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AVAILABLE STUDY FOR SSC CRYOGENIC SYSTEM--MODELS

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We study the availability of systems with characteristics representative of large-scale cryogenic systems for the SSC. By large-scale we mean an SSC system composed of units roughly equivalent, or at least comparable to the whole Tevatron cryosystem.

In order to exhibit the essense of the computations and results in analytic form, we consider a system of 9 units for a first-pass study. The aspects of a larger 12-unit system that are distinctly different in the 9-unit system will be explained after we first see them in the smaller system. We will study 2 models. The first model is a repairable system with one repair crew and the system down when any 2 units are down. The second model has 3 repair crews and the system is down when any 3 units are down (3-unit kill).

Model I 9-Unit Repairable System with One Repair Crew and 2-Unit Kill

Assumptions

We assume 9 identical refrigerator units around a closed ring. Each unit operates with an independent probability-of-failure rate λ (all same) in its normal, unstressed operating model. When any individual unit is down its function load may be taken over by the excess operating capacity of the 2 units adjacent to the down unit. We assume that excess operating capacity of an operating unit can be switched on immediately and distributed to take up only 1/2 the load of a single adjacent down unit. That is, we assume each unit has operating capacity at least 50 percent in excess of its load in its normal, unstressed state, and ignore any possibility of distributing excess capacity beyond an adjacent 1/2 unit. We assume a probability-of-failure rate for a unit operating in the stressed state where it is picking up 1/2 the load of an adjacent down unit to be λ'. Presumably $\lambda' \geq \lambda$. We assume that there is a single repair crew that starts repair immediately when the first unit goes down and works with a repair rate μ until that unit is again operative and carrying its own load.

If a second unit goes down before restoration of a previously down unit the whole system is shut down until the single repair crew finishes repairing the first down unit and starts repairing the other down unit while the system is again up and operating. Note that 1/2 of the 36 states of the 9-unit system with 7 up units and 2 down units could leave the system operational even if only one were being repaired. That is a different model that we can also study.

Up-down Configurations or States

States of our system corresponding to particular configurations of distributions of up and down units are illustrated in Fig. 1. There is a single state with all 9 units up. This state is by itself class-0. There are 9 states of the class-1 in which the system is operating with one unit down and being repaired. There are 9 states with 2 adjacent units down plus 9 states with 2 next-adjacent units down that form class-2'; in which, by our assumptions the system is necessarily down. There are also 18 states with 2 units down that could leave the system operating in class-2; except that in this model we agree to shut down in this case.

Probability Equations

The probability that the system is in a state of class-j, we call P_j. The probability transition rate equations are then the coupled set,

$$\dot{P}_0 = -9\lambda\, P_0 + \mu\, P_1 \qquad \dot{P}_0 = \frac{d}{dt} P_0$$

$$\dot{P}_1 = -(6\lambda + 2\lambda')\, P_1 + 9\lambda\, P_0 + \mu\, P_2 + \mu\, P_{2'} - \mu\, P_1$$

$$\dot{P}_2 = -\mu\, P_2 + 4\lambda\, P_1$$

$$\dot{P}_{2'} = -\mu\, P_{2'} + (2\,\lambda' + 2\lambda)\, P_1 \;.$$

Because of our stipulation that the system be shut down when any 2 units are down, this is actually only 3 equations in 3 unknowns. The last two equations can be added to obtain an equation for the single variable $P_2 + P_{2'}$; which is all that occurs in the remaining equations.

Solutions

Subject to the boundary conditions: $P_0(t = 0) = 1$ and $P_0(t) + P_1(t) + P_2(t) + P_{2'}(t) = 1$, the solution is

$$P_0(t) = \frac{\mu^2}{\lambda_1\lambda_2} + \frac{1}{(\lambda_2 - \lambda_1)}\left\{\lambda_2\left[1 - \frac{\mu^2}{\lambda_1\lambda_2} - \frac{9\lambda}{\lambda_2}\right] e^{-\lambda_1 t} - \lambda_1\left[1 - \frac{\mu^2}{\lambda_1\lambda_2} - \frac{9\lambda}{\lambda_1}\right] e^{-\lambda_2 t}\right\}$$

$$P_1(t) = \frac{9\lambda\mu}{\lambda_1\lambda_2} - \frac{9\lambda}{(\lambda_2 - \lambda_1)}\left\{(\frac{\mu}{\lambda_1} - 1)\, e^{-\lambda_1 t} - (\frac{\lambda}{\lambda_2} - 1)\, e^{-\lambda_2 t}\right\}$$

$$P_2(t) + P_{2'}(t) = \frac{9\lambda(6\lambda + 2\lambda')}{(\lambda_2 - \lambda_1)}\left\{\frac{1}{\lambda_1} - \frac{1}{\lambda_2} - \frac{e^{-\lambda_1 t}}{\lambda_2} + \frac{e^{-\lambda_2 t}}{\lambda_2}\right\} \;,$$

where the eigenvalues are

$$\lambda_{1,2} = \frac{15}{2}\,\lambda + \lambda' + \mu \;\mp\; \left[(\frac{3}{2}\,\lambda - \lambda')^2 + \mu(6\lambda + 2\,\lambda')\right]^{1/2} .$$

Availability

Availability is the probability that the system is up at time t after virgin start in class 0:

$$A(t) = P_0(t) + P_1(t) = 1 - P_2(t) - P_{2'}(t)$$

$$= \frac{1}{1 + \frac{9(6 + 2\lambda'/\lambda)}{\frac{\mu}{\lambda}(\frac{\mu}{\lambda} + 9)}} - \frac{9(6 + 2\lambda'/\lambda)}{(\lambda_2 - \lambda_1)/\lambda}\left[-\frac{\lambda}{\lambda_1} e^{-\lambda_1 t} + \frac{\lambda}{\lambda_2} e^{-\lambda_2 t}\right]$$

$$\equiv A_\infty + A_1 e^{-\lambda_1 t} + A_2 e^{-\lambda_2 t} \quad .$$

Stationary Availability is the long-time limit:

$$A_\infty \equiv \lim_{t \to \infty} A(t) = \frac{1}{1 + \frac{9(6 + 2\lambda'/\lambda)}{\frac{\mu}{\lambda}(\frac{\mu}{\lambda} + 9)}} \quad .$$

Availability for all $\mu/n\lambda$ remains large for times $t \leq 5/\lambda$ and for times $t >> 10/\lambda$ approaches the stationary availability

$$A_\infty = \frac{1}{1 + \frac{n\lambda[(n - 3)\lambda + 2\lambda']}{\mu(\mu + n\lambda)}} = \frac{1}{1 + \frac{n\lambda[1 - (3 - 2\lambda'/\lambda)/n]}{\mu(\frac{\mu}{n\lambda} + 1)}} \quad .$$

For $\mu/n\lambda >> 1$ this goes like

$$A_\infty \approx \frac{1}{1 + (\frac{n\lambda}{\mu})^2} \approx 1 - (\frac{n\lambda}{\mu})^2 \quad .$$

Maintenance Function and Mean Time to Repair

The maintenance function is the probability of the system coming up at t under the condition that it was initially down. This requires solving a different set of equations:

$$\dot{P}_{b0} = \mu P_{b1}$$

$$\dot{P}_{b1} = -\mu P_{b1} + \mu(P_{b2} + P_{b2'})$$

$$\dot{P}_{b2} = -\mu P_{b2}$$

$$\dot{P}_{b2'} = -\mu P_{b2'} \quad .$$

The condition that the system is initially down means: $P_{b0}(t = 0) = P_{b1}(t = 0)$ and $P_{b2}(t = 0) \neq 0 \neq P_{b2'}(t = 0)$. The solution is:

$$P_{b2}(t) = P_{b2}(0)\ e^{-\mu t}$$

$$P_{bs'}(t) = P_{b2'}(0)\ e^{-\mu t}$$

$$P_{b1}(t) = \mu[P_{b2}(0) + P_{b2'}(0)]\ te^{-\mu t}$$

$$P_{b0}(t) = [P_{b2}(0) + P_{b2'}(0)][1 - (1 + \mu t)\ e^{-\mu t}],$$

$$1 = P_{b2}(0) + P_{b2'}(0).$$

The maintenance function M(t) is then

$$\begin{aligned} M(t) &= P_{b0}(t) + P_{b1}(t) \\ &= 1 - (1 + \mu t)\ e^{-\mu t} + \mu t\ e^{-\mu t} \\ &= 1 - e^{-\mu t} \quad . \end{aligned}$$

This is the same maintenance function as for a single unit; reflecting the fact that there is no more than one unit under repair by the single maintenance crew. Note: these equations and M(t) are independent of λ and λ'.

Mean Repair Time is

$$\mathrm{MRT} = \int_0^\infty [1 - \mathrm{MIt})]\ dt = \frac{1}{\mu} \quad ,$$

which is the same as for a single unit.

Mean Time Before Failure is

$$\mathrm{MTBF} = \int_0^\infty t[\dot{P}_2 + \dot{P}_{2'}]_{\mu=0}\ dt$$

$$= \int_0^\infty t\ \frac{9\lambda(6\lambda + 2\lambda')}{(\lambda_1\lambda_2)^2}\ [e^{-\lambda_1 t} - e^{-\lambda_2 t}]_{\mu=0}\ dt$$

$$= \left.\frac{9\lambda(6\lambda + 2\lambda')(\lambda_1 + \lambda_2)}{(\lambda_1\lambda_2)^2}\right|_{\mu=0} = \frac{1}{9\lambda} + \frac{1}{6\lambda + 2\lambda'} \quad .$$

Note: we call this "mean time before failure," which is the mean time to the first failure starting from the totally-up initial state 0. A mean time between failures cannot be computed this way for this system because the distribution function for the first up times is not the same as the distribution of second up times, etc. An alternative for Mean Time Between Failure in Stationary Operation for very long operating times may be obtained from the (supposedly general, but unproved) equation for the stationary availability and mean repair time calculated earlier:

$$A_\infty = \frac{1}{1 + \frac{9\lambda(6\lambda + 2\lambda')}{\mu(\mu + 9\lambda)}} = \frac{1}{1 + \frac{\mathrm{MTF}}{\mathrm{MRT}}} \quad .$$

This relation generalized for $9 \geq n$ gives

$$\mathrm{MTF} = \left.\frac{\mu + n\lambda}{n\lambda[n - 3)\ \lambda + 2\ \lambda']}\right|_{n=9} .$$

This expression has the desirable feature that it increases MTF with increasing repair rate μ. However, it does not go to MTBF as $\mu \geq 0$. For large $\mu/\lambda >> n$ we have MTF ~ $\mu/(n\lambda)^2$.

Parameter Values

We expect $\lambda' = 2\lambda$ or $\lambda' = 2$ will serve as typical, i.e., that the independent failure rate of a unit operating in its stressed mode might be twice as great as for operation in the unstressed mode. This might be a reasonable range, since at least 50 percent excess unit capacity is required for this model.

The failure-rate ratio λ'/λ is speculative at this stage, whereas the repair-rate ratio μ/λ should be indicated in the Tevatron experience. However, at the present, speculation on typical values for μ/λ covers quite a broad range of uncertainty, depending on what aspects of the data are critical. We compute for $\lambda'/\lambda = 2$ and 1, and μ/λ = 200, 100, 30, 20, 10, 1, 0.1, 0.

Tevatron I Experience recently indicates a mean down-time interval for repair of the cryosystem is ~0.5 to 1 hour, with mean time between failures ~4 occurrences per month. Typical repair jobs are stuck values or broken valve stems, etc. This suggests

$$\frac{\lambda}{\mu} = \frac{T^{\text{repair}}}{T^{\text{operate}}} \simeq \frac{0.75}{\frac{30}{4}\ (24)} = \frac{1}{240} .$$

The major maintenance item is malfunction of electrical transducers, but these cause no down time--although they might at SSC, because they may be in the SSC tunnel.

Central Helium Liquefier experience has indicated 17 to 24 weeks between failures, and ~100 hours for typical repair time. This suggests

$$\frac{\lambda}{\mu} = \frac{T_{CHL}^{repair}}{T_{CHL}^{operate}} \simeq \frac{100}{17(7)24} \simeq \frac{1}{30} \quad .$$

These data suggest we consider values of μ/λ = 20, 30, 100, 200 for n = 9, 12 and λ'/λ = 2, 1. The mean times between failures and associated stationary availabilities for these parameters are shown in the following table for both n = 9 and n = 12.

Mean Time Between Failure and Stationary Availability
For One Repair Crew and Two-Unit Kill

$(\frac{\mu}{\lambda},\frac{\lambda'}{\lambda})$	(20,2)	(30,2)	(100,2)	(200,2)	(20,1)	(30,1)	(100,1)	(200,1)
For n = 9								
	HTBF =	0.235			MTBF = 0.254			
MTF	0.322	0.433	1.211	2.322	0.403	0.542	1.514	2.908
A	0.866	0.929	0.992	0.998	0.890	0.942	0.993	0.998
For n = 12								
	MTBF =	0.160			MTBF = 0.174			
MTF	0.205	0.269	0.718	1.359	0.242	0.318	0.848	1.606
A	0.804	0.890	0.986	0.996	0.829	0.905	0.988	0.997

For $\mu/\lambda \geq 20$ the eigenvalue rate $\lambda_1 \geq 15\lambda$ and $(\lambda_2 - \lambda_1)/\lambda \geq \sqrt{(n-1)\ \mu/\lambda} \geq \sqrt{160}$ $= 12.6$. So the system is at its stationary operating state, i.e., has dissipated more than 95 percent of the transient $|A - A_\infty|$ in its approach to A_∞, by $t \approx \frac{3}{15}\frac{1}{\lambda} = 0.2\,\frac{1}{\lambda}$; which is of the order of MTBF.

Eigenvalues and Availability Coefficients are given in the Table:

λ'/λ	μ/λ	λ/λ_1	λ/λ_2	A	A_1	A_2
2	200	164.8	254.22	0.9978	0.00611	-0.00396
	100	77.87	141.13	0.9918	0.01827	-0.01008
	20	15.35	43.65	0.8657	0.2072	-0.07285
	10	9.487	29.51	0.6786	0.4737	-0.1523
	1	7.298	13.70	0.1000	1.926	-1.026
	0.1	8.482	10.72	0.01001	4.745	-3.755
	0	9.	10.	0	10.	-9.
1	200	168.5	248.5	0.9983	0.005341	-0.003621
	100	80.21	136.79	0.9934	0.01587	-0.009303
	20	15.84	41.15	0.8896	0.1795	-0.6909
	10	9.542	27.45	0.7252	0.4212	-0.1463
	1	6.628	12.37	0.1219	1.891	-1.013
	0.1	7.575	9.624	0.01248	4.638	-3.650
	0	8.	9.	0	9.	-8.

Model II 9-Unit Repairable System with Three Repair Crews System Off when Three Units Down

Assumptions

We assume identical refrigerator units around a closed ring, each operating independently with failure rate λ in its normal, unstressed state and λ' in its stressed mode when taking over 1/2 the load function of an adjacent unit that is down. We assume that each unit has excess capacity to take over 1/2 load of a single adjacent down unit only, and that switching into this mode requires no time. Down units are assumed to begin repair immediately with repair rate μ. We assume there are 3 repair crews, and the system is turned off when any 3 units are down, inspite of the fact that not all states with 3 down units are necessarily down states of the system. The set of such states is small for $n = 9$, but sizeable for $n = 12$. This is the principal distinction between $n = 9$ and $n = 12$.

Classes of States

States of the 9 unit ring that we must consider include again the single state of class 0 with all units up; 9 states of class 1 operating with one unit down; and 18 states of class 2' in which the system must be down with 2 units down. In addition, we consider the 3 states of class 3, in which the system could operate with 3 units down, but does not, due to stipulation of our model. We must also consider the 72 states of class 3', which can be reached by failure from operating states of class 2 and which do not allow the system to operate because at least one pair of the down units do not have at least 2 up units between them to take up the interior 1/2-loads of the pair of down units. Actually, this does not completely specify the classes of states that must be considered for exact solution of this model. This is because some states of class 3' can repair back into the non-operational class 2' rather than the operating class 2; we will call this class 3'. Also the subclass of states of class 2 which decay into states of class 3 must be distinguished as a concommitant of the need to distinguish among various 3-down-unit states. However, as part of our computational model we shall finesse these details, as described more conveniently below along with the probability transition equations.

Transition Probability Equations

$$\dot{P}_0 = -9\lambda P_0 + \mu P_1$$

$$\dot{P}_1 = -(6\lambda + 2\lambda' + \mu) P_1 + 9\lambda P_0 + 2\mu(P_2 + P_{2'})$$

$$\dot{P}_2 = -(3\lambda + 4\lambda' + 2\mu) P_2 + 4\lambda P_1 + 3\mu P_3 + 2\mu P_{3'} - \lambda P_{\overline{3'}}$$

$$\dot{P}_{2'} = -2\mu P_2 + (2\lambda + 2\lambda') P_1 + \mu P_{3'} + \mu P_{\overline{3'}}$$

$$\dot{P}_3 = -3\mu P_3 + \lambda P_{\overline{2}}$$

$$\dot{P}_{3'} = -3\mu P_{3'} + (3\lambda + 4\lambda') P_2 - \lambda P_{\overline{2}}$$

For n = 9 there are only 3 states in class 3; and only 9 states of class 2 having the jth and (j + 3)th units down can decay into class 3, each at a rate λ. We finesse this by defining the effective failure rate $\overline{\lambda}$ in the assumption that $\lambda P_{\overline{2}}(t) \equiv \overline{\lambda}\, P_2(t)$ for all t. $P_{\overline{2}}$ is the sum of probabilities of states with jth and (j + 3)th units down:

$$P_{\overline{2}} = \sum_{j=1}^{9} P(j,j+3) = 9P(1,4) \quad ,$$

by symmetry. $P_2 - P_{\overline{2}}$ is the sum of probabilities of states with jth and (j + 4)th units down: $P_2 - P_{\overline{2}} = 9P(1,5)$. Thus we expect that the time-average value of $\overline{\lambda}$ is $\overline{\lambda} = 1/2\ \lambda$.

We finesse the $P_{\overline{3'}}$ the same way. $P_{\overline{3'}}$ is the sum of probabilities of 9 states with j, j + 1 and j + 3th units down plus 9 states with j, j + 2 and j + 3th units down plus 9 states with j, j + 2 and j + 4th units down. Again set $\mu P_{\overline{3'}} \equiv \overline{\mu} P_{3'}$, with $\overline{\mu} = (27/72)\ \mu = (3/8)\ \mu$ on average.

With these finesses the above equation set is complete for determining P_0, P_1, P_2, $P_{2'}$, P_3, $P_{3'}$ as functions of time. The equations leave $P_0 + P_1 + P_2 + P_{2'} + P_3 + P_{3'}$ = constant and we set $P_0 = 1$ and $P_1 = P_{2'} = P_3 = P_{3'} = 0$ at $t = 0$. We are not interested in the individual P_i's so much as the probability of the whole system being up: $A \equiv P_0 + P_1 + P_2 = 1 - (P_{2'} + P_3 + P_{3'})$.

It is convenient to describe our technique for solving the probability transition equations in the Laplace transform domain where $P_i(t)$ is mapped into $\rho_i(q)$ and $\dot{P}_i(t)$ into $[q\rho_i(q) - P_2(0)]\ \lambda$. Denominating all rates in units of λ so that λ' $\alpha\lambda$, λ $\alpha\lambda$, μ $\beta\lambda$, μ $\beta\lambda$ the Laplace transform equations are

$$(q + 9)\ \rho_0 = \beta\rho_1 + 1$$

$$(q + 6 + 2\alpha + \beta)\ \rho_1 = 9\rho_0 + 2\beta(\rho_2 + \rho_{2'})$$

$$(q + 3 + 4\alpha + 2\beta)\ \rho_2 = 4\ \rho_1 + 3\beta\rho_3 + (2\beta - \overline{\beta})\ \rho_{3'}$$

$$(q + 2\beta)\ \rho_{2'} = (2 + 2\alpha)\ \rho_1 + (\beta + \beta)\ \rho_{3'}$$

$$(q + 3\beta)\ \rho_3 = \bar{\alpha}\rho_2$$

$$(q + 3\beta)\ \rho_{3'} = (3 + 4\alpha - \bar{\alpha})\ \rho_2\ .$$

The solution for the combined probabilities that the system will be in an up state, called the availability,

$$A = P_0 + P_1 + P_2 = 1 - (P_{2'} + P_3 + P_{3'})\quad ,$$

is of the form

$$A(t) = A_\infty + \sum_{i=1}^{4} A_i e^{q_i \lambda t}\quad ,$$

where q_i are the eigenvalues of the matrix of coefficients of q for the Laplace transform rate equations. For $\alpha \equiv \lambda'/\lambda = 2$, $\bar{\alpha} = \frac{1}{2}\lambda$, $\beta \equiv \mu/\lambda = 20$, $\bar{\beta} = \frac{3}{8}\beta$,

$$q_1 = -21.13427$$
$$q_2 = -41.0752$$
$$q_3 = -50.0126\ .$$
$$q_4 = -77.7780\ .$$

Note: the 2 lowest eigenvalues differ by $q_2 - q_1 = 41.1 - 21.1 = 20.0$, so for times greater than about

$$t \approx \frac{-3 + \ln \frac{0.088}{0.00024}}{20\ \lambda} \approx \frac{2.7}{20}\frac{1}{\lambda} = 0.2\ \frac{1}{\lambda}$$

all the transients of $|q_i| > |q_1|$ are negligible compared to the q_1 transient. Furthermore at $t = 0.2/\lambda$

$$e^{-21(.2)} = e^{-4} \approx 0.02$$

is all that is left of the $A_1 e^{q_1 \lambda t}$ transient as $A(t)$ approaches the stationary availability, A_∞. Our previous model indicates that as μ/λ gets larger the dominance of the lowest eigenvalue becomes even more pronounced. This is also the case here.

Maintenance Function M(t)

The maintenance function for this system is

$$M(t) = P_{b0} + P_{b1} + P_{b2} \quad ,$$

where P_{bi} are solutions of the transition equations with $\lambda = \lambda' = \bar{\lambda} = 0$ and $(P_{b2} + P_{b3} + P_{b3'}) = 1$ at $t = 0$. That is,

$$\dot{P}_{b0} = \mu P_{b1}$$

$$\dot{P}_{b1} = -\mu P_{b1} + 2\mu(P_2 + P_{2'})$$

$$\dot{P}_{b2} = -2\mu P_2 + 3\mu P_3 + 2\mu P_{3'} - \mu P_{\overline{3'}} \qquad (\mu P_{\overline{3'}} = \bar{\mu} P_{b3'})$$

$$\dot{P}_{b2'} = -2\mu P_{2'} \qquad + \mu P_{3'} + \mu P_{\overline{3'}}$$

$$\dot{P}_{b3} = -3\mu P_{3'}$$

$$\dot{P}_{b3'} = -3\mu P_{3'} \quad .$$

Solution of these equations is

$$P_{b3} = P_{b3}(0)\, e^{-3\mu t} \quad , \qquad P_{b3'} = P_{3b'}(0)\, e^{-3\mu t}$$

$$P_{b2'} = (1 + \bar{\mu}/\mu)\, P_{b3'}(0)\, [e^{-\mu t} - e^{-3\mu t}]$$

$$P_{b2} = [3P_{b3}(0) + (2 - \bar{\mu}/\mu)\, P_{b3'}(0)][e^{-2\mu t} - e^{3\mu t}]$$

$$P_{b1} = 3[P_{b3}(0) + P_{b3'}(0)][e^{-\mu t} - 2e^{-2\mu t} + e^{-3\mu t}]$$

$$P_{b0} = 3[P_{b3}(0) + P_{b3'}(0)][\tfrac{1}{3} - e^{-\mu t} + e^{-2\mu t} - \tfrac{1}{3}\, e^{-3\mu t}] \quad .$$

Since $P_{b2'}(t = 0) = 0$ we have $P_{b3}(0) + P_{b3'}(0) = 1$ and

$$P_{b3} + P_{b3'} = e^{-3\mu t}$$
$$P_{b2} + P_{b2'} = 3(e^{-2\mu t} - e^{-3\mu t})$$
$$P_{b1} + P_{b0} = 3(\tfrac{1}{3} - e^{2\mu t} + \tfrac{2}{3}\, e^{-3\mu t}).$$

The Maintenance Function is

$$M(t) = P_{b0} + P_{b1} + P_{b2}$$

$$= 1 - [3(1 - P_{b3}(0)) - (2 - \bar{\mu}/\mu)\, P_{b3'}(0)]\, e^{-2\mu t}$$

$$+ [2 - 3P_{b3}(0) - (2 - \bar{\mu}/\mu)\, P_{b3'}(0)]\, e^{-3\mu t} \quad .$$

The Mean Repair Time is

$$MRT = \int_0^\infty [1 - M(t)]\, dt$$

$$= [\tfrac{5}{6} - 1/2\, P_{b3}\,(0) - (2 - \bar{\mu}/\mu)\, \tfrac{1}{6}\, P_{b3'}(0)]\, \tfrac{1}{\mu} \quad .$$

We note that this expression depends on the distribution among classes 3, 3', $\overline{3'}$ in the initial down system. If we choose a distribution weighted equally for each state in these classes, i.e., $P_{b3}(0)/P_{b3'}(0) = 3/72 = 1/24$, then

$$\text{MRT} = \frac{1}{\mu}\, 0.5533 \quad .$$

Using this in the relation

$$A_\infty = \frac{1}{1 + \frac{\text{MRT}}{\text{MTBF}}} = 0.949$$

gives

$$\text{MTBF} = \text{MRT}\,\frac{0.949}{1 - 0.949} = 18.69\ \text{MRT} = 10.35\,\frac{1}{\mu} = \frac{10.35}{20}\,\frac{1}{\mu}$$

$$= 0.5173\,\frac{1}{\lambda}$$

for mean time between failures. This is 0.5173 times the mean time between failures of single independent units.

Availability

n = 9 , 3-Repair Crews , 3-Unit Kill

$\lambda' = 2\lambda$

μ/λ	A	λ_1/λ	A_1	λ_2/λ	A_2	λ_3/λ	A_3	λ_4/λ	A_4
0.1	0.0225	0.2112	-1.054	8.421	29.67	10.24	-47.39	11.93	19.741
1	0.2055	2.114	-0.004432	7.154	2.279	12.05	-1.815	16.68	0.3339
10	0.8510	12.26	0.2546	20.65	-0.003376	30.03	-0.1032	47.06	0.0010010
20	0.9492	21.13	0.08776	41.11	-0.000239	50.01	-0.03651	77.78	-0.0002438
30	0.9753	30.60	0.04385	61.34	0.00007620	70.00	-0.01884	108.6	-0.0002586
100	0.9974	99.59	0.004859	202.0	0.00007187	210.0	-0.002353	318.4	-0.00001345
200	0.9993	199.3	0.001278	402.2	0.00002613	410.0	-0.006437	618.5	-0.000002083

Mean Repair Time: MRT = 0.5533 $1/\lambda$, for various μ/λ.

Mean Time Between Failures: $\text{MTBF} = \text{MRT} \dfrac{A_\infty}{1 - A_\infty}$

μ/λ	λ x MTBF
0.1	0.1276
1	0.1431
10	0.3160
20	0.5173
30	0.7241
100	2.152
200	4.201

Rules of Thumb for interpreting results for model II shown in the table for $\mu/\lambda > 1$ and $n = 9$ are:

1) $A \simeq 1 - (\frac{n\lambda}{\mu})^2 \frac{1}{5}$ holds well, and corresponds to $A \simeq 1 - (\frac{n\lambda}{\mu})^2$ in model I. The n-dependence here is presumed on basis of model I.

or

$$A \simeq 1 - 0.6\ A_1$$

and

2) $A_1 \simeq (\frac{n\lambda}{\mu})^2\ 8$

3) $\lambda_1/\lambda \simeq \mu/\lambda$

4) $\lambda_2/\lambda_1 \simeq 2$

5) $A_2/A_1 \sim 1/100$

6) $\lambda_4/\lambda_1 \simeq (3\text{-}4)$

All of these are consistent with corresponding suggestions from model I.

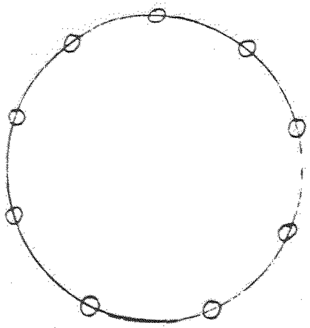
General Observations and Conclusions

The ratio of repair rate μ to failure rate λ for the individual units determines the availability of the n-ring system; mostly in the form

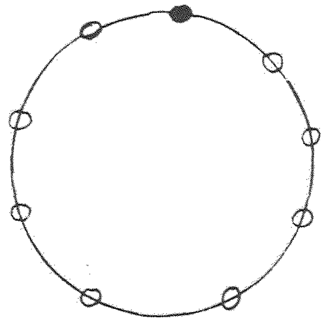
$$\left(\frac{n\lambda}{\mu}\right)^2 \frac{1}{2r}$$

where r is the number of repair crews. Models I and II have a couple of characteristics in common which could be quite different from the actual operating SSC. First, both of these models assume a continuous repair rate, μ, characterizing the probability $1 - e^{-\mu t}$ that a down unit will become repaired and operable in time t after it went down. Immediate commencement of repair and switching into the crippled operating mode were assumed. These assumptions may be incompatible with a 60-km-diameter ring that operates with a continuous (non-pulsed) beam, in a very deep tunnel. The second assumption that may differ from reality is our limitation in these models that excess refrigerator capacity cannot be distributed beyond an adjacent 1/2 refrigerator sector. If this limitation on the distribution of excess capacity can be lifted it would have a salutary effect on the availability in the resulting models in comparison with the present more restricted models.

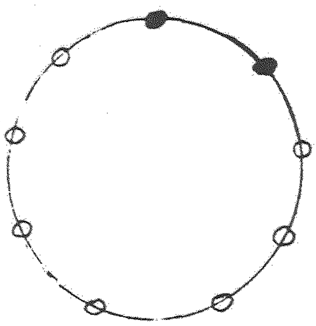
FIG.



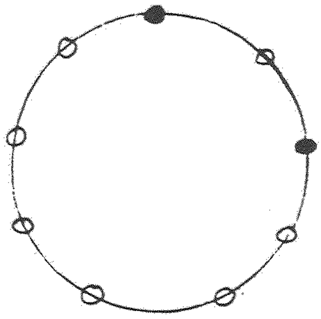
CLASS-0 (1 STATE)
ALL UNITS UP (o)



CLASS-1 (9 STATES)
ONE UNIT DOWN (•)

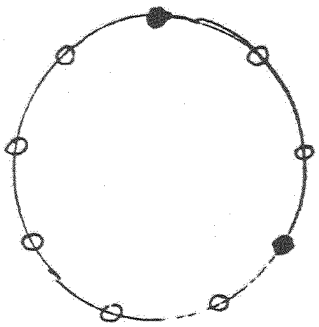


(9 STATES)

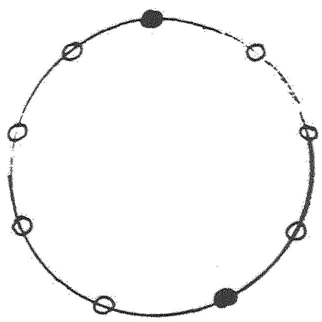


(9 STATES)

CLASS-2′ (18 STATES)
TWO UNITS DOWN
SYSTEM NECESSARILY DOWN



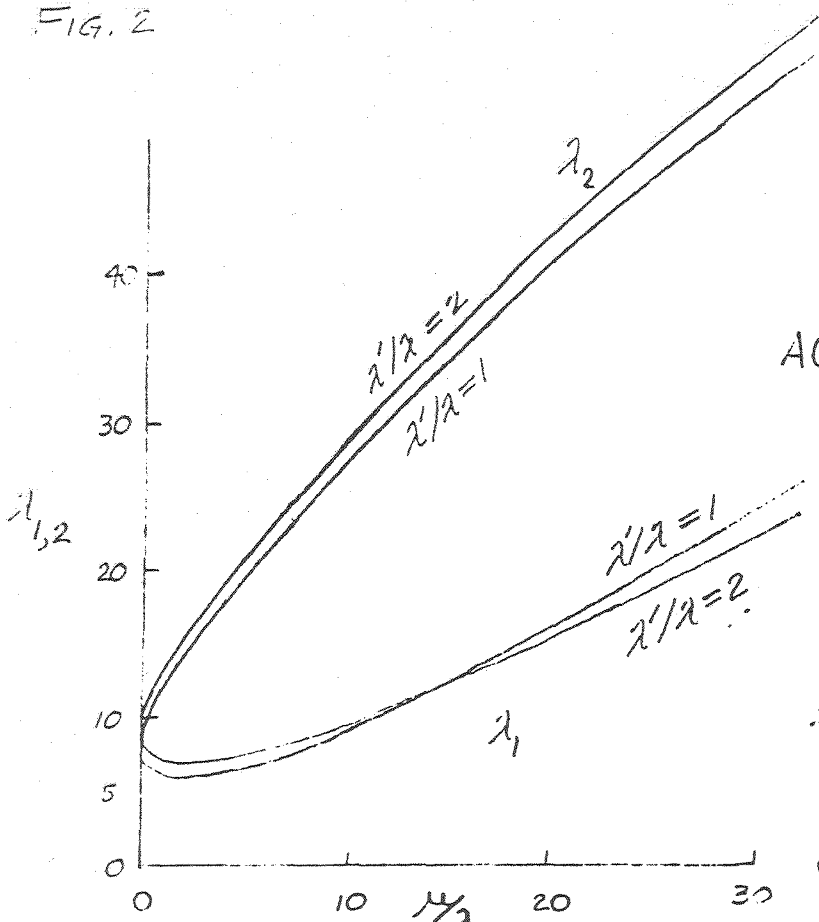
(9 STATES)



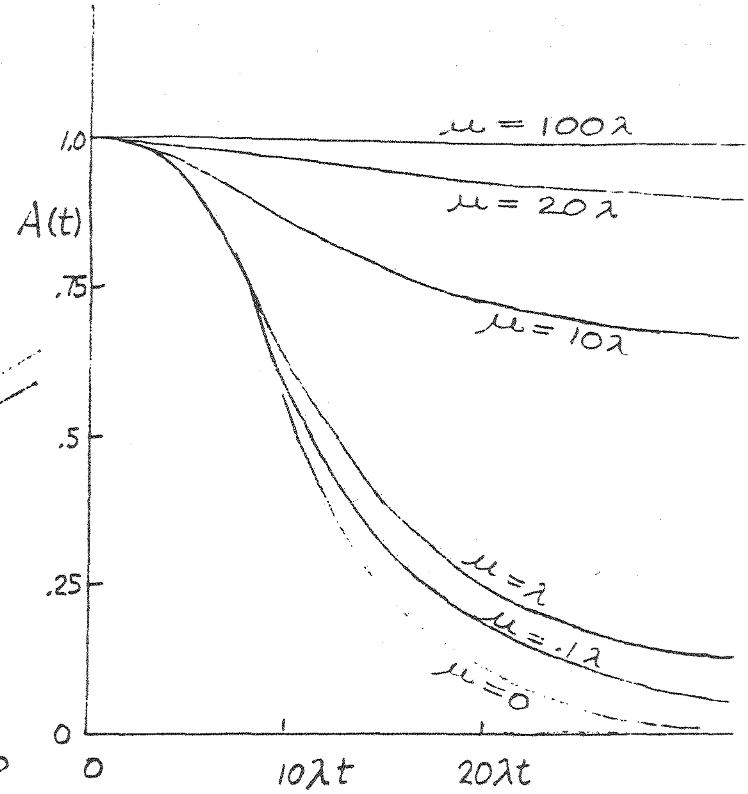
(9 STATES)

CLASS-2 (18 STATES)
TWO UNITS DOWN
SYSTEM COULD RUN, BUT IS DOWN BY STIPULATION

FIG. 2



RATE EIGENVALUES λ_1 AND λ_2
FOR $n=9$, $\lambda'/\lambda = 1, 2$



AVAILABILITY VS TIME
FOR $n=9$, $\lambda' = 2\lambda$