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# AN ESTIMATE OF THE CONTRIBUTIONS OF BELLOWS TO THE IMPEDANCES AND BEAM INSTABILITIES OF THE SSC

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# AN ESTIMATE OF THE CONTRIBUTIONS OF BELLOWS TO THE IMPEDANCES AND BEAM INSTABILITIES OF THE SSC

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## INTRODUCTION

Between sections of the vacuum chamber, bellows are needed to compensate for thermal expansion and transverse offsets. For beampipe made of stainless steel with a coefficient of linear expansion $19\times10^{-6}/^{\circ}C$ and a temperature variation of $\sim316^{\circ}C$, the allowance for bellows is ~1.2% of the total length of the beampipe, if we assume that the bellows are 50% compressible[1]. This implies 1.08 km of bellows for Design A of the SSC which has a circumference of 90 km, Such a length of bellows will certainly contribute

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to the longitudinal and transverse impedances of the accelerator and will therefore affect the stability of the beam. In the Reference Designs[2], the actual impedances of the bellows have not been calculated; only an allowance of $Z_{||}/n = 0.05\ \Omega$ and $Z_{\perp} = 7\ \mathrm{M}\Omega/\mathrm{m}$ is made for miscellaneous discontinuities because all the bellows and pumping ports are assumed totally shielded. It is the purpose of this article to examine the actual contributions by the bellows to the longitudinal and transverse impedances assuming that they are not shielded.

## COMPUTATIONS

For the ease of computation, we assume the corrugations of the bellows be rectangular with period 2g = 3 mm, depth $\tau$ = 4.875 mm as shown in Figure 1; the beampipe radius is taken to be b = 1.5 cm.

The code TCBI[3] solves directly Maxwell's equations in the time domain and calculate the wake potential $\hat{W}(t)$ of a Gaussian bunch with RMS length $\sigma_\ell$ and one unit of charge,

$$\hat{W}(t) = \int d\tau\, q(\tau) W(t-\tau)\,, \tag{1}$$

where W is the wake potential due to a point charge and $q(\tau)$ the charge distribution of the bunch which, in reality, is a truncated Gaussian (in our computation we truncated it at $\pm 5\sigma_\ell$). A Fourier transformation of Eq. (1) gives us $\hat{Z}(\omega)$

the effective impedance seen by the bunch, which is related to the actual impedance seen by a point charge $Z(\omega)$ by

$$\hat{Z}(\omega) = Z(\omega)\, e^{-\frac{1}{2}(\omega\sigma_\ell/c)^2} . \qquad (2)$$

In doing the Fourier transform, care must be taken to set the time coordinate correctly so that $\hat{W}(0)$ represents the wake at the center of the bunch. In our computation, we truncated the wake $\hat{W}(t)$ at 60 cm so that the impedance could have a resolution of 0.25 GHz. As will be seen in our resulting plots, ripples of period 0.5 GHz are seen in the curves.

A small bunch length should be used so that the actual impedance at high frequencies will not be smeared away according to Eq. (2). We took $\sigma_\ell$ = 1.5 mm so that the impedance would be attenuated to 80% at 21 GHz, 50% at 37.5 GHz and 5% at 78 GHz. The mesh was taken to be 0.375 mm so that there would be four inside each corrugation. Further reduction of $\sigma_\ell$ is not possible unless the mesh size is reduced also. We had tried to reduce the mesh size by half; not many changes in the results were observed but the computing time was increased by several times.

We ran TBCI for a bellow of 5 corrugations as shown in Figure 1 for the longitudinal mode m = 0 and the transverse mode m = 1. The results are shown in Figures 2 to 7.

## IMPEDANCES AT LOW FREQUENCIES

When the frequency f approaches zero, from Figures 4 and 7, the longitudinal and transverse impedances for one corrugation are respectively

$$Z_{\parallel} = j\,0.53\times10^{-9} f\ \Omega, \tag{3}$$

$$Z_{\perp} = j\,0.19\ k\Omega/m. \tag{4}$$

Here, we assume that the impedance of N corrugations is equal to N times the impedance of one corrugation. The above values can be checked by two existing formulas at low harmonics $n \ll 2R/g$ or $R/d$ where R is the radius of the accelerator ring and $d = b+\tau$ is the bigger radius of the bellows. The longitudinal[4] and transverse[5] impedances are

$$Z_{\parallel} = j\left(\frac{Z_0 g}{c}\ln S\right) f\ , \tag{5}$$

$$Z_{\perp} = j Z_0\left(\frac{g}{\pi b^2}\right)\left(\frac{S^2-1}{S^2+1}\right), \tag{6}$$

with $S = d/b$ and f the frequency in Hz. These give $Z_{\parallel} = j0.53\times10^{-9}\ f\ \Omega$ and $Z_{\perp} = j219\ \Omega/m$. Equation (5) is valid when $g/b \ll \pi$ which is certainly satisfied. Equation (6) is valid when $g/b \ll \pi^2/32$ and is not so well satisfied. A more accurate numerical calculation[5] shows that $Z_{\perp} = j198\ \Omega/m$. As a whole, our computation reproduces the correct results.

## RESONANCES

The real part of the longitudinal impedance in Figure 3 and the real part of the transverse impedance in Figure 6 are similar in shape. They have a big resonance near 13 GHz. For the longitudinal mode the lowest resonance is at 7.62 GHz which is just below the cutoff frequency of the beampipe $f_{cutoff} = 2.405c/2\pi b = 7.655$ GHz. As a result, the fields are trapped inside the corrugations and cannot flow out into the beampipe thus causing little influence to the wake of the bunch. Therefore, this resonance is not seen in the impedance plot. A computation had been done by changing the parameters of the corrugation to $\tau = 4.6$ mm and $g = 1.48$ mm so that the fundamental mode is at exactly $f_{cutoff}$ and this peak did appear in the impedance plot. For the transverse impedance, the resonance near 13 GHz is the fundamental resonance.

There is another smaller resonance near 38 GHz both for the longitudinal mode and the transverse mode. However, because the RMS bunch length $\sigma$ of Design A is 7 cm, this resonance will not have any influence on the stability of the beam.

## BEAM INSTABILITIES

For a RMS bunch length of $\sigma_\ell = 7$ cm, the bunch bandwidth is ~1 GHz. Thus the impedances of the bellows are of broad band and will therefore drive the single-bunch instabilities. In the Reference Designs[2], the most severe limitation on beam current is set by the transverse mode-coupling instability which arises when the real frequency shift of any mode becomes equal to the synchrotron frequency. Assuming that the largest shift is due to mode $\mu = 0$, the limit on $Z_\perp$ is[2]

$$\bar{Z}_\perp \leq \frac{4\sqrt{\pi}\,\eta\,(E/e)(\sigma_E/E)}{I\,\bar{\beta}} , \tag{7}$$

where

$$\bar{Z}_\perp = \frac{\sigma_\ell}{c\sqrt{\pi}} \int_{-\infty}^{\infty} \mathrm{Im}\, Z_\perp(\omega)\, e^{-(\omega\sigma_\ell/c)^2} d\omega \tag{8}$$

for a Gaussian bunch. In above, $\eta = 1.3\mathrm{x}10^{-4}$ is the frequency-slip factor, $\sigma_E/E = 1.5\mathrm{x}10^{-4}$ the RMS energy spread at injection energy $E = 1$ TeV, $\bar{\beta} = 150$ m the average betatron function and $I = 7.7$ μA the average single-bunch current. We get for the threshold $\bar{Z}_\perp$ =120 MΩ/m. For 10.8 km of bellows with a period of 3 mm, there are in total 360,000 corrugations. Using Figure 7, if the integration of Eq. (8) is performed, we get $Z_\perp$ = 68 MΩ/m. If a 100% safety factor is included, the bellow contribution alone will be higher than the threshold. We note that for the whole machine, the estimate[2] of $Z_\perp$ is only 47 MΩ/m.

The next dangerous instability is the transverse microwave which has a risetime fast compared with a synchrotron period and is driven by disturbances of wavelengths much shorter than the bunch length. The impedance limit[6] for a broad band at f ~13 GHz is

$$|Z_\perp| \leqslant \frac{4\eta(E/e)(\sigma_E/E)(\sigma_\ell/R)(2\pi Rf/c)}{I\bar{\beta}} = 1287\ M\Omega/m. \tag{9}$$

This limit will be very much lower if the traditional cutoff frequency is used for f instead. Figures 6 and 7, $|Z_\perp|$ has a maximum of 3.8 kΩ/m for 5 corrugations or 274 MΩ for 360,000 corrugations which is dangerously to high.

As for longitudinal microwave instability[7], the limit on $Z_{||}/n$ is

$$|Z_{||}/n| \leqslant \frac{\sqrt{2\pi}\,\eta(E/e)(\sigma_\ell/R)(\sigma_E/E)^2}{I} = 4.7\ \Omega. \tag{10}$$

From Figures 3 and 4, we get, at ~13 GHz, $|Z_{||}/n| = 2.3\ \Omega$ for all the corrugations which is rather too high.

The longitudinal mode-coupling instability occurs when two modes collide as the real frequencies shift. For a short bunch the lowest mode $\mu = 1$ is shifted most, the stability limit for $Z_{||}/n$ is given by[7]

$$\mathrm{Im}\, Z_{||}/n \leqslant \frac{8\sqrt{\pi}\,\eta(E/e)(\sigma_E/E)^2(\sigma_\ell/R)}{I} = 27\ \Omega. \tag{11}$$

The bellow contribution from Figure 4 is 0.64 Ω which is very much lower than the above limit.

## DISCUSSIONS

1. We learn from above that the bellow contributions to the impedances will upset the single-bunch stabilities, among which the dangerous ones are the transverse mode-coupling, transverse microwave, and longitudinal microwave stabilities. Therefore, the corrugations must be shielded in some way to preserve beam stability.

2. In our analysis, we make the assumption that the impedance of N corrugations is N times the impedance of one corrugation. Strictly speaking, this is true only when the wavelength is much shorter than the period of the corrugations. We had run TBCI for 1 corrugation, 3 corrugations, 5 corrugations and 9 corrugations and found that the impedance per corrugation decreased slightly with the number of corrugations. For example, near zero frequency, the imaginary parts of the transverse impedance per corrugation are respectively 0.198, 0.190, 0.186 and 0.185 kΩ/m when 1, 3, 5 and 9 corrugations are considered. For this reason, we believe that our estimates for 360,000 corrugations might have been slightly too high but nevertheless of the correct order of magnitude.

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6. R.D. Ruth, Proceedings of the Workshop on Accelerator Physics Issues for a Superconducting Super Collider, Ann Arbor, MI, 1983, p. 151
7. S. Krinsky and J.M. Wang, BNL-33867

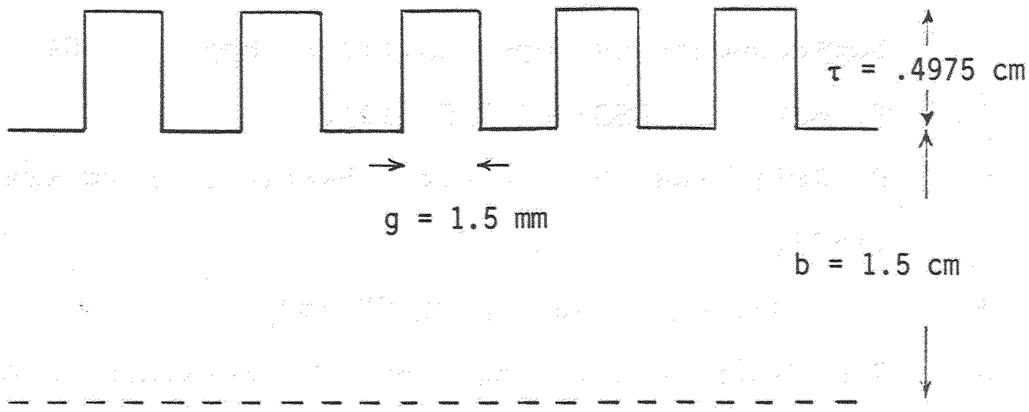


Figure 1. Five corrugations of a bellow

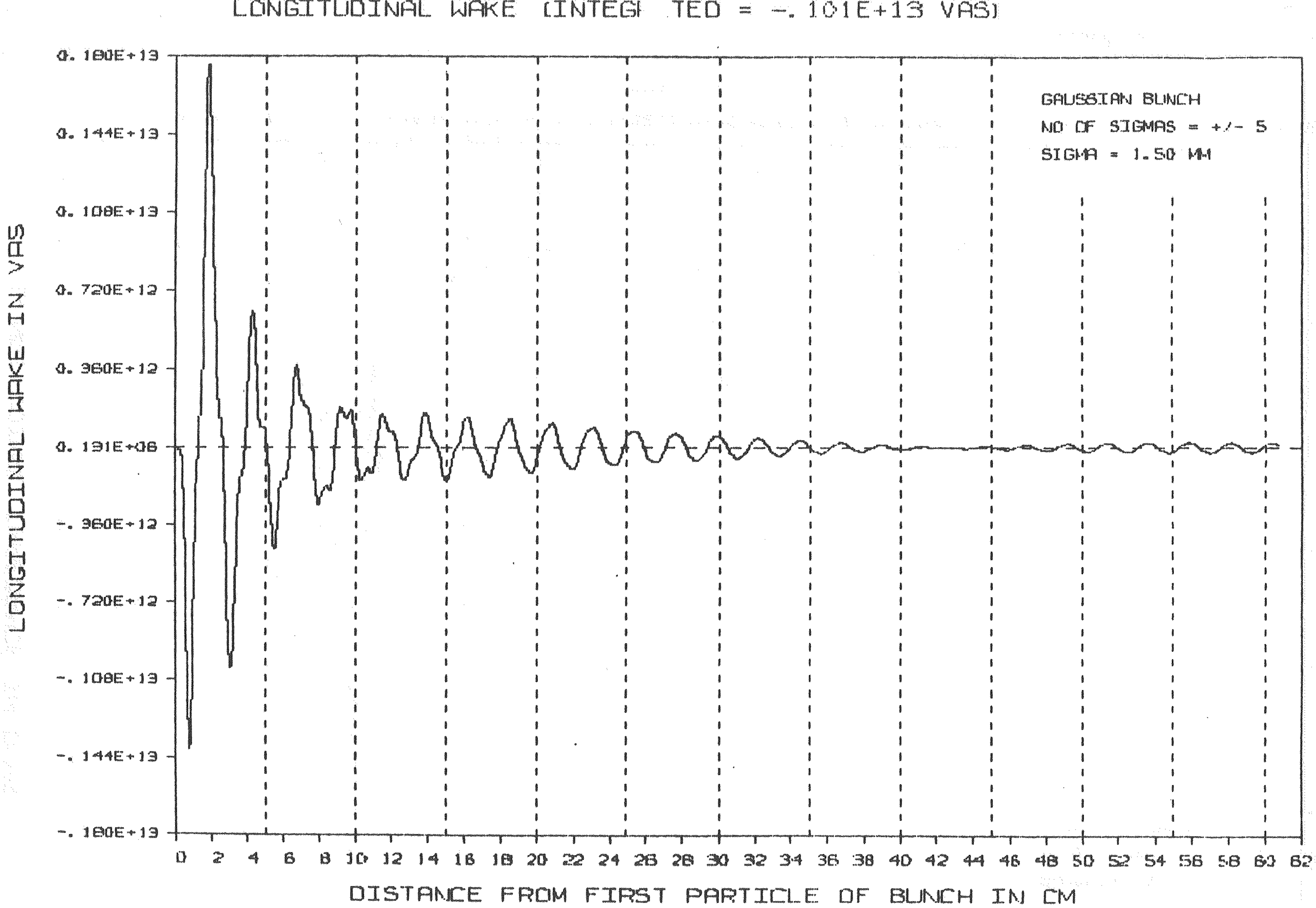
LONGITUDINAL WAKE (INTEGI TED = -.101E+13 VAS)
GAUSSIAN BUNCH
NO OF SIGMAS = +/- 5
SIGMA = 1.50 MM
0.180E+13
0.144E+13
0.108E+13
0.720E+12
0.360E+12
0.191E+06
-.360E+12
-.720E+12
-.108E+13
-.144E+13
-.180E+13
LONGITUDINAL WAKE IN VAS
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50 52 54 56 58 60 62
DISTANCE FROM FIRST PARTICLE OF BUNCH IN CM

FIGURE 2

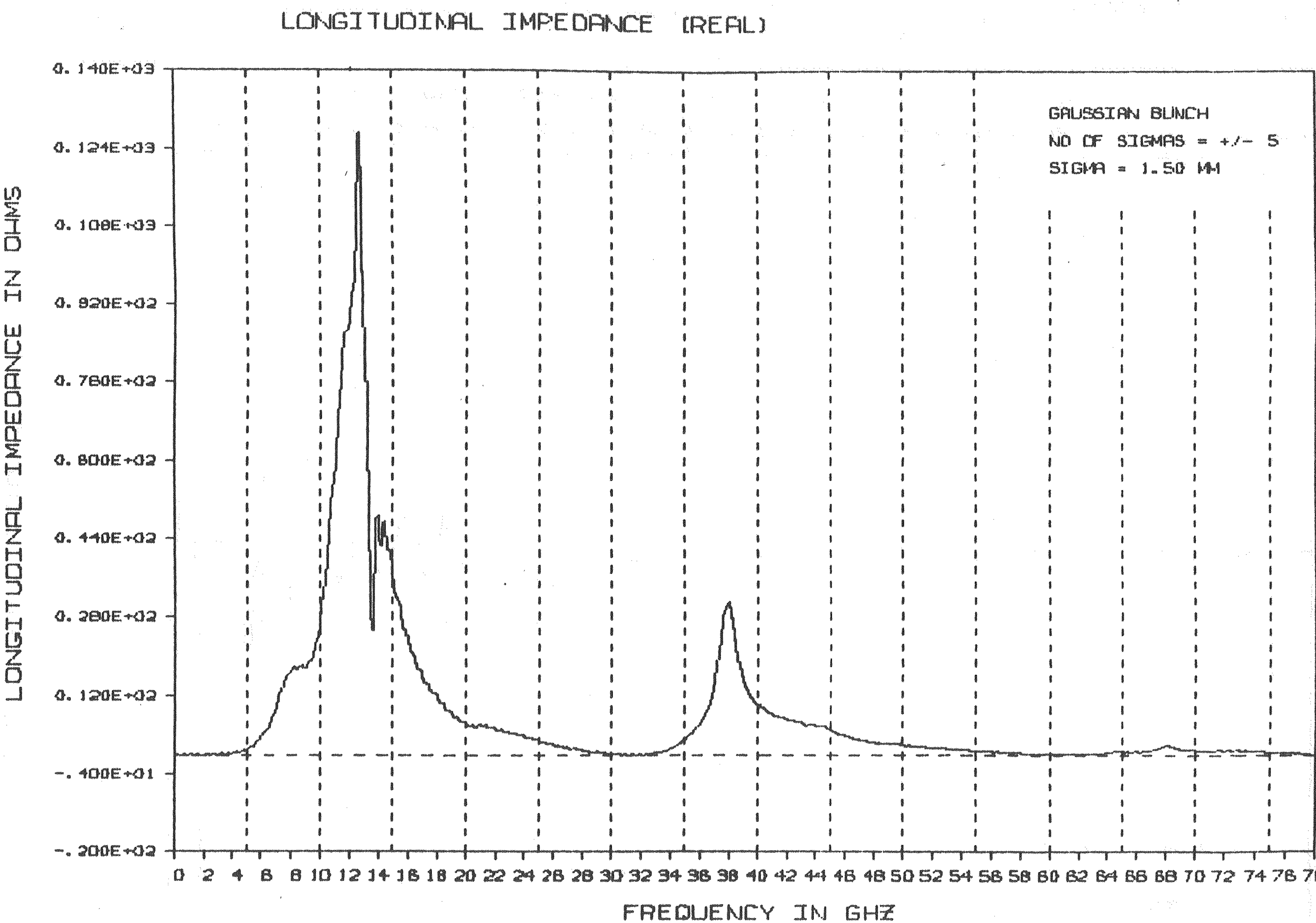
LONGITUDINAL IMPEDANCE (REAL)
0.140E+03
0.124E+03
0.108E+03
0.920E+02
0.760E+02
0.600E+02
0.440E+02
0.280E+02
0.120E+02
-.400E+01
-.200E+02
LONGITUDINAL IMPEDANCE IN OHMS
GAUSSIAN BUNCH
NO OF SIGMAS = +/- 5
SIGMA = 1.50 MM
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50 52 54 56 58 60 62 64 66 68 70 72 74 76 78
FREQUENCY IN GHZ

FIGURE 3

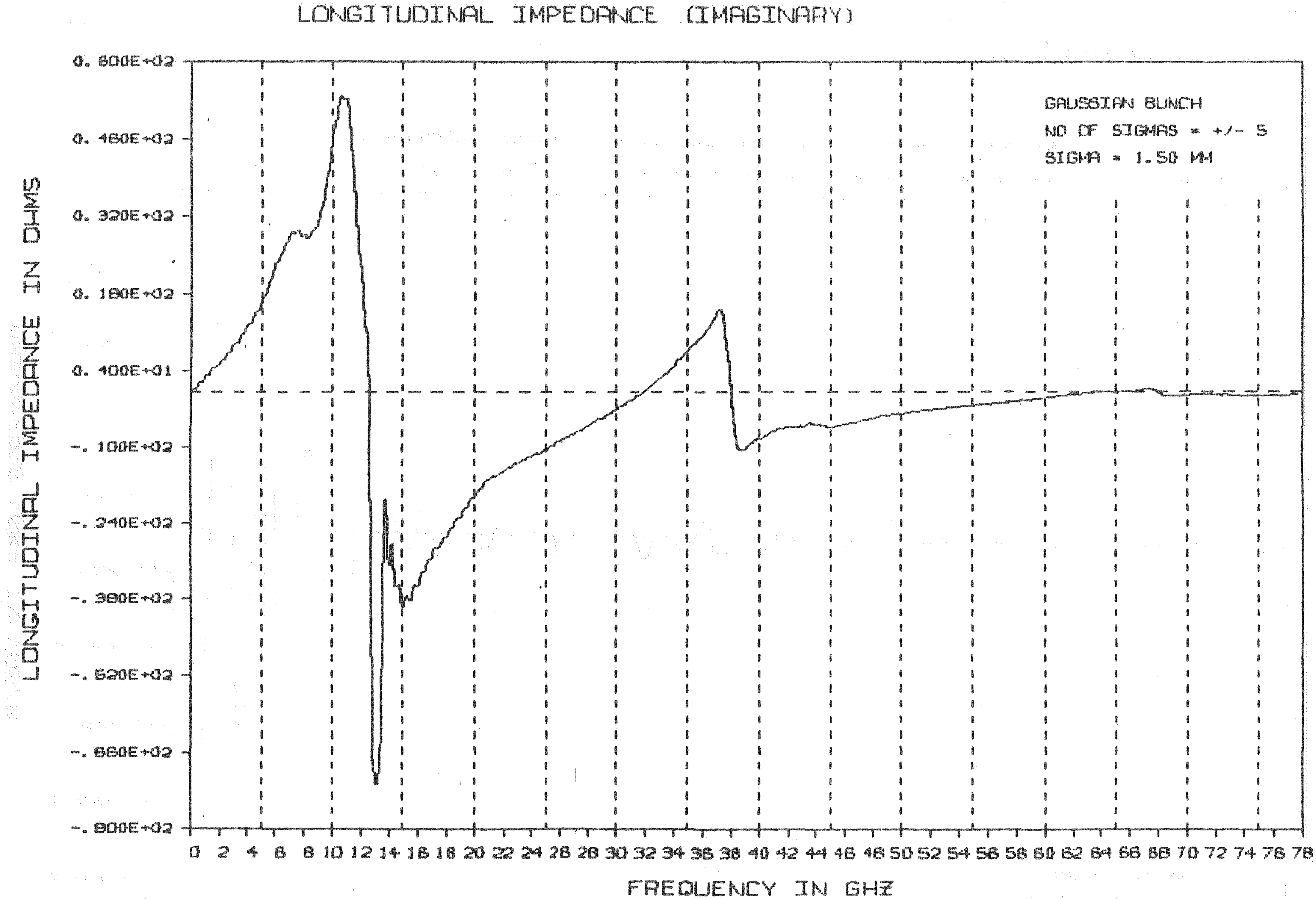
LONGITUDINAL IMPEDANCE (IMAGINARY)
GAUSSIAN BUNCH
NO OF SIGMAS = +/- 5
SIGMA = 1.50 MM
LONGITUDINAL IMPEDANCE IN OHMS
0.600E+02
0.460E+02
0.320E+02
0.180E+02
0.400E+01
-.100E+02
-.240E+02
-.380E+02
-.520E+02
-.660E+02
-.800E+02
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50 52 54 56 58 60 62 64 66 68 70 72 74 76 78
FREQUENCY IN GHZ

FIGURE 4

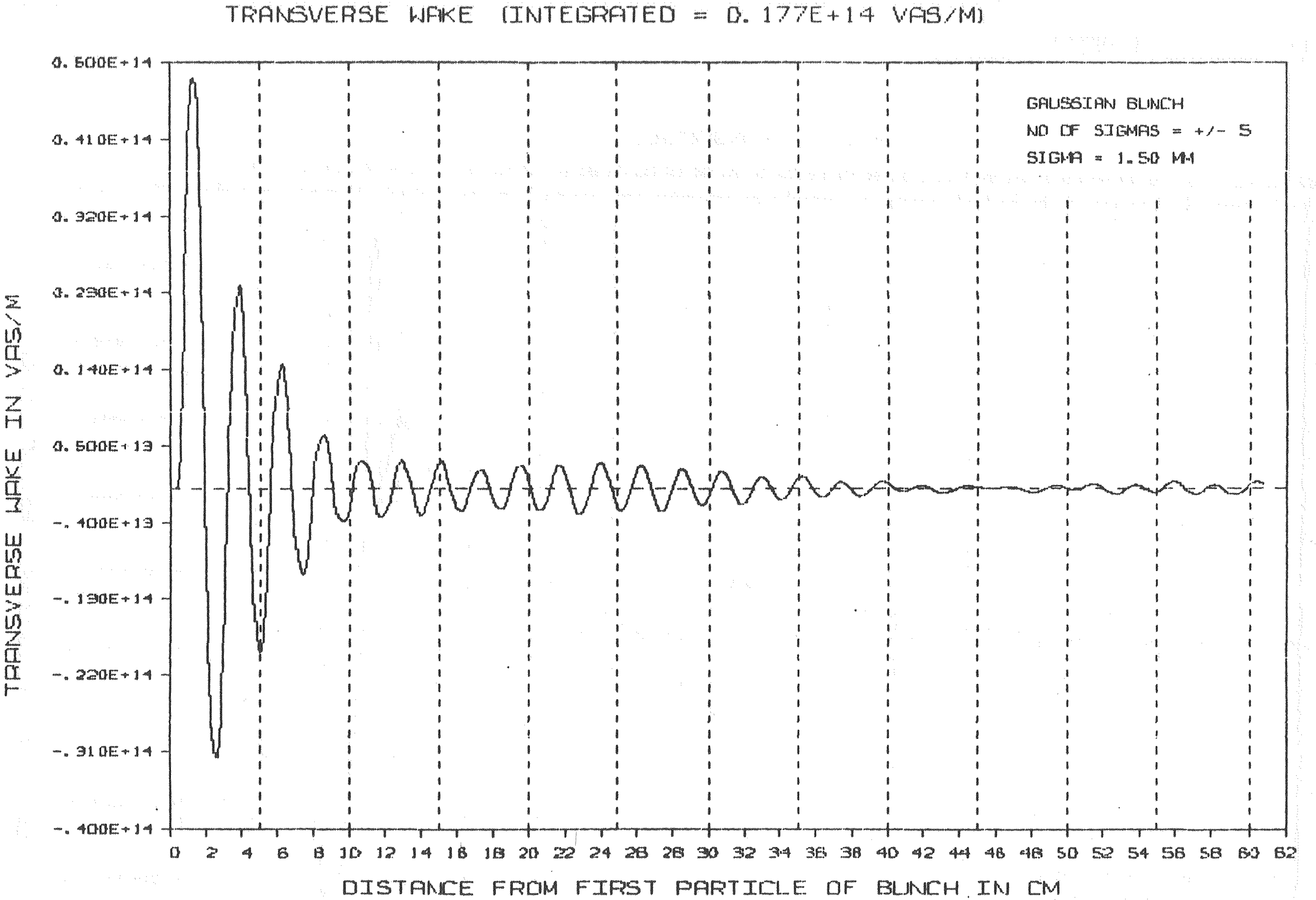
TRANSVERSE WAKE (INTEGRATED = 0.177E+14 VAS/M)
GAUSSIAN BUNCH
NO OF SIGMAS = +/- 5
SIGMA = 1.50 MM
TRANSVERSE WAKE IN VAS/M
0.500E+14
0.410E+14
0.320E+14
0.230E+14
0.140E+14
0.500E+13
-.400E+13
-.130E+14
-.220E+14
-.310E+14
-.400E+14
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50 52 54 56 58 60 62
DISTANCE FROM FIRST PARTICLE OF BUNCH IN CM

FIGURE 5

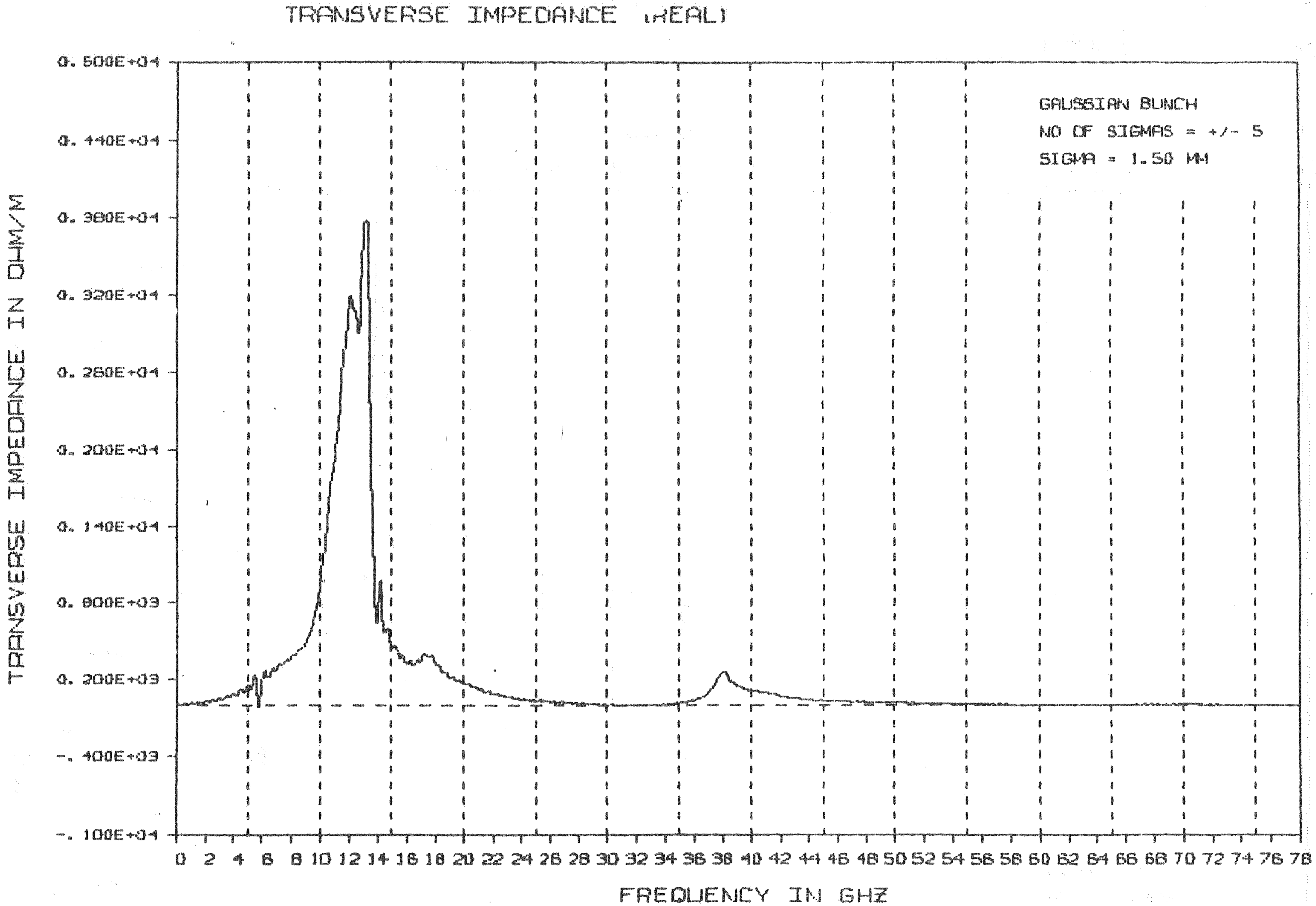
TRANSVERSE IMPEDANCE (REAL)
0.500E+04
0.440E+04
0.380E+04
0.320E+04
0.260E+04
0.200E+04
0.140E+04
0.800E+03
0.200E+03
-.400E+03
-.100E+04
TRANSVERSE IMPEDANCE IN OHM/M
GAUSSIAN BUNCH
NO OF SIGMAS = +/- 5
SIGMA = 1.50 MM
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50 52 54 56 58 60 62 64 66 68 70 72 74 76 78
FREQUENCY IN GHZ

FIGURE 6

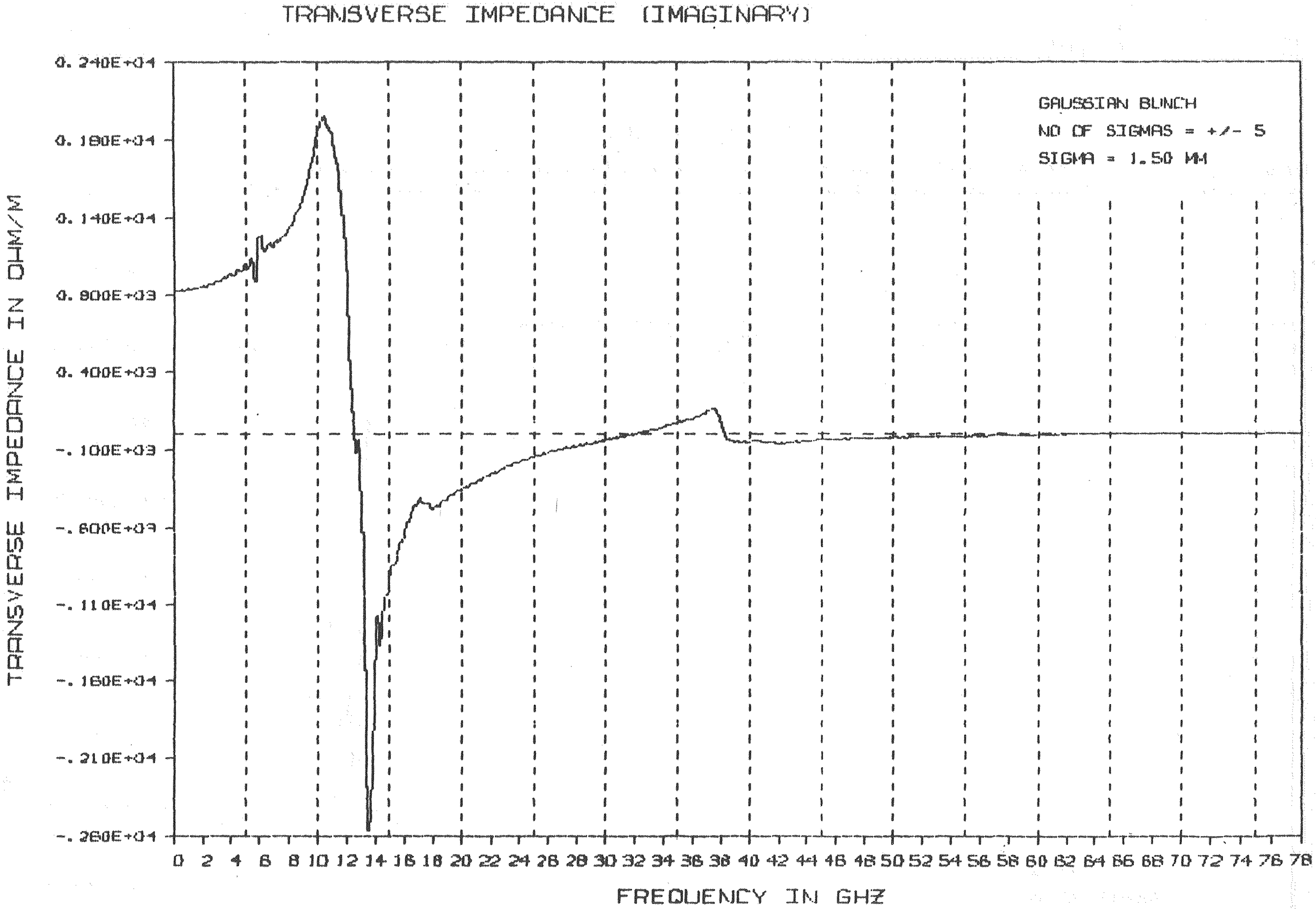
TRANSVERSE IMPEDANCE (IMAGINARY)
GAUSSIAN BUNCH
NO OF SIGMAS = +/- 5
SIGMA = 1.50 MM
TRANSVERSE IMPEDANCE IN OHM/M
0.240E+04
0.180E+04
0.140E+04
0.800E+03
0.400E+03
-.100E+03
-.600E+03
-.110E+04
-.160E+04
-.210E+04
-.260E+04
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50 52 54 56 58 60 62 64 66 68 70 72 74 76 78
FREQUENCY IN GHZ

FIGURE 7