acceptable linear apertures. The differences represent trade-offs between advantages and disadvantages that do not seem overwhelming. In case a more detailed study is called for in the future, we suggest that a detailed cost-aperture optimization like that done on the 60° lattice in Ref.1 be performed on the 90° lattice.

References

1. SSC-SR-1013, SSC Aperture Estimate for Cost Comparisons (1985).
2. Magnet Errors Group, Aperture Task Force, edited by H.E.Fisk and J. Peterson, SSC-7 (1985).
3. A. Dragt communicated the information that the fringe fields of the IR quadrupoles may contribute similar amount of nonlinearities as the chromaticity sextupoles in a bare lattice. This does not change our point here.