Preliminary Estimate of Emittance Growth Due to Position Jitter and Magnet Strength Noise in Quadrupole and Sextupole magnets

SSC-N-34
9/9/85
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Summary

The problem of beam response to position jittering and magnet strength fluctuations is an extensive subject involving detailed knowledge of the noise spectrum and the beam natural frequencies. In case of unfortunate frequency combinations, resonance beam response may cause instability of the beam.

This calculation is not an attempt to deal with the difficult resonant behavior, assuming they are avoided by carefully choosing the relevant frequencies. The position jitter and magnet strength fluctuations are assumed to be random from magnet to magnet and from turn to turn. A statistical calculation then suffices for an estimate of the problem.

Magnet position can jitter in the transverse and the longitudinal directions. The following table shows the dominating effect of position jittering and strength fluctuations in the various types of magnets:

	transverse position jitter	longitudinal position jitter	strength fluctuations
dipole	no effect	diffusion on orbit	diffusion on orbit
quadrupole	diffusion on orbit	emittance growth	emittance growth
sextupole	emittance growth	higher order effect	higher order effect

The diffusion effect on orbit can be controlled straightforwardly by a feedback system. In this calculation, we present only some estimates concerning the three emittance growth entries. We found that the white noise tolerances are

sextupole transverse jitter $< 2.2\ \mu m$
quadrupole longitudinal jitter $< 40\ \mu m$
quadrupole strength fluctuation $< 1.0 \times 10^{-5}$

For the quadrupole strength fluctuation tolerance, we assume on the average one fluctuation per 1000 revolutions and each fluctuation propogates for the distance of one circumference before it is damped. These results are preliminary and are to be replaced by more careful calculations when available.

Transverse position jitter of sextupoles

Suppose a sextupole located at $s=0$ is displaced horizontally by x_0 due to its position jittering as a particle passes by. It acts like a quadrupole with strength

(1) $$\Delta k(s) = S\, x_0\, \delta(s)$$

where $S = B''\ell/B\rho$ is the sextupole strength. The equation of motion of the particle is

(2) $$d^2x/ds^2 + K(s)\, x = \Delta k(s)\, x$$

Making a Courant-Snyder transformation gives

(3) $$x = \sqrt{\beta(s)}\; u$$

$$d^2u/d\theta^2 + \nu^2 u = \nu^2 \beta^2 \Delta k\, u$$

Before and after the action by the sextupole, the particle motion is a pure betatron oscillation

(4) $$u_1 = A_1 \cos(\nu\theta + \varphi_1) \qquad s < 0$$

$$u_2 = A_2 \cos(\nu\theta + \varphi_2) \qquad s > 0$$

At $s=0$, the boundary conditions are

(5) $$u_2 = u_1$$

$$du_2/d\theta = du_1/d\theta + \nu\beta\, S\, x_0\, u_1$$

These conditions then connect the amplitude and phase variables A_1, A_2, φ_1 and φ_2. In particular,

$$(6) \qquad A_2^2 = A_1^2 \left[1 - \beta S x_0 \sin 2\varphi_1 + (\beta S x_0)^2 \cos^2\varphi_1 \right]$$

Assuming the sextupole dispacement is random from magnet to magnet and from turn to turn with a rms $\langle x_0^2 \rangle^{1/2}$, the expectation value of the particle amplitudes grows by the amount

$$(7) \qquad \Delta A^2/A^2 = \beta^2 S^2 \langle x_0^2 \rangle/2$$

per crossing of a sextupole.

For the SSC, we assume two types of sextupoles in the normal arc cells. The emittance growth per cell is then

$$(8) \qquad \Delta A^2/A^2 = \langle x_0^2 \rangle/2 \left[(\beta S)_F^2 + (\beta S)_D^2 \right]$$

Taking β_F = 346m, β_D = 115m, S_F = 0.0013 m^{-2}, S_D = -0.0021 m^{-2}, we find that $\Delta A^2/A^2 = 0.13\times10^{-12} \langle x_0^2 \rangle$ per traversal of a cell where x_0 is in units of µm. During 30 hours of storage time, the beam traverses about 1.6×10^{11} cells. If we set an emittance growth limit of about 10%, the position jittering tolerance is found to be 2.2µm.

Longitudinal Position Jittering in Quadrupoles

Consider a quadrupole of length ℓ jitters in its longitudinal position by an amount $\Delta\ell$. The jittering introduces two effective error quadupoles at the entrance and the exit of the quadrupole,

$$(9) \qquad \Delta k(s) = (K\,\Delta\ell/\ell\,)\,[\,\delta(s) - \delta(s-\ell)\,]$$

where $K= B'\ell/B\rho$ is the quadrupole strength. Following a similar procedure as above, we describe the betaron motion of a particle by

$$(10) \qquad u = \begin{cases} A_1 \cos(\,\nu\theta + \varphi_1\,) & s < 0 \\ A_2 \cos(\,\nu\theta + \varphi_2\,) & 0 < s < \ell \\ A_3 \cos(\,\nu\theta + \varphi_3\,) & s > \ell \end{cases}$$

Matching the boundary conditions at $s=0$ and $s=\ell$, averaging over the initial phase φ_1 and assuming the β-function at the entrance and the exit of the quadrupole are equal, the emittance increase per quadrupole traversal is found to be

$$(11) \qquad \Delta A^2/A^2 = (\beta\,K\,\Delta\ell/\ell\,)^2\,(\,1 - \cos 2\psi\,)$$

where ψ is the betatron phase advance across the quadrupole. Since $\psi \approx \ell/\beta$, the emittance growth per cell traversal is

$$(12) \qquad \Delta A^2/A^2 = \langle\Delta\ell^2\rangle/2\,\left[\,K_F^{\,2} + K_D^{\,2}\,\right] \qquad \text{per cell.}$$

Taking $K_F = -K_D = 0.01/\text{m}$ and again imposing a maximum of 10% increase of emittance duing 30 hours beam storage time, the tolerance on $\Delta\ell$ is found to be 40µm.

Strength Fluctuation in quadrupoles

The analysis of the sextupole transverse position jitter also applies to quadrupole strength fluctuations. The result is

(13) $\Delta A^2/A^2 = \langle(\Delta K/K)^2\rangle (\beta K)^2/2$ per quadrupole crossing

where $\Delta K/K$ is the white noise relative error in quadrupole strength. For a simplistic model, we assume that (a) the quadrupole strength fluctuation comes from power supply misfiring (Peter Limon) that occurs on the average of once per 1000 beam revolutions, (b) each fluctuation lasts for a distance around one circumference, and (c) these fluctuations represent white noise as far as the beam motion is concerned. Taking the values used so far, the condition that the beam emittance does not grow more than 10% during storage requires $\Delta K/K$ to be less than 1.0×10^{-5}. This tolerance is relaxed by a factor of $\sqrt{f}$ if the power supply misfiring occurs less frequently by a factor of f.