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AVAILABILITY STUDY FOR SSC CRYOGENIC SYSTEM---MODEL III

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MODEL--III 12-Unit Repairable System with Three Repair Crews and 3-Unit Kill

Assumptions

We assume 12 identical refrigerator units around a closed ring. Each unit operates with an independent probability-of-failure rate
λ (all same) in its normal, unstressed operating mode. When any individual unit is down its function load may be taken over by the excess operating capacity of the two units adjacent to the down unit. We assume that each unit has operating capacity at least 50 percent in excess of its load in its normal, unstressed state, and that when a single adjacent unit goes down this excess capacity can be switched immediately to take over one half the load of an adjacent down unit. We assume that only a single adjacent 1/2 unit load can be handled by this excess capacity. We assume a probability-of-failure rate for a unit operating in the stressed mode where it is picking up the adjacent 1/2 load of a down unit to be λ'. We presume $\lambda' \cong \lambda$ or 2λ. We assume there are three repair crews that begin repair on any down unit immediately when it fails and works with repair rate μ until that unit is again up.

If any three units are down we stipulate that the whole system be turned off in spite of the existence of an appreciable fraction of states with three units down in which the whole system could operate under our other assumptions. This describes what we mean by "three- unit kill".

Up/Down Configuration States

States of the 12-unit ring include the classes:
- class 0 = a single state with all 12 units up,
- class 1 = 12 states with one unit down and 11 units up,
- class 2 = 42 states with 2 units down and 10 units up, and the whole system up,
- class 2'= 24 states with 2 units down and 10 units up, but the whole system necessarily down,
- class 3 = 40 states 3 units down and whole system down by stipulation, although it could otherwise be up, and
- class 3'= 168 states with 3 units down in which the whole system is necessarily down.

We also distiquish the subclasses:
- subclass $\overline{3'}$ = 36 states of 3' which can repair back into the non-operational class 2', and
- subclass $\overline{2}$ = the fraction 2/7 of class 2 which decays into class 3.

Transition Probability Rate Equations

$\dot{P}_i$ = the time rate of change of the probability that the whole system is in class i at time t. The rate equations are:

$$\dot{P}_0 = -12\lambda P_0 + \mu P_1 \quad ,$$

$$\dot{P}_1 = -(2\lambda' + 9\lambda)P_1 + 2\mu(P_2 + P_{2'}) - \mu P_1 + 12\lambda P_0 \quad ,$$

$$\dot{P}_{2'} = \mu(P_{3'} + P_{\overline{3'}}) + (2\lambda' + 2\lambda)P_1 - 2\mu P_{2'} ,$$

$$\dot{P}_2 = -(4\lambda' + 6\lambda)P_2 + 7\lambda P_1 + \mu(2P_{3'} - P_{\overline{3'}}) + 3\mu P_3 - 2\mu P_2 ,$$

$$\dot{P}_3 = -3\mu P_3 + \lambda P_{\overline{2}} ,$$

$$\dot{P}_{3'} = -3\mu P + (4\lambda' + 6\lambda)P_2 - \lambda P_{\overline{2}} .$$

These rate equations plus the assumption that we can replace

$$\lambda P_{\overline{2}} \equiv (2/7)\lambda P_2 , \text{ and}$$

$$\mu P_{\overline{3'}} \equiv (3/14)\mu P_{3'} ,$$

completely specify MODEL III.

Solution of these equations for the initial condition P (t=0) =1 give the

Availibility = probability system is up at time t

$$= A(t) = A_\infty + \sum_{i=1}^{4} A_i e^{-\lambda_i t} ,$$

in terms of the eigenvalues λ_i and coefficients A_i . The Stationary Availability is the long-time limit

$$A_\infty = \lim_{t\to\infty} A(t) .$$

Maintenance Function M(t)

The maintenance function M(t) is the probability that the system is up at time t starting from the initial condition that it was down at t = 0, and evolves by the above rate equations with $\lambda = \lambda' = \overline{\lambda} = 0$. Solution of this set of equations is

$$P_{b3} = P_{b3}(0)e^{-3\mu t},$$

$$P_{b3'} = P_{b3'}(0)e^{-3\mu t},$$

$$P_{b2'} = (17/14)P_{b3'}(0)[e^{-\mu t} - e^{-3\mu t}] ,$$

$$P_{b2} = [3P_{b3}(0) + (25/14)P_{b3'}(0)][e^{-2\mu t} - e^{-3\mu t}] ,$$

$$P_{b1} = 3[P_{b3}(0) + P_{b3'}(0)][e^{-\mu t} - 2e^{-2\mu t} + e^{-3\mu t}] ,$$

$$P_{b0} = 3[P_{b3}(0) + P_{b3'}(0)][\tfrac{1}{3} - e^{-\mu t} + e^{-2\mu t} - \tfrac{1}{3}e^{-3\mu t}] .$$

The Maintenance Function is

$$M(t) = P_{b0} + P_{b1} + P_{b2}$$

$$= 1 - [3(1 - P_{b3}(0)) - \tfrac{25}{14}P_{b3'}(0)]\, e^{-2\mu t}$$

$$+ [2 - 3\,P_{b3}(0) - \tfrac{25}{14}P_{b3'}(0)]\, e^{-3\mu t} .$$

Mean Repair Time is

$$\begin{aligned} MRT &= \int_0^\infty [1 - M(t)]\, dt \\ &= [\tfrac{5}{6} - \tfrac{1}{2} P_{b3}(0) - \tfrac{25}{14} P_{b3'}(0)]/\mu \quad . \end{aligned}$$

Since $P_{b3}(0) + P_{b3'}(0) = 1$ does not specify $P_{b3}(0)$ and $P_{b3'}(0)$, we choose an initial distribution weight equally among states of each class, i.e. $P_{b3}(0)/P_{b3'}(0) = 40/168$; and then

$$MRT = .4968/\mu \quad .$$

This compares with $MRT = .5533/\mu$ for MODEL II, which differs from the present model only by having $n = 9$.

Using the above value of MRT in the relation for the stationary availability, (which is supposed to be general), gives the
Mean Time Between Failures

$$MTBF = \frac{MRT}{1 - 1/A_\infty} = \frac{.4968 \quad A_\infty}{\mu(1 - A_\infty)} \quad .$$

Results

Results are summarized in Table I for $\lambda' = 2\lambda$ and various values of μ/λ . We saw in model I that $\lambda'/\lambda = 1$ and 2 are similar for $\mu/\lambda \geqslant 20$. For large $\mu/\lambda \geqslant 50$ there is oscillatory time depend-ence, as indicated by the complex eigenvalues $\lambda_3 = \lambda_2^*$. However, the oscillatory behavior damps at twice the rate of the slowest secular decay eigenmode, i.e. $Re\lambda_2/\lambda_1 \cong 2$; so it is negligible for most t. The table shows that μ/λ for the individual units determines the availability for the whole system, and secular relaxation goes roughly as we found in models I and II, viz:

$$A(t) \cong 1 - .2(n\lambda/\mu)^2 + .3(n\lambda/\mu)^2 e^{-\mu t} \quad ,$$

for $\mu/\lambda \geqslant 20$.

$n = 12$, $\lambda' = 2\lambda$, $MRT = .4968/\mu$

μ/λ	$MRT \times \lambda$	$MTBF \times \lambda$	A_∞	λ_4/λ	A_4	λ_3/λ	A_3	λ_2/λ	A_2	λ_1/λ	A_1
1	.4968	.1009	.1688	20.63	.6534	14.75	−2.815	9.340	3.003	2.278	−.0108
10	.04968	.2077	.8070	52.65	.008944	31.20	−.1561	21.64	−.02419	13.52	.3644
20	.02484	.3383	.9316	83.93	.001193	49.62	−.05781	43.55	.001381	21.90	.1237
30	.01656	.4719	.9661	114.48	.0002532	67.20	−.08574	66.17	.05751	31.15	.06186
100	.004968	1.414	.9965	325.2		207.00 + 4.860i		207.00 − 4.960i		99.77	
200	.002484	2.788	.99911	625.3		407.1 + 5.685i		407.1 − 5.685i		199.4	