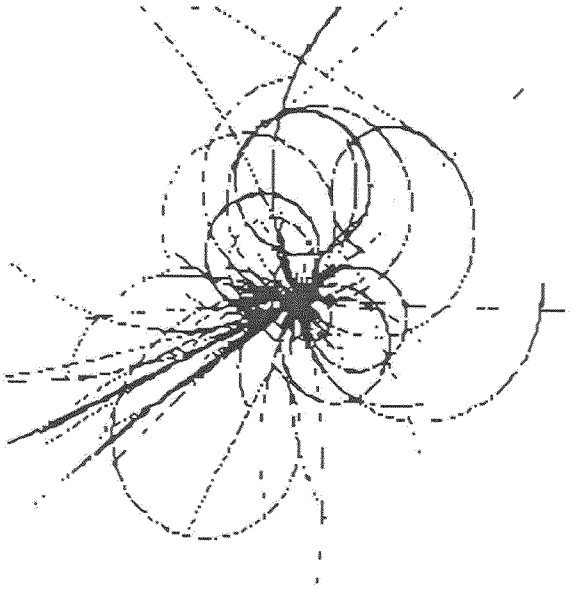
THE SUPERCONDUCTING SUPER COLLIDER LABORATORY



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NOTES

OFF-MOMENTUM LINEAR APERTURE

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The purpose of this short note is (1) to check the applicability of the linear aperture concept for off-momentum particles and (2) to obtain the linear aperture for $\delta = \pm 0.001$ to see if there is enough room for off-momentum beam operations.

To examine the linear aperture, we use the program Racetrack. This program includes multipoles up to order 20. We generated a random set of multipole values (a_n, b_n; $n \geq 2$) and found the new closed orbit for various values of δ. Around this fixed point, it is possible to define a matrix $M\delta$ which describes the paraxial optics around the new orbit. This symplectic matrix has two invariants I^1_δ and I^2_δ. These are <u>linear generalized</u> Courant-Snyder invariants. They properly include the coupling resulting from the random multipoles.

It is clear that the smear of these invariants will be the results of non-linear effects alone.

We proceed as follows for a given random seed:

1. We selected δ and then obtained the fixed point and the invariants.
2. We tracked the initial ray $\vec{Z}^o = (X_o, 0, X_o, 0)$ for 100 turns for 5 values of X_o. (X_o = 0.05, 0.1, 0.15, 0.2, 0.25 mm starting from the interaction point). The vector $\vec{Z}^o$ is measured with respect to the fixed point.
3. For each $\vec{Z}_o$, we computed the invariants as a function of the turn number n. M is the mapping for one turn.

$$(\vec{Z}^n = M^n \vec{Z}^o \Rightarrow I_\delta^{i=1,2} (\vec{Z}^n) \ .$$

For a given value of δ and $\vec{Z}^o$, the smear S_δ defined following the prescription in the aperture report SSC-SR-1013 (including the magnet errors quoted therein with coil inner diameter of 4-cm).

$$S_\delta(\vec{Z}^o) = \underset{\substack{\text{over}\\ i=1,2}}{\text{Max}} \left\{ \frac{\underset{\text{over } n}{\max} \sqrt{I_\delta{}^i(\vec{Z}^n)} - \underset{\text{over } n}{\min} \sqrt{I_\delta{}^i(\vec{Z}^n)}}{< \sqrt{I_\delta^i} >_{\text{over } n}} \right\}$$

The linear aperture is the value of $\vec{Z}_o = \vec{Z}_L$ for which S_δ reaches 30%.

For the D lattice, we found the following linear apertures. (We used only one random seed).

$\delta = 0 \qquad X_L \approx 0.27$ mm

$\delta = 0.001 \quad X_L \approx 0.15$ mm $\qquad\qquad \vec{Z}_L = (X_L, 0, X_L, 0)$

$\delta = -0.001 \quad X_L \approx 0.13$ mm

We also tried a different initial ray for $\vec{Z}^o$. We launched a particle with no initial displacement: $\vec{Z}^o = (0, X'_o, o, X'_o)$. We tried this at $\delta = 0.001$ and obtained $X'_L \approx 0.176$ mr. Obviously, due to the shift from M_o to M_δ, the β function which was initially unity has lost its meaning. Therefore $X'_L = 0.176$mr and $X_L = 0.15$ mm might correspond to a similar generalized emittance since β is about 1m.

To check if $X_L = 0.15$mm and X'_Lmr $= 0.176$mr do correspond to the same generalized emittance to a higher accuracy, we calculated

$$\frac{<I^1(X'_L) + I^2(X'_L)>}{<I^1(X_L) + I^2(X_L)>} \quad \text{and found it to be 1.07.}$$

This is more consistent to the assumption of the "smear" being independent of the initial phases.

Conclusion:

The concept of "smear" is applicable to off-momentum particles. The linear aperture decreases by a factor of 2 from the on-momentum value for $\delta = \pm 0.001$.

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