Orbit Distortion and Tune Shifts Due to the Long Range Beam-Beam Interactions

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The long range beam-beam interaction (LRBBI) has been studied before.[1-4] In this note, we will supplement these studies by a few remarks.

Orbit Effects

We first consider the orbit distortion effect. We assume the LRBBI takes place in a drift space ±L around each interaction point (IP). The orbit distortion outside the interaction region (IR) is given by (same expression applies to horizontal or vertical crossings)

$$\Delta x(s) = -d\ [\beta(s)/\beta^*]^{1/2}\ \sin[\pi\nu-\psi(s)]/\sin(\pi\nu) \qquad (1)$$

where

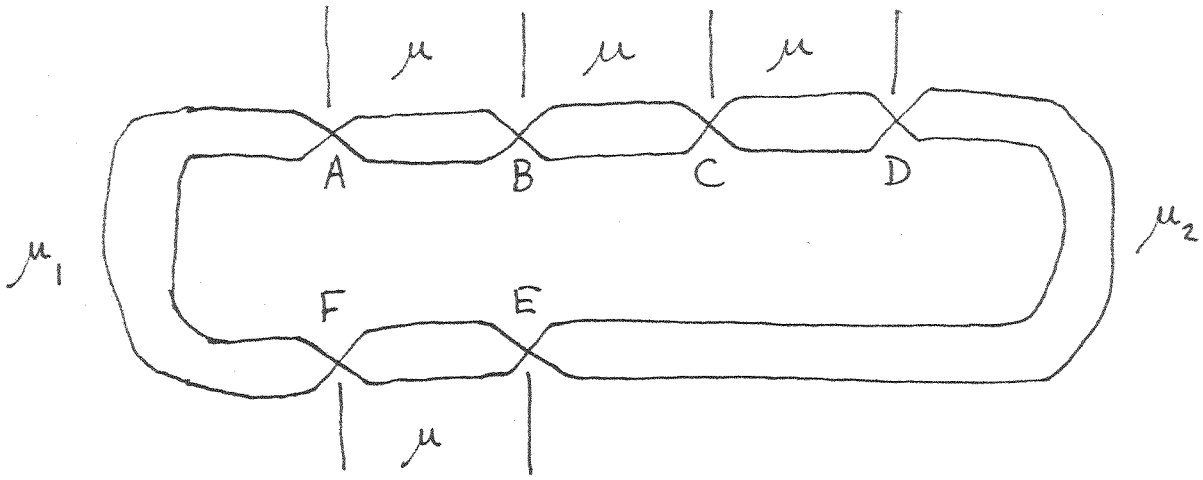
$$d = 4\,Nr_pL\,/\,(\alpha\gamma\,S_B)$$

and

N = number of protons per bunch = 1.5×10^{10}
$r_p = 1.53 \times 10^{-18}$ m
α = total crossing angle = 50 μrad
S_B = bunch spacing = 10 m
$\gamma = 2 \times 10^4$ (20 TeV)
$\beta^* = 1$ m
$L = 70$ m.

We have assumed the LRBBI's take place at locations at $s \gg \beta^*$. We also ignore the deviation of beam trajectory from a straight line in the quadrupole triplet. For simplicity, we assume all IP's have the same β^*. The orbit distortion at an IP due to the LRBBI in its own IR is zero, but the other 5 IR's will contribute. Substituting the numerical values, we find d= 0.64 μm.

It has been suggested that the IR's are arranged as sketched below:[5]



where μ is the phase advance between adjacent IP's, μ_1 is the phase advance from F to A, μ_2 is the phase advance from D to E. We have the dondition that $\mu_1+\mu_2+4\mu=2\pi\nu$. The betatron phase advance between adjacent IP's is chosen to be an odd multiple of $\pi/2$, i.e. $\mu=(m+1/2)\pi$.[5] This arrangement has a beneficial effect on the LRBBI orbit distortion because the contributions from A and C cancel each other outside the region to first order, and so do the contributions from B and D. In addition, the $\pi/2$ phase advance between adjacent IP's also makes the sextupole effects of the LRBBI cancel between A,C and between B,D.

At the IP's, the orbit distortions are obtained by summing the contributions from all 6 LRBBI's taking into account of proper phases. This gives

$$\Delta x^* = \begin{array}{l} -d + [(-1)^m\cos(\pi\nu-\mu_1) + \sin(\pi\nu-\mu_1)]\, d/\sin(\pi\nu) \\ d - [(-1)^m\cos(\pi\nu-\mu_1) - \sin(\pi\nu-\mu_1)]\, d/\sin(\pi\nu) \\ d - [(-1)^m\cos(\pi\nu-\mu_1) + \sin(\pi\nu-\mu_1)]\, d/\sin(\pi\nu) \\ -d + [(-1)^m\cos(\pi\nu-\mu_1) - \sin(\pi\nu-\mu_1)]\, d/\sin(\pi\nu) \\ (-1)^m\, d\cot(\pi\nu) \\ (-1)^m\, d\cot(\pi\nu) \end{array} \qquad (2)$$

respectively for A, B, C, D, E and F. Note that the crossing angles have alternating signs from IP to IP. If the tune is $\nu = 86.28$, $\mu_1 = 34.0\times 2\pi$ and $m=7$,[6] we obtain

	A	B	C	D	E	F	
$\Delta x^* =$	-0.53	1.81	0.53	-1.81	-0.53	-0.53	μm

(3)

We will suggest later to arrange the crossings so that IP's are crossed horizontally and vertically in alternation. (*) If this is adopted, the orbit distortions at the 6 IP's become (starting with horizontal crossing at A)

	$\Delta x^* \sin(\pi\nu)/d$	$\Delta y^* \sin(\pi\nu)/d$
A	$-\sin(\pi\nu) + (-1)^m \cos(\pi\nu-\mu_1)$	$\sin(\pi\nu-\mu_1)$
B	$\sin(\pi\nu-\mu_1)$	$\sin(\pi\nu) - (-1)^m \cos(\pi\nu-\mu_1)$
C	$\sin(\pi\nu) - (-1)^m \cos(\pi\nu-\mu_1)$	$-\sin(\pi\nu-\mu_1)$
D	$-\sin(\pi\nu-\mu_1)$	$-\sin(\pi\nu) + (-1)^m \cos(\pi\nu-\mu_1)$
E	$(-1)^m \cos(\pi\nu)$	0
F	0	$(-1)^m \cos(\pi\nu)$

(4)

Assuming $\nu_x = \nu_y = 86.28$, $\mu_{1x} = \mu_{1y} = 34.0 \times 2\pi$ and m=7, we have

$$\Delta x^* = -1.17,\ 0.64,\ 1.17,\ -0.64,\ -0.53,\ 0.0$$
$$\Delta y^* = 0.64,\ 1.17,\ -0.64,\ -1.17,\ 0.0,\ -0.53 \qquad (5)$$

Values in Eqs.(3) and (5) are small compared with the beam size at IP, $\sigma = 7\mu m$ with $\beta^* = 1m$ and normalized emittance of 10^{-6}m-rad. On the other hand, fine tuning of orbit at the IP's can be provided if needed, especially if a smaller emittance or smaller β^* is introduced.

(*) Beams are separated vertically. The 50 μrad crossing, however, can be in either plane.[6]

The tune shifts per crossing caused by the LRBBI are

$$\Delta\nu_x = -\Delta\nu_y = \pm\ 2\,Nr_pL\ /\ (\gamma\pi\beta^*S_B\alpha^2) \qquad (6)$$

where + sign is for vertical crossing, - sign is for horizontal crossing. The tune shifts are about 0.0017 per crossing.

If crossings are either all horizontal or all vertical, the LRBBI tune shifts are 6x0.0017=0.01, which requires a current dependent compensation. Similar attention has to be made during injection even though the beams are separated. However, by alternating horizontal and vertical crossings, the total tune shifts is zero. We suggest that this does not cause significant complications on the lattice design and that it might simplify the operations. In case all IP's are not operated under the same conditions, the partial cancellation may still help.

Conclusions

1. Clustered IR's with $\pi/2$ phase advance between adjacent IR's help minimizing the orbit distortion and sextupole effects caused by LRBBI. The orbit distortions at the 6 IP's are given by eqs.(3) or (5) depending on the crossing pattern chosen.

2. A crossing pattern with alternating horizontal and vertical crossings help minimizing the LRBB tune shift effects.

References

1. SSC Reference Design Studies, May 1984.
2. P.Morton and J.Shonfeldt, Ann Arbor Workshop on SSC, 1983, p.54.
3. S.Peggs, 1984 SSC Summer Study, Snowmass, p.423.
4. A.Ruggiero, 1984 SSC Summer Study, Snowmass, p.426.
5. SSC-SR-1014, Clustered Interaction Region Option for the SSC, 1985.
6. A. Garren, private communications.