Conductance Calculation - Molecular Flow, Long Tube of Circular Cross Section

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A formula for the conductance calculation in molecular flow is presented. The formula gives the conductance of pipes which are constant cross section, as well as in which the circumference and cross section of the pipes are increasing or decreasing function of the pipe length.
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A formula for the conductance calculation in molecular flow is presented. The formula gives the conductance of pipes which are constant cross section, as well as in which the circumference and cross section of the pipes are increasing or decreasing function of the pipe length.

References:

1) Knudsen, M., 1910, Ann. Phys. 31, 205; 32, 890; 33, 1435

Distributions:
1. MOLECULAR VELOCITIES

\[ V_{av} = 1.45 \times 10^4 (T / M)^{\frac{1}{2}} \text{ cm/sec.} \]  
\[ \text{at air molecule } M = 29, \text{ temperature } T = 300^\circ \text{K} \]

The average air molecule has a velocity of about \( V_{av} = 4.6 \times 10^4 \text{ cm/sec.} \)

2. MOLECULAR FLOW - LONG TUBE OF CONSTANT CROSS SECTION

\[ q = \Phi BL \quad \text{(Knudsen)} \]  
\[ q \] is the number of molecules striking the wall each second  
\[ \Phi \] is the number of molecules impinging on the unit surface per unit time (\( \Phi = n V_{av} / 4 \) \( n \) is the molecular density)  
\[ B \] is the periphery of the cross section  
\[ L \] is the length of the tube

\[ q' = qmV = BLnV_{av}mv / 4 \]  
\[ q' \] is the momentum transferred by all the molecules to the wall

Assume the number \( N \) of molecules crossing the cross section \( A \) of the pipe per unit time is \( N = Avn \) and the pressure difference \( \Delta P \) achieved corresponds to a force (4)

\[ \Delta F = A\Delta P = AkT\Delta n \]  
\[ k \] is Boltzmann's constant

For equilibrium condition \( q' = \Delta F \), then

\[ 4AkT\Delta n = BLnV_{av}mv \]  
\[ m \] is the mass (of molecule)

From eqs. (4) & (6), we have

\[ N / \Delta n = [4A^2 / (BL)][kT / (mV_{av})] \]
By using definition of the conductance \( N / \Delta n = C \), also
\[
V_w = \left( \frac{2}{\sqrt{\pi}} \right) \left( \frac{2kT}{m1^1} \right)
\]

Therefore, we can obtain
\[
C = \left[ \frac{2A^2}{(BL)\pi kT / (2m)} \right] = \left[ \frac{2A^2}{(BL)\pi R_0 T / (2M)} \right]
\]
(9)

\( R_0 \) is the gas constant (per mole)
\( M \) is molecular weight

The equation (9) contains the assumption that a uniform drift velocity \( v \) is superimposed on the random Maxwell - Boltzmann distribution of the molecules. Knudsen modifies the above equation by multiplying a factor \( \frac{8}{(3\pi)} \) and assume the superimposed drift velocity of a molecule is proportional to its random velocity. The conductance will become
\[
C = \frac{8}{3\sqrt{\pi}} \left( \frac{2kT}{m} \right) \left( \frac{A^2}{BL} \right) = \frac{3.44 \times 10^4}{\sqrt{\pi}} \left( \frac{T}{M} \right) \left( \frac{A^2}{BL} \right)
\]
(10)
in CGS units.

For the uniform circular cross section, \( A = \pi D^2 \), and \( B = \pi D \). The conductance of a tube is
\[
C = 3.81 \left( \frac{T}{M} \right) \left( D^3 / L \right)
\]
(11)

where \( D \) (cm), \( L \) (cm) and \( C \) (liter/sec.)

For air at 20\(^\circ\)C, \( \left( \frac{T}{M} \right) = 3.18 \), therefore
\[
C_{air} = 12.1 D^3 / L
\]
(12)

It can be seen that conductance (molecular flow) is independent of the pressure.

3. MOLECULAR FLOW - TAPERED TUBES (CONICAL SHAPE TYPICALLY)

We can rewrite the equation (10) by using equation (1)
\[
C = \frac{4}{3} V_w K / (BL / A^2)
\]
(13)

\( K \) is the shape factor

If the conductance results from a series connection of conductance of length \( dL \), then
\[
\frac{1}{C} = \left[ \frac{3}{4} / (V_w K) \right] \int_{L}^{L} (B / A^2) \, dL
\]
(14)
or \( C = \frac{4}{3} V_{sv} K / \int_{x}^{L} \left( \frac{B}{A^2} \right) dL \) (15)

For the constant cross section, \( B \) and \( A \) are not functions of \( L \), \( \int_{x}^{L} dL = L \) and 

\text{equation (15) results in equation (13) or (10).} 

It is the short conclusion that equation (15) is a general formula which gives the 

conductance of pipes which are constance cross section, as well as in which \( B \) and 

\( A \) are continuously increasing or decreasing function of \( L \).

For a conical pipe, \( B_{1} \) and \( A_{1} \), represent the parameter of cross section at the 

small end while \( B_{2} \) and \( A_{2} \) are the parameter of cross section at the large end. 

At a distance \( x \), \( B_{x} \) and \( A_{x} \) will be 

\( B_{x} = B_{1} + (B_{2} - B_{1})(x/L) = K_{B}[a_{1} +(a_{2} - a_{1})(x/L)] \)

\( A_{x} = K_{A}[a_{1} +(a_{2} - a_{1})(x/L)]^2 \) (16)

\( a \) is the radius

\( K_{B} \) is the constant ratio between the circumference and radius

\( (K_{B} = 2\pi a / a = 2\pi) \)

\( K_{A} \) is the constant ratio between the cross section area and the square of

the radius \( (K_{A} = \pi a^2 / a^2 = \pi) \)

Substitute eq. (16) into eq. (15)

\[
\int_{x}^{L} \frac{B_{x}}{A_{x}} dL = \frac{K_{B}}{K_{A}^2} \int_{x}^{L} \frac{dx}{[a_{1} + (a_{2} - a_{1})(x/L)]^2} = \left( \frac{K_{B}}{K_{A}^2} \right) \left( \frac{L}{2} \right) \left( \frac{a_{1} + a_{2}}{a_{1}^2 \cdot a_{2}^2} \right)
\]

Therefore the conductance (eq 15) will be given by

\( C = \frac{8}{3} V_{sv} \left( K_{A} / K_{B} \right) \left[ a_{1}^2 \cdot a_{2}^2 / (a_{1} + a_{2}) \right] (K / L) \) (18)

For a circular cross section

\( B = 2\pi r ; A = \pi r^2 ; K_{B} = 2\pi ; K_{A} = \pi ; K_{A}^2 / K_{B} = \pi^2 / (2\pi) = \pi / 2; \) and \( K = 1 \)

Then the conductance of a tapered pipe of circular cross section will become

\( C = (4\pi / 3) \left[ r_{1}^2 r_{2}^2 / (r_{1} + r_{2}) \right] V_{sv} \) (19)

or for \( D = 2r \)
\[ C = 7.62 \left( \frac{\tau}{M} \right)^\frac{4}{3} D_1^2 D_2^2 / \left[ (D_1 + D_2) L \right] \]  

(20)

Where \( D, \ L \) (cm), and \( C \) (liter/sec.), \( D_1 \) and \( D_2 \) being the diameters of the tapered pipe at its end.

By comparing eq (20) with eq (11), it results that the equivalent diameter for a tapered tube is

\[ D_e = \left[ \frac{2D_1^2 D_2^2}{(D_1 + D_2)} \right]^{\frac{1}{3}} \]  

(21)

4. THE CONCLUSION

The equation of the conductance of long tubes for molecular flow with the circular cross section is

\[ C = 3.81 \left( \frac{\tau}{M} \right)^\frac{4}{3} \frac{(D_e)}{L} \]  

or \( C_{pr} = 12.1 \frac{D_e^3}{L} \)

where \( D_e \) (cm), \( L \) (cm), \( C \) (liter/sec.), \( \left( \frac{\tau}{M} \right)^\frac{4}{3} = 3.18 \) at 20°C for air

in case of constant crass section:

\( D_e = D = 2r \)

in case of a tapered tube with a circular crass section

\[ D_e = \left[ \frac{2D_1^2 D_2^2}{(D_1 + D_2)} \right]^{\frac{1}{3}} \]

5. REFERENCE

1) Knudsen, M., 1910, Ann. Phys. 31, 205; 32, 890; 33, 1435