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DIPOLE LENGTH and MULTIPOLES

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There is some confusion about the "proper" relation between dipole length and multipole coefficients. When we have settled upon a particular design there will be no confusion - the coefficients will be averages over the specified length - but in the meantime we must compare the quality of different dipole types, with different length, based largely on measurements of shorter test magnets. Orbit theorists, on the other hand, are proceeding on the basis that beam quality depends on the basic magnet quality and the total magnet length per cell but not on the number of dipoles per cell.

Suppose we have a half-cell of length L, with bend angle θ, containing a dipole "slot" L_m (slot lengths include ends), $L_m/L \sim 0.85$; and we choose m units with $l_m = L_m/m$, and $\vartheta_m = \theta/m$. For one dipole

$$\Delta x' = \vartheta_m \, \Delta B/B = \vartheta_m \sum (f_k a_k + g_k b_k)$$

where $f_k(x,y)$ and $g_k(x,y)$ are the usual multipole polynomials. For N cells

$$\text{rms } \langle \Delta a/a \rangle \sim N^{1/2} m^{1/2} \vartheta_m \beta_0 \, (\text{amp})^{k-1} \langle a_k \rangle$$
$$\sim (L\theta)^{1/2} (\text{amp})^{k-1} (l_m^{1/2} \langle a_k \rangle) \qquad (N\theta = \pi, \;\; \beta_0 = L/.3)$$

which is a measure of the effect of a_k, containing all of the lattice dimensions but not the particular array of dipole errors. The *scaling variable* $(L\vartheta) \sim 1\text{m}$. The combination amplitude (amp) indicates the proper dimensions (cm^{k-1}).

The orbit theorists are assuming that, for the same magnet quality, the rms $\langle a_k \rangle$ for the dipole unit varies with $1/l_m^{1/2}$, with a constant not easily related to measurements on short test magnets. The assumption is reasonable for long dipole units because we expect $\Delta B \sim l_m^{1/2}$ and $B \sim l_m$.

It is usually assumed that the choice of unit length for the dipoles is solely a matter of construction convenience - short dipoles mean too many ends, very long dipoles are difficult to handle etc. - but clearly this requires the square-root scaling. If the coefficients were independent of unit length, as is often assumed, then a dipole twice as long would require smaller coefficients (.7 times) for the same beam quality, and this can be obtained only by increasing the radius (say from 5 to 6cm.), and much of the advantage of longer length would be lost.

Square-root scaling cannot be used for very short lengths. It would imply that short coil measurements would give large random coefficients and this is not the case. Apparently the individual physical errors causing the multipoles are of some length. We expect that short coil meaurements made in different magnets, or very far apart in long magnets, have an rms σ_0 that is the basic quality of construction. On the other hand we expect measurements made close together will be the same. This implies a *correlation* which is *one* for short distance and shifts smoothly to *zero* for long distance. A simple form would be *corr* = $\exp(-x^2/2x_0^2)$, where x_0 is a *correlation length.*

For a dipole of length L, the average σ is given by

$$\sigma^2 = \frac{\sigma_0{}^2}{L^2} \int_0^L \int_{-x}^{L-x} \exp(-u^2/2x_0{}^2)\, du\, dx$$

and the *correlation* between lengths L1 and L2 of the same dipole, spaced 0 - L1 and s - (s+L2), is

$$\frac{\sigma_0{}^2}{L1\ L2\ \sigma_{L1}\sigma_{L2}} \int_0^{L1} \int_{s-x}^{s+L2-x} \exp(-u^2/2x_0{}^2\ du\, dx$$

for long L $\quad \sigma^2 = \sigma_0{}^2\ (2\pi)^{1/2} x_0/L.$

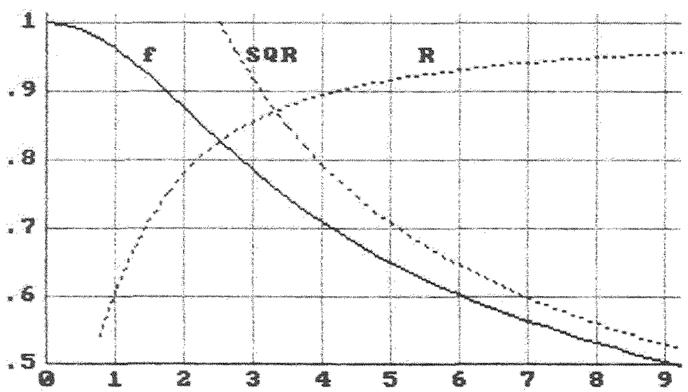

Fig.1

$f = \sigma/\sigma_0$ is shown as a function of L/x_0, also the "long L" formula (SQR). A square-root form does fit the tail adequately. Curve R is the ratio f/SQR and provides a relative measure of the advantage of short dipoles. Consider dipole lengths $4x_0$ and $6x_0$: R is 0.89 and 0.93 which indicates an insignificant advantage. However a longer x_0 such that the lengths are 1 and $1.5x_0$ gives R's .6 and .72 .

The most important problem involving the concept of *correlation length* is the difficulty of extending short test dipole measurements to long lengths. If the long length is $4x_0$, as I suspect it might be, then the short coil measurements should be multiplied by 0.7, but until we build several long dipoles we won't really know what multiplier to use. This problem is exacerbated for iron dominated dipoles where we might expect rather short correlation length. Direct comparison of short coil measurements between radically different magnet styles is not reliable. It is useful to see if the extensive Doubler measurements give any basis for an estimate of x_0.

DOUBLER MEASUREMENTS

This discussion is based on results given in a paper by R. Hanft[1], however the conclusions are not the same. The numerical values are based on production measurements of 656 dipoles, excluding earlier production, and are somewhat better than the rms. values for all the dipoles installed in the Doubler.

For technical reasons the 20' dipoles were measured in three segments using 8' probes. The sements are labelled *upstream, center,* and *downstream* and are effectively 6', 8', and 6' long. The outer measurements also include end structure field errors. The two ends do differ physically, particularly in the direction of winding and in the lead arrangement.

The normal values quoted for the dipoles are a weighted sum of the three segments for each dipole, which are then summarized as a mean and an rms. One can also collect separately summarized values for the three segments, and for the difference between upstream and downstream for each dipole. If the errors are constant throughout the length of the dipole (or a very long x_0) then full-length and segment values will be the same and the end difference will be approx. zero. We will attempt to evaluate x_0 based on the segment values being larger, but our success depends very much on the real end effects being small and independent.

Figure 2 shows the full-length rms values. The three lines are all of the form $(1.6")^k$. The multipoles are split into upper and lower groups which are the right-left symmetry groups. Simple collar or shim errors always generate strong correlations between multipoles and none are observed, so the right-left symmetry must be a correlation between many random errors. Quadrupole errors were removed by bolt adjustment. Before the new support system the skew quad value was on the extension of the upper line, and normal quad was a little below the lower line.

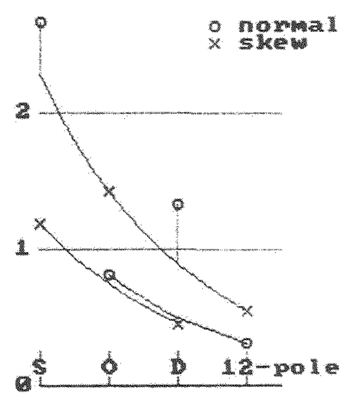

Figure 2

Figure 3 shows the segment rms values. The same lines are used as in figure 2. At first glance the segments are the same as full-length, except for the skew sextupole. A closer inspection shows that almost all values are above the lines and that the order is upstream above downstream above center. Let me try a simple calculation. First I will average the normal and skew values for each multipole in order to remove the right-left symmetry, then I assume a value for x_0 (4m) and find the 6', 8', and 20' averages (.991, .985, and .919 σ_0). I use these values to establish σ_0 for each sector, find a weighted average, and calculate a full-length value. As shown in the table an x_0 = 4m gives a good fit. It is not a sensitive process but is good within a total range of one meter.

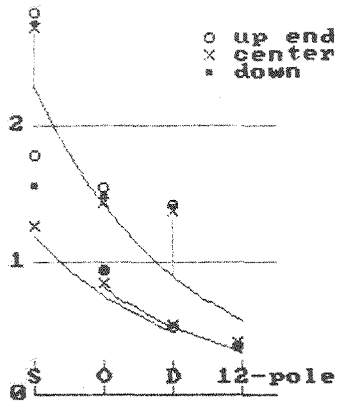

Figure 3

	up	cent	dwn	long	4m calc
sext	2.30	1.98	2.15	1.92	1.96
oct	1.24	1.14	1.20	1.12	1.10
dec	.96	.95	.96	.89	.89
12	.48	.48	.48	.44	.45

The paper of Ray Hanft includes scatter diagrams for various segment combinations. I have converted these into correlation coefficients by comparison with computer generated examples. The skew sextupole correlations are approx. -.3 for upstream-downstream and .6 for center-downstream. A correlation length of 4m would predict .52 and .83. If one simply assumes additional random error at each end sufficient to give the observed increase in the rms then the correlation would be diluted, *but not nearly enough*! It is necessary to assume a strong *anti-correlation* between the additional values at opposite ends of the same dipole in order to approach the observed scatter patterns.

An anti-correlation between opposite end errors would mean that the rms difference between upstream and downstream segments would contain a large end term (the ends add), whereas the full-length rms would contain very little additional end effect (the ends cancel). Both of these effects are observed. In addition one finds upstream and downstream *mean values* of -.43 and +.63 (one expects <.1). There is a skew sextupole end contribution which is as large as 6' of normal dipole error and it is highly structured.

After a little reflection one realizes that there cannot be only a large skew sextupole end error. One would have to build six carefully proportioned windings. There must be other components. There are a number of hints that they do indeed exist, for example there are other significant end-to-end differences in the mean values. In fact one finds patterns which are generally opposite for the "upper" and "lower" groups, indicating a right-left type symmetry. One comes to the reluctant conclusion that there is much more, highly structured, end-error than is initially apparent.

CONCLUSIONS

The best that we can say is that the correlation length for the Doubler dipoles appears to be approximately 4 meters. If this value should apply to the SSC dipoles then square-root scaling is a reasonable approximation. This is a great convenience because it means that the beam quality does not depend on how we split the cell bending into unit dipoles. Should the correlation length be very long then we should probably be using much shorter units.

The relation between short coil measurements on short experimental units and the proper average values for long units remains vague.

All statements concerning magnet quality should contain an explicit reference to magnet length and how the reporter assumes quality and length are related. One must not ignore the basic fact that beam quality independence from unit dipole length presumes $1/l_m^{1/2}$ scaling.

[1] Lengthwise Variation in Field Harmonics of Tevatron Dipoles, R. Hanft, Fermilab.