SSC-N-37

12 September 1985

Heating Load in SSC Final Focus Quadrupoles Due to p-p Inelastic Scattering

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1. Introduction

A numerical integration (in contrast to Monte Carlo) has been made to calculate the energy caried by charged mesons leaving a round bore in the quadrupole triplet near an SSC beam crossing point (IR) The present version of the calculation is considered preliminary, but it is easily modified to include different optics and dimensions, improved cross sections, other scattering processes, other collimation schemes, and a beam separation dipole. In addition, the meson-decay muon spectrum can be calculated, but because of the nature of the calculation its angular distribution cannot.

2. Cross Sections

For present purposes, we have simply used the inelastic cross sections given in the Reference Designs Study (RDS):

$$x_R \frac{d^3N}{dx_R dp_\perp^2} = \left[\sum_{mesons} A_i(1-x_r)^{n_i}\right]\left[\frac{C_m}{(1+p_\perp/p_{\perp 0})^m}\right]$$

Coefficients and exponents are as given in the RDS. C_m is chosen such that the integral over $d(p_\perp^2)$ is unity, and m varies from 6.5 to 9.0, depending upon the pseudorapidity.

It is useful to check the total energy and multiplicity:

1. *Energy.* The total energy per collision going into charged mesons in one direction is the beam energy, E_b, times

$$\int x_R \frac{dN}{dx_R} dx_R = \sum_{mesons} \frac{A_i}{1+n_i}$$
$$= 0.515$$

The number is a little high: neutral mesons ($\approx \frac{1}{3}$ of the total?) are neglected, as are leading baryons. A number closer to 0.4 might be more reasonable. On the other hand, the overestimate compensates in part for the neutrals.

If the total cross section is 120 mb and the luminosity is $10^{33}\text{cm}^{-2}\text{s}^{-1}$, 768 W goes into scattered particles. We take $\frac{3}{4}$ of this (90 mb) to be inelastic. Half goes each way, and of this 52% goes into charged mesons. 150 W thus goes into charged mesons in each direction due to inelastic scattering.

2. *Charged meson multiplicity.* For the ith meson species, we may write

$$N_i = \int_{m_i/E_b}^{1} \frac{dN_i}{dx_R} dx_R$$
$$= A_i \int_{m_i/E_b}^{1} \left[\frac{1}{x} - \frac{(1-(1-x)^{n_i})}{x}\right] dx$$
$$= A_i \ln(E_b/m_i) + f(n)$$

Here E_b is the beam energy (20 TeV), m_i the meson mass (0.139 GeV or 0.495 GeV), and $f(n)$ is a benign function of n; it varies from 1.78 to 2.20 over the region of interest. The multiplicity is totally dominated by the $\ln(E_b/m_i)$ ($\propto \ln s$) term—there are lots of soft mesons! Numerically, we find

$$N = 11.0 + 10.9 + 1.1 + 1.0_5$$
$$= 24.2$$

where the contributions are from π^+, π^-, K^+, and K^- production. The total charged multiplicity (in both directions) is thus 48.4.

Straightforward numerical integration to find the multiplicity is difficult because of the behavior of $1/x$. Programs were written to do this in the process of checking the function generators, and reasonable agreement with the above analytic calculation was found. Before an early program bug was removed, the lower integration limit was taken as m_i rather than m_i/E_b; we then got 17.5, in agreement with the RDS. Either an error of this sort or the difficulty with numerical convergence could be responsible for the discrepancy.

In any case, the cross sections appear to be sufficiently well-behaved for our present purposes. A more careful modeling is planned for the future, as is inclusion of singly diffractive scattering.

3. Quadrupoles

Since in general the particles being lost have far less than beam momentum, it is simplest to treat the trajectory from first principles. At a distance z from the front of a quadrupole, a particle's displacement in the focusing direction is

$$x = x_0 \sin(\kappa z + \phi_0)$$

and in the defocusing direction by

$$y = a_+ \exp(\kappa z) + a_- \exp(-\kappa z) \quad ,$$

where x_0, ϕ_0, a_+, and a_- are trivially related to the four initial displacements and slopes. As usual,

$$\kappa^2 = \frac{p_b}{p} \left(\frac{1}{B\rho_b} \left| \frac{\partial B}{\partial x} \right| \right) \quad ,$$

where the subscript b indicates the parameter value at beam momentum. It is also convenient to define

$$f(z) = x^2 + y^2 - R^2 \quad ,$$

where R is the beam pipe radius. f behaves as the sum of trigonometric functions and exponentials. It is negative for a particle inside the beam pipe, and crosses zero with positive slope when a particle hits the pipe.

The idea is to find the smallest positive root. If it is less than the physical length of the quadrupole, it is interpreted as the point the particle leaves the pipe. Since the nuclear interaction length in the coil or nearby metal is about 15 cm, this z is also within a meter of where the energy is actually deposited.

For a triplet, one merely cascades the calls to this program; the input parameters of one quad (if the particle is still in the pipe) are easily related to the exit displacements and slopes from the last. The program is told whether each element is focusing or defocusing in the horizontal plane, but internally reverses things for negative mesons.

One steps through particle type, x_R, θ, and ϕ. For a given choice, the energy assigned to a hit is

$$\Delta E_i = \left(x_R \frac{d^3 N_i}{dx_R dp_\perp^2} \right) \Delta x_R \Delta p_\perp^2 \frac{\Delta \phi}{2\pi} \quad .$$

4. Other Elements.

A collimator is easily inserted by depositing the energy at the collimator's location if $x^2 + y^2 \geq r_{col}^2$.

Dipoles can be implemented with fair ease, but so far we have not done so.

5. Results.

Results for the RDS triplet (p. 112), a 1 cm radius collimator just before the first quadrupole, and a 1.5 cm radius beam pipe, are shown in Fig. 1 (differential distribution) and Fig. 2 (integral distribution). The units are normalized to the fraction of the inelastic energy going into charged mesons, so 1.0 corresponds to 150 W at the canonical luminosity.

The differential distribution is much as one might expect: The front part of QF1 is "shadowed" by the collimator. The growing exponentials in one direction finally dominate, leading to the increased energy deposition near the end of the quadrupole. The large displacements and slopes in this direction continue to result in particle loss in the next quad, even though this direction is now focusing. The pattern continues in the following quads, but with diminished amplitudes because lower-momentum particles have already been swept out.

A total of 37 W is deposited in the triplet. (The remarkable agreement with the RDS result is regarded as fortuitous!) It is our impression that inclusion of neutrals and particles from other processes will increase this number to no more than 50 W. Of the total, 7 W comes from particles deflected in QF1 but which hit in QD2. These could possible be removed by a collimator between QF1 and QD2, but the 7 W maximum gain is probably not worth it.

18%, or 28 W, hits the 1 cm radius collimator in front of QF1. This collimator is clearly of great importance, and its design must be carefully optimized. As presently conceived, it is the most restrictive aperture in the machine (or at least between clustered IR's), and as such will be hit by *incoming* "halo" particles produced in adjacent IR's. It is virtually certain that a more restrictive aperture must be put somewhere else, possibly in the outgoing beam soon after beam separation.

6. Future work.

In the near future, we plan to

1. Step the radius of the first collimator.
2. Investigate the effect of a collimator between QF1 and QD2.
3. Introduce the first separation dipole, with collimators in the long drift space after it.
4. Produce the secondary muon spectrum.

On a longer scale, we plan to look into better cross section parameterizations, include processes such as single diffraction dissociation, and to follow the IR optics design as it evolves. In particular, the design of the first separation dipole is very much affected: Mesons should go into warm metal, and should go into it as soon as possible in order to suppress muon production.

We have also not addressed the global problem of masking and collimation. If the effective cross section for a scattering which will produce debris in a detector somewhere is 10 μb and the detector contains elements which are sensitive for 3 μs, then the detector will contain extraneous junk in 3% of the events it records. In this respect we are particularly concerned about slightly off-momentum protons which also have large betatron amplitudes; a subset of these particles are "almost" in aperture and are candidates for a collision as they approach a detector.

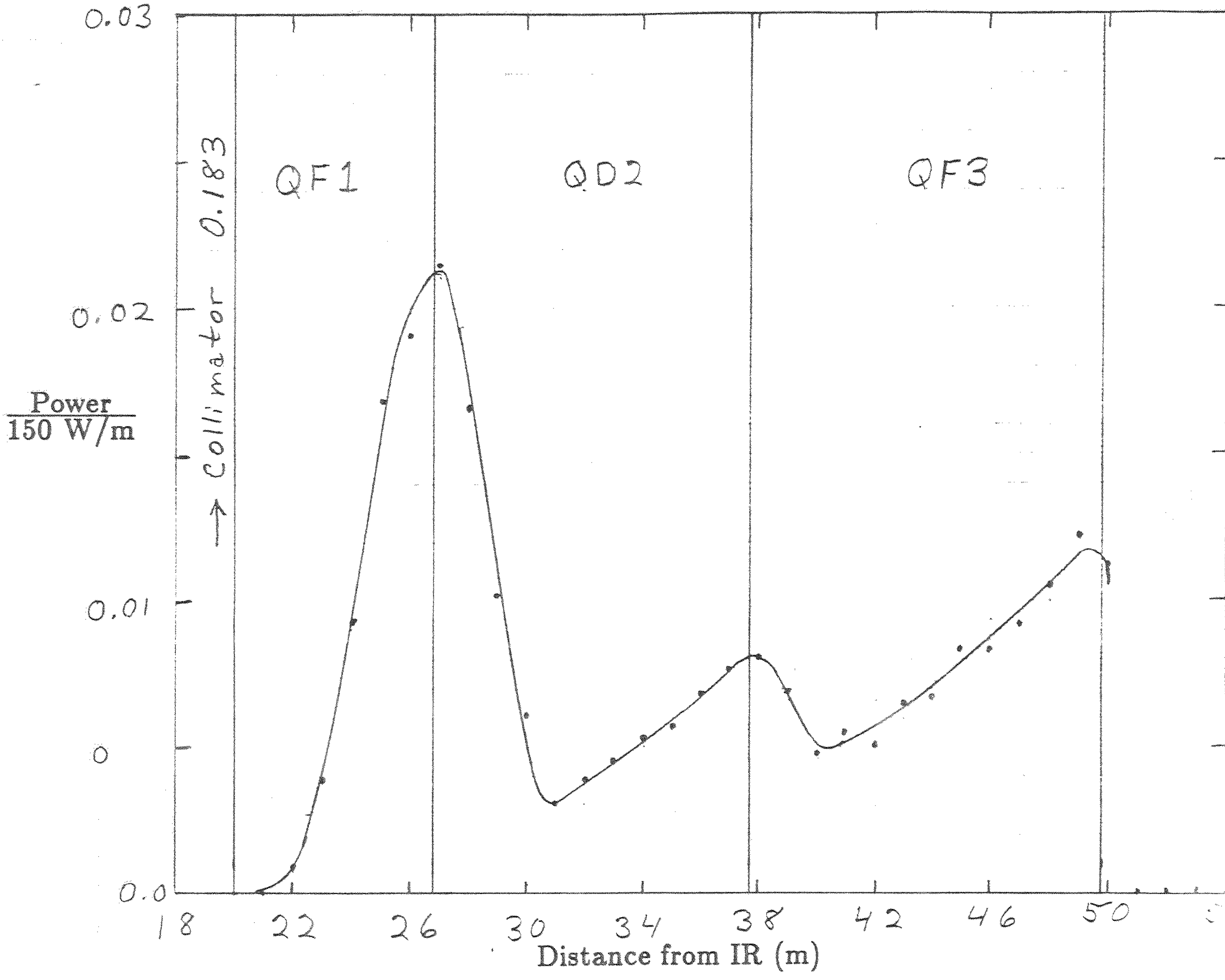

Fig. 1. Differential energy deposition in quadrupoles in charged meson inelastic production

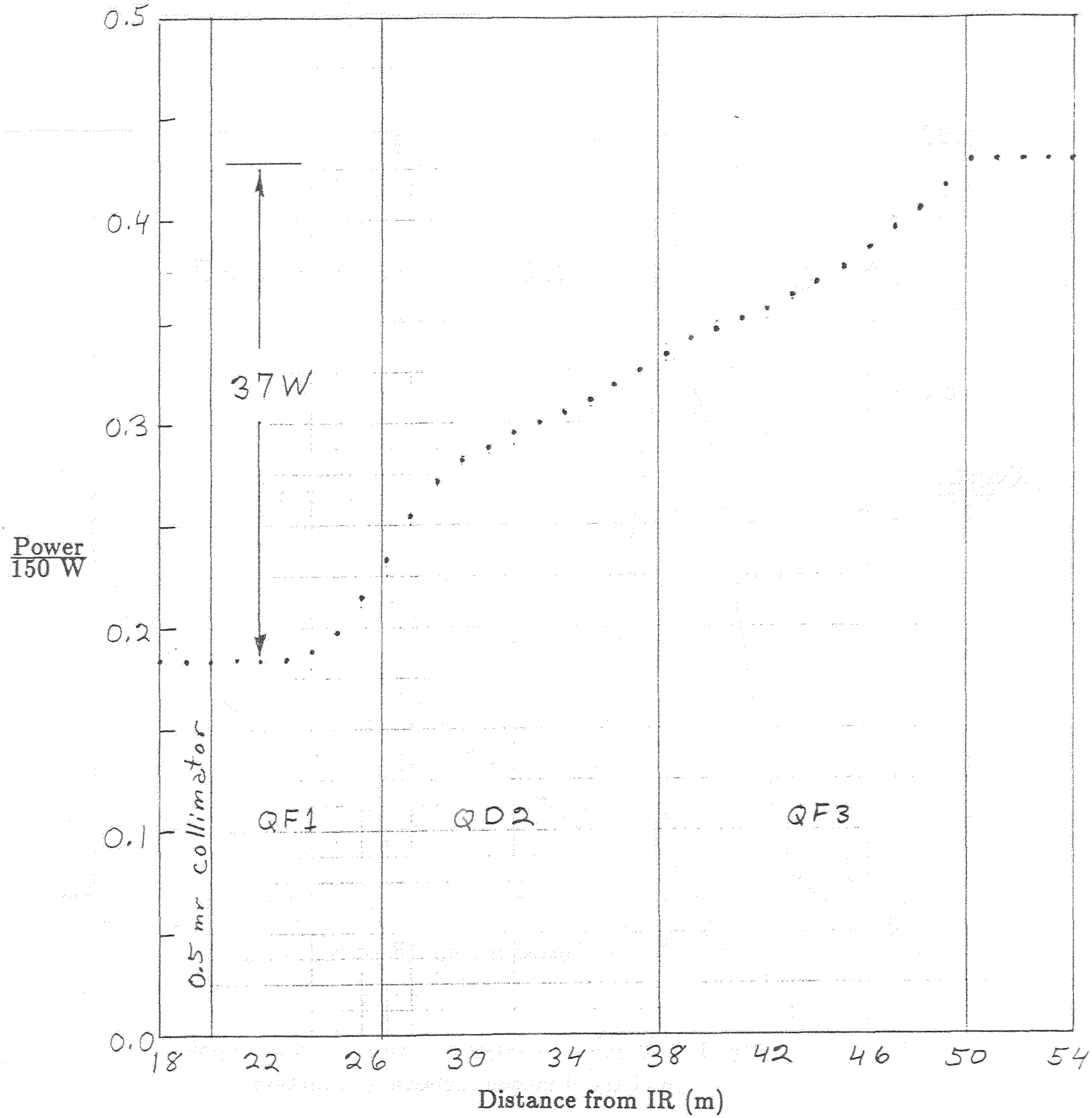

Fig. 2. Integral energy deposition in quadrupoles in charged meson inelastic production