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AVAILABILITY STUDY FOR SSC CRYOGENIC SYSTEM---MODELS IV and VI,2

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MODEL IV 12-Unit Repairable System With 2 Repair Crews and 3-Unit Kill and Open Feedthrough of Excess Capacity

Assumptions

We assume 12 identical refrigerator units around a closed ring. Each unit operates with an independent probability-of-failure rate λ (all same) in its normal unstressed operating mode. When any individual unit is down its function load may be immediately taken over by the excess operating capacity of other units. Repair at rate μ is assumed to begin immediately on any unit when it goes down. We assume the whole system is shut down when any 3 units are down. This much is the same as in model III. The distinction from model III is that whereas it was restricted to distributing excess refrigerating capacity only to the adjacent half unit load, in the present model IV we assume that the excess capacity can be distributed freely around the ring. This treats the whole cryosystem analogously to a toroidal pipe with all sections open to each refrigerator. This removes the restriction that the system must be down when any pair of down units are not separated by more than two up units; without changing the other assumption that not more than 50 percent excess capacity will be demanded of any uinit. In order to maintain comparability with previous models, we assume that, although the excess load can be shared among all the up units in this model, we assign them collectively the same failure rate for operation in the stressed modes as would apply collectively in previous models. That is: The two units adjacent to a down unit were assumed to operate in the stressed mode with failure rate λ' in previous models. In the present model we assume the failure rate for operation of the n-unit system with j units down to be $2j\lambda' + (n-3j)\lambda$; as it would also be in the previous models. We still presume $\lambda' = \lambda$ or 2λ.

Up/Down Configuration States

States of the12-unit ring are included in the classes:

class 0 = a single state with all units up,
class 1 = 12 states with one unit down and 11 units up,
class 2 = 66 states with two units down and 10 units up,
class 3 = 220 states with three units down and the system shut down until at least one unit is restored to operation.

Probability Transition Rate Equations

$\dot{P}_i$ = the time rate of change of probability that the system be in class i at time t. The rate equations that characterize this model are:

$$\dot{P}_0 = -12\lambda P_0 + \mu P_1 \quad ,$$

$$\dot{P}_1 = -(2\lambda' + 9\lambda)P_1 - \mu P_1 + 2\mu P_2 + 12\lambda P_0 \quad ,$$

$$\dot{P}_2 = -(4\lambda' + 6\lambda)P_2 - 2\mu P_2 + 3\mu P_3 + (2\lambda' + 9\lambda)P_1 \quad ,$$

$$\dot{P}_3 = -3\mu P_3 + (4\lambda' + 6\lambda)P_2 \quad .$$

With the initial condition $P_0(t=0) = 1$ the solution gives the

Availability = probability system is up at time t

$$= A(t) = A_\infty + \sum_{i=1}^{4} A_i\, e^{-\lambda_i t} ,$$

in terms of eigenvalue rates λ_i and coefficients A_i .

The Stationary Availability is the long-time limit

$$A_\infty = \lim_{t \to \infty} A(t) \quad .$$

Maintenance Function and Mean Repair Time

The maintenance function M(t) is the probability that the system is up at time t after being down and beginning repair at t=0 . M(t) is obtained from the characteristic rate equations with $\lambda = \lambda' = 0$ and the initial condition $P_3(t=0) = 1$. The solution is

$$M(t) = 1 - P_3(t) = 1 - e^{-3\mu t}.$$

The Mean Repair Time is

$$MRT = \int_0^\infty [1 - M(t)]\, dt = 1/3\mu \ .$$

This compares with MRT $\cong 1/2\mu$ for model III, which does not have the feedthrough distribution of excess capacity, but does have the same 3-unit kill.

Using the above MRT in a general relation for it, the stationary availability A_∞ , and the Mean Time Between Failures, MTBF we obtain

$$MTBF = MRT/(1/A_\infty - 1) = A_\infty/[3\mu(1 - A_\infty)] \quad .$$

Results

Results are summarized in Table I for $\lambda'/\lambda = 1$ and 2 and various values of μ/λ . Again we can summarize by observing; (1) it is unimportant whether λ'/λ is 1 or 2 as long as $\mu/\lambda >> \lambda'/\lambda$, and (2) $\lambda_1 \sim \mu$, $\lambda_2 \sim 2\lambda_1$, for $\mu/\lambda > 10$, and (3) the secular availability is given roughly by

$$A(t) \cong [1 - (\lambda/\mu)^2] + 2(\lambda/\mu)^2 e^{-\mu t}.$$

MODEL IV

$n = 12$, $MRT = 1/3\mu$

μ/λ	λ'/λ	$MRT \times \lambda$	$MTBF \times \lambda$	A_∞	λ_1/λ	A_1	λ_2/λ	A_2	λ_3/λ	A_3
1	2	.3333	.08333	.2000	8.654	3.329	14.33	−3.493	22.02	.9646
	1	"	.1196	.2642	7.432	2.865	13.00	−3.275	18.57	1.145
10	2	$3.333\ 10^{-2}$	.2728	.8911	11.70	.2249	29.83	−.1462	57.46	.03006
	1	"	.4335	.9286	12.36	.1639	28.90	−.1209	51.74	.02837
20	2	$1.667\ 10^{-2}$	.6581	.9753	20.32	.05531	47.90	−.03855	90.79	.007960
	1	"	1.073	.9847	21.59	.03789	47.47	−.02947	83.94	.006917
30	2	$1.111\ 10^{-2}$	1.223	.9910	29.78	.02149	66.51	−.01591	122.7	.003407
	1	"	2.009	.9945	31.33	.01428	66.53	−.01160	115.1	.002814
100	2	$3.333\ 10^{-3}$	10.03	.9997	99.11	$8.954\ 10^{-4}$	202.3	$-7.734\ 10^{-4}$	337.6	$2.006\ 10^{-4}$
	1	"	17.07	.9998	101.0	$5.590\ 10^{-4}$	204.1	$-5.070\ 10^{-4}$	327.9	$1.433\ 10^{-4}$
200	2	$1.667\ 10^{-3}$	38.90	.99996	199.0	$1.236\ 10^{-4}$	400.5	$-1.133\ 10^{-4}$	639.4	$3.247\ 10^{-5}$
	1	"	64.35	.99997	201.0	$7.593\ 10^{-5}$	403.2	$-7.177\ 10^{-5}$	628.8	$2.175\ 10^{-5}$

AVAILABILITY STUDY FOR SSC CRYOGENIC SYSTEM---MODEL IV,2

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AVAILABILITY STUDY FOR SSC CRYOGENIC SYSTEM---MODEL IV,2

MODEL IV 12-Unit Repairable System With 2 Repair Crews and 2-Unit Kill and Open Feedthrough of Excess Capacity

Assumptions

We assume 12 identical refrigerator units around a closed ring. Each unit operates with an independent probability-of-failure rate λ (all same) in its normal unstressed operating mode. When any individual unit is down its function load may be immediately taken over by the excess operating capacity of other units. Repair at rate μ is assumed to begin immediately on any unit when it goes down. We assume the whole system is shut down when any 2 units are down. This much is the same as in model I. The distinction from model I is that whereas it was restricted to distributing excess refrigerating capacity only to the adjacent half unit load, in the present model IV,2 we assume that the excess capacity can be distributed freely around the ring. This treats the whole cryosystem analogously to a toroidal pipe with all sections open to each refrigerator. This removes the restriction that the system must be down when any pair of down units are not separated by more than two up units; without changing the other assumption that not more than 50 percent excess capacity will be demanded of any unit. In order to maintain comparability with previous models, we assume that, although the excess load can be shared among all the up units in this model, we assign them collectively the same failure rate for operation in the stressed modes as would apply collectively in previous models. That is: The two units adjacent to a down unit were assumed to operate in the stressed mode with failure rate λ' in previous models. In the present model we assume the failure rate for operation of the n-unit system with j units down to be $2j\lambda' + (n-3j)\lambda$; as it would also be in the previous models. We still presume $\lambda' \cong \lambda$ or 2λ .

Up/Down Configuration States

States of the12-unit ring are included in the classes:

class 0 = a single state with all units up,
class 1 = 12 states with one unit down and 11 units up,
class 2 = 66 states with two units down and and the system shut down
until at least one unit is restored to operation.

Probability Transition Rate Equations

$\dot{P}_i$ = the time rate of change of probability that the system be in class i at time t. The rate equations that characterize this model are:

$$\dot{P}_0 = -12\lambda P_0 + \mu P_1 \quad ,$$

$$\dot{P}_1 = -(2\lambda' + 9\lambda)P_1 - \mu P_1 + 2\mu P_2 + 12\lambda P_0 \quad ,$$

$$\dot{P}_2 = -2\mu P_2 + (2\lambda' + 9\lambda)P_1 \quad .$$

With the initial condition $P_0(t=0) = 1$ the solution gives the

Availability = probability system is up at time t

$$= A(t) = A_\infty + \sum_{i=1}^{2} A_i e^{-\lambda_i t} ,$$

in terms of eigenvalue rates λ_i and coefficients A_i .

The Stationary Availability is the long-time limit

$$A_\infty = \lim_{t\to\infty} A(t) \quad .$$

Maintenance Function and Mean Repair Time

The maintenance function M(t) is the probability that the system is up at time t after being down and beginning repair at t=0 . M(t) is obtained from the characteristic rate equations with $\lambda' = \lambda = 0$ and the initial condition $P_2\ (t=0) = 1$. The solution is

$$M(t) = 1 - P_2\ (t) = 1 - e^{-2\mu t}.$$

The Mean Repair Time is

$$MRT = \int_0^\infty [1 - M(t)]\ dt = 1/2\mu \ .$$

This compares with MRT = $1/\mu$ for model I, which does not have the feedthrough distribution of excess capacity, but does have the same 2-unit kill.

Using the above MRT in a general relation for it, the stationary availability A_∞ , and the Mean Time Between Failures, MTBF we obtain

$$MTBF = MRT/(1/A_\infty - 1) = A_\infty/[2\mu(1 - A_\infty)] \quad .$$

Results

Results are summarized in Table I for $\lambda'/\lambda = 1$ and 2 and various values of μ/λ . Again we can summarize by observing; (1) it is unimportant whether λ'/λ is 1 or 2 as long as $\mu/\lambda >> \lambda'/\lambda$, and (2) $\lambda_1 \simeq \mu$, $\lambda_2 \simeq 2\lambda_1$, for $\mu/\lambda \geqslant 10$.

MODEL IV, 2 $n = 12$, FEEDTHROUGH AND 2-UNIT KILL

$MRT = 1/2\mu$

μ/λ	λ'/λ	$MRT \times \lambda$	$MTBF \times \lambda$	A_∞	λ_1/λ	A_1	λ_2/λ	A_2
1	2	.5	.08324	.1428	10.26	2.032	17.74	-1.175
	1	"	.09852	.1646	9.683	2.055	16.31	-1.220
10	2	.05	.1369	.7325	14.84	.4152	40.16	-.1534
	1	"	.1666	.7692	15.08	.3833	37.91	-.1525
20	2	.025	.2052	.8914	23.26	.1743	61.74	-.06566
	1	"	.2424	.9065	23.89	.1569	59.11	-.06339
30	2	.01667	.2693	.9417	32.39	.09591	82.60	-.03761
	1	"	.3181	.9502	33.26	.08538	79.74	-.03561
100	2	.005	.7196	.9931	100.4	.01251	224.5	-.005598
	1	"	.8425	.9941	101.9	.01087	221.1	-.005010
200	2	.0025	1.386	.9982	199.8	.003464	425.2	-.001628
	1	"	1.560	.9984	201.5	.002978	421.5	-.001423