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One More Time About Spatial Resolution in Drift Chambers

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An importance of triggering on the first electron for the ultimate possible spacial resolution in drift chambers is re-analyzed. Strong influence of ionization density on a diffusion contribution is shown. Presented calculations and experimental data confirm that, beyond a certain distance, the triggering on the first electron is of no importance (although, this is crucial for achieving the best resolution in the vicinity of anode wire).

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Summary:

An importance of triggering on the first electron for the ultimate possible spacial resolution in drift chambers is re-analyzed. Strong influence of ionization density on a diffusion contribution is shown. Presented calculations and experimental data confirm that, beyond a certain distance, the triggering on the first electron is of no importance (although, this is crucial for achieving the best resolution in the vicinity of anode wire).

1. Introduction.

Two of the contributions to the spatial resolution of drift chambers (primary ionization and diffusion) are obvious; however, the lack of understanding of the intimate relationship of these factors seems to be rather common. One may find that the diffusion limit is often estimated as σ_x*sqrt(x), where σ_x is a diffusion coefficient, and this is not quite correct. As was shown [1], triggering on the first electron can give a much better resolution in comparison with this naive estimation. On the other hand, the direct comparison with a case of triggering on other than the first electron has yet to be done and, therefore, the essentials of the ultimate possible resolution have remained unclear.

Presented calculations and experimental data are intended to clarify the existing confusion.

First, simple math examples are given to explain how diffusion is intertwined with ionization statistics in terms of resolution, and, then, Monte Carlo simulation shows what one may expect from a realistic gas. Data obtained in a test drift tube by using a laser beam complete the discussion.

2. Math.

Let us assume N electrons at a distance x from a wire, each drifting with a constant velocity v_{dr}. Also, let σ_x denote a diffusion constant. Then, each electron arrives at the anode wire at time T, which has a probability distribution of a Gaussian shape $\rho_0(T)$ with an average $T_0=x/v_{dr}$ and sigma $\sigma_0=\sigma_x/v_{dr}$ (fig.1):

$$\rho_0(T)=\frac{1}{\sqrt{2\pi\sigma_0}}\exp\left(-\frac{(T-T_0)^2}{2\sigma_0^2}\right) \tag{1}$$

In a real drift wire detector the ionization electrons are spread along a particle track (fig.2); however, a few of them come to the wire essentially at the same time so that the model is quite relevant and must reveal the main

features. (Notice that the farther the track is from the wire, the larger the number of electrons arriving at about the same time.)

In the model the first electron out of N would have an obvious distribution $\rho_1(T)$:

$$\rho_1(T) = N \cdot \rho_0(T) \cdot F(T)^{N-1} \tag{2}$$

The term $\rho_0(T)$ is a probability of one particular electron to arrive at time T, the next term $F(T)^{N-1}$ gives a probability for the other N-1 electrons coming later, and multiplication by N reflects that any electron out of N could be the first one. This distribution is shown in fig.3 and has been known already [2]. It is obviously narrower than the original Gaussian for a single electron. Also, one can notice that the distribution's peak is shifted towards earlier times. The RMS improvement with N is shown in fig.4 and has a simple asymptotic expression [1]:

$$RMS_1(N) \sim \sigma_0 \cdot \frac{\pi}{2\sqrt{3\ln N}} \tag{3}$$

One can see from fig.4 that for N~10 or larger the resolution is about two times better and there is almost no further improvement due to the slow sqrt(log) asymptotic.

It should be noted here that operation in limited streamer mode [3] provides a natural way to have a signal triggered on the first electron. On the other hand, it should be difficult to trigger on the first electron in proportional mode due to the small signal and large amplitude fluctuations in an avalanche.

Before analyzing the later contribution, let us look at what happens if the electronics is triggered on exactly the second, third and so forth electron. The arrival time distribution $\rho_k(T)$ for the k-th electron out of N is as follows:

$$\rho_k(T) = N \cdot \rho_0(T) \cdot C(N-1, k-1) \cdot (1 - F(T))^{k-1} \cdot F(T)^{N-k} \tag{4}$$

In this expression N again means that any electron could be the k-th electron; C(N-1,k-1) gives the number of possible combinations for (k-1) electrons out of the rest (N-1) electrons which have to come before the k-th electron, the probability for each of them being equal to 1-F(T). The remaining (N-k) electrons have to come later: $F(T)^{N-k}$.

The distributions for different k's are shown in fig.5. One can see that the gain in resolution could be even larger if it were possible to tune a threshold on a particular arriving electron, the best result being for k=N/2. Is

it possible to tune a threshold, say, on the third electron in proportional mode? No. What prevents triggering by exactly the k-th electron are large amplitude fluctuations in the avalanche corresponding to one electron. That means that if one has a threshold corresponding, for instance, to a tripled average amplitude for a single electron avalanche, the discriminator might be triggered on the second and the forth electrons as well as on the third one. Thus, the final distribution will be a superposition of all distributions $\rho_k(T)$ with weights depending on avalanche amplitude fluctuations. Since each of the distributions has a different systematic shift of the mean, the resulting distribution spread depends very much on amplitude fluctuations.

There is a number of papers devoted to theoretical and empirical analysis of fluctuations corresponding to a single electron avalanche (see, e.g., [4] and references there in). For example, one of the first estimations [5], being in reasonable agreement with an experiment [6], results in exponential amplitude fluctuations:

$$\rho(A) = \frac{1}{A_0}\exp\left(-\frac{A}{A_0}\right) \tag{5}$$

This simple distribution can be easily implemented into analytical calculations.

Assuming exponential amplitude fluctuations (5) and a threshold level equal to a, one can immediately obtain the probability to reach this threshold with the k-th electron (see Appendix A):

$$p_k(a) = \frac{1}{(k-1)!}\left(\frac{a}{A_0}\right)^{k-1} \cdot \exp\left(-\frac{a}{A_0}\right) \tag{6}$$

This in turn means that the resulting time distribution $\rho_a(T)$ will be:

$$\rho_a(T) = \sum_{k=1}^{N} p_k(a) \cdot \rho_k(T) \tag{7}$$

The distributions for different N and a are shown in fig.6(a,b). Note that a=0 corresponds to triggering on the first electron. One can notice that the time distribution gets slightly wider when a increases; however, with further increasing of a (especially for big N) there is a slight improvement in time resolution (the distribution finally becomes narrower).

Thus, although triggering on the first electron does make resolution better than the *naive* estimation mentioned earlier, the resolution is not

much better when compared to the case of the alternative high threshold option (and could be even slightly worse).

To complete the comparison, one needs to look at a track very close to the wire. There is obviously no diffusion in this case. With the average distance between clusters equal to λ, the resolution for triggering on the first electron will be $RMS_1=\lambda/2$. If one could trigger exactly on the k-th electron, the resolution would be equal to (see Appendix B):

$$RMS_k = \lambda / 2 \cdot \sqrt{k} \tag{8}$$

To get the resolution for different thresholds in proportional mode, one should use an equation (7). In this case, the resolution gets even worse (Appendix C):

$$RMS_a = \lambda / 2 \cdot \sqrt{1 + 2\frac{a}{A_0}} \tag{9}$$

There is no doubt that triggering on the first electron results in the best possible resolution in this region and any increase of threshold leads to resolution degradation. For example, for threshold equal to A_0 (mean amplitude for a single electron avalanche) the resolution is $\sqrt{3}$ times worse than the one for the case of triggering by always the first electron.

3. Monte Carlo Calculations.

To understand what one might expect from a real detector, we must implement in calculations the fact that all ionization electrons are spread along a particle track. We ran a simple Monte Carlo assuming a Poisson distributed spread of clusters along the track with a mean distance between them λ, a constant drift velocity and σ_x independent of electric field. The detector geometry has been shown in fig.2 (25 mm wide square tube with an anode wire in the center). Gas mixture of $Ar+C_4H_{10}=1+3$ was assumed, which gave the parameter values λ~250 μm [7] and $\sigma_x \sim 100\,\mu m/\sqrt{cm}$ [8].

Fig.7 shows resolution versus distance for different threshold levels.

A curve corresponding to triggering on the first electron has a rather flat behavior, a fact which is quite often observed. This fact is a manifestation of the intimate connection between contributions of diffusion and primary statistics. A priori one might expect increasing degradation of the resolution with distance due to increasing diffusion. Indeed, the diffusion does increase with distance, but so does the number of electrons arriving at

about the same time. This number results in the statistical suppression of the diffusion (see fig.4) so that the resolution does not degenerate so quickly. Finally, this suppression saturates (fig.4), and the resolution gets worse; however, it is about a factor of 2 better than the naive estimation.

Another -- now somewhat expected -- feature is that the resolution is almost independent of threshold at large distances.

One may ask an interesting question. What kind of resolution improvement should we expect for a chamber working at high pressure? Far away from the wire σ_x gets better as $p^{-0.5}$, plus we should unticipate that this diffusion contribution is additionally suppressed by increasing primary ionization statistics. To check this, we looked at the resolution at different distances from a wire as a function of pressure. Results shown in fig.8 can be fitted by a surprisingly simple function: $\sigma \sim \sigma_0/p^{\alpha}$. At x=0 α is obviously equal to 1; however, even far away from the wire, where the resolution is thought to be determined by diffusion only, α is still larger than 0.5 which means that the role of primary ionization is not diminished yet. This was also observed experimentally [9].

Fig.9 dramatizes the effects of ionization density on the diffusion suppression. Assuming absence of other contributions such as misalignment, δ-electrons, electronics jitter and etc., the chamber at high pressure (i.e. when ionization density is really high) would give a fantastic resolution. And the best diffusion suppression would be at higher thresholds (compare with Fig.6(b)).

4. Experimental Data.

The data obtained with a laser beam [10] can be used to verify the ideas presented above. A laser beam focused and tuned to produce the same amount of electron-ion pairs as a minimum ionizing particle was used to ionize the gas in a square one inch tube with a 100 μm anode wire. Data were obtained in limited streamer mode at different high voltages beginning from the singles rate plateau knee. A few millivolt discriminator threshold (at 50 Ω) was high enough so that only a streamer signal could trigger the electronics and low enough to eliminate fluctuations connected with streamer amplitude variations. Fig.10 presents the data. At the knee of a singles rate plateau a streamer obviously is not triggered by the first avalanche yet, and one may see a dramatic degradation of the space resolution in the vicinity of the anode wire. The resolution in this region improves when the high voltage increases, reaching the limit apparently connected with primary ionization statistics. On the other hand, the resolution at large distances does not improve with high voltage increase, so

triggering on the first electron in this region is not crucial for achieving the ultimate resolution.

It is worthwhile mentioning that the resolution obtained with a laser beam could be too optimistic an estimation of drift chamber performance due to a very uniform distribution of ionization (no clustering) and due to the absence of δ-electrons.

The resolution obtained with a prototype consisted of similar square tubes in a muon beam is shown in fig.11 (detailes can be found elsewhere [10]). The major features (resolutions near the wire and far away and, also, very slow worsenning of the resolution with increasing distance) coincide very well with the predictions (compare with fig.7).

5. Conclusion.

The calculations and data presented above result in the conclusion that the triggering of the electronics for drift readout chambers on the first arriving electron is really essential *only* for tracks going nearby the anode wire. If a track is away from the anode wire, the triggering on the first electron is of no importance.

It has been also shown why the resolution should improve with pressure increase faster than $p^{-0.5}$ even far away from a wire.

Acknowledgments.

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Appendix A. Calculation of the probability to reach a certain threshold with the k-th electron.

The eq.(6) can be verified by induction.

The equation is obviously correct for k=1; probability that the first electron will give an amplitude exceeding a threshold a is as follows:

$$p_1(a) = \int_a^{+\infty} \rho(A)dA = \int_a^{+\infty} \frac{1}{A_0} e^{-\frac{A}{A_0}} dA = e^{-\frac{a}{A_0}}.$$

Assuming that eq.(6) is valid for the k-th electron, one can easily see that it is correct for the (k+1)-th electron as well.

Indeed, if the first electron gives an amplitude a' ($0 \le a' < a$; the probability is $\rho(a')da'$), then the rest k electrons should add a charge larger than $a - a'$ (the probability is $p_k(a - a')$) so that the sum of the two amplitudes would exceed the threshold.

Thus,

$$p_{k+1}(a) = \int_0^a \rho(a') \cdot da' \cdot p_k(a - a') = \ldots = \frac{1}{k!}\left(\frac{a}{A_0}\right)^k e^{-\frac{a}{A_0}},$$

which proves eq.(6).

Appendix B. Calculation of RMS of the distribution in the case when the track goes in the vicinity of wire and electronics is triggered on exactly the k-th electron.

The probability that the k-th electron is at distance x from the wire (assuming that the track goes through the wire) is as follows:

$$\rho_k(x)dx = P_{k-1}(x)\cdot\frac{2dx}{\lambda}\ , \tag{B1}$$

where $P_{k-1}(x)$ is a probability to get k-1 electrons within ±x around the wire (the Poison distribution):

$$P_{k-1}(x) = \frac{1}{(k-1)!}\left(\frac{2x}{\lambda}\right)^{k-1} e^{-\frac{2x}{\lambda}}\ . \tag{B2}$$

One can easily obtain that RMS of the distribution given by eq.(B1) is

$$RMS_k = \frac{\lambda}{2}\sqrt{k}. \tag{B3}$$

Appendix C. Calculation of the resolution (RMS) in the case when the track goes in the vicinity of wire and electronics is tuned on a fixed threshold *a*.

The probability to reach a threshold *a* with the k-th electron $p_k(a)$ is given by eq.(6). The k-th electron in turn has an arrival time distribution $\rho_k(x)$ as given by eq.(B1). Thus, the combination of the both gives the distribution corresponding to the case of a fixed threshold (see eq.(7)):

$$\rho_a(x) = \sum_{k=1}^{+\infty} p_k(a)\cdot\rho_k(x) = \sum_{k=0}^{+\infty}\frac{2}{\lambda}\frac{1}{k!}\left(\frac{2x}{\lambda}\right)^k e^{-\frac{2x}{\lambda}}\cdot\frac{1}{k!}\left(\frac{a}{A_0}\right)^k e^{-\frac{a}{A_0}}. \tag{C1}$$

The rest calculations of the RMS are trivial:

$$RMS = \sqrt{\overline{x^2} - \overline{x}^2} = \frac{\lambda}{2}\left(1 + 2\frac{a}{A_0}\right). \tag{C2}$$

References.

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2. e.g., A.H.Walenta, in Proceedings of Summer Institute on Particle Physics, July 1983 (SLAC-267, 1984), p.323.
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9. G.D.Alekseev, private communication.
10. A.Korytov et al., to be published (some of the results can be found in Yu.Bonushkin et al., NIM A315 (1992) 55; R.Sumner et al., GEM Internal Note TN-92-203, SSC Lab, 1992; A.Korytov, GEM Internal Note TN-93-282, SSC Lab, 1993, p. 243)

Figure Captions.

Fig.1. A Gaussian resolution function for a single electron (abscissa in σ_0-units).

Fig.2. A drift cell with a track in it. Note that for a track 1 cm away from the anode wire and assuming mean distance between clusters λ=250 μm, electrons from about 8 clusters are at about the same distance from the wire (within 50 μm).

Fig.3. Arrival time distribution for the first electron out of N (abscissa in σ_0-units).

Fig.4. Resolution improvement with respect to a single electron resolution versus N (triggering on the first electron).

Fig.5. Arrival time distribution for the k-th electron out of N=16 (abscissa in σ_0-units).

Fig.6. Trigger time distribution in proportional mode and for a discriminator set at different thresholds (in units of an average amplitude of a single electron avalanche): (a) for N=8, (b) for N=32.

Fig.7. Spatial resolution versus distance for different threshold levels (in units of an average amplitude of a single electron avalanche). Monte Carlo.

Fig.8. Resolution vs pressure for different distances between track and anode wire. Triggering on the first electron. Monte Carlo.

Fig.9. Modification of resolution presented in Fig.7 for the high pressure condition. Monte Carlo.

Fig.10. Resolution versus distance for a drift tube operated in streamer mode for different high voltages. Ionization was produced by a laser beam.

Fig.11. Spatial resolution obtained with a prototype based on drift tubes and operated in limited streamer mode. Muon beam data.

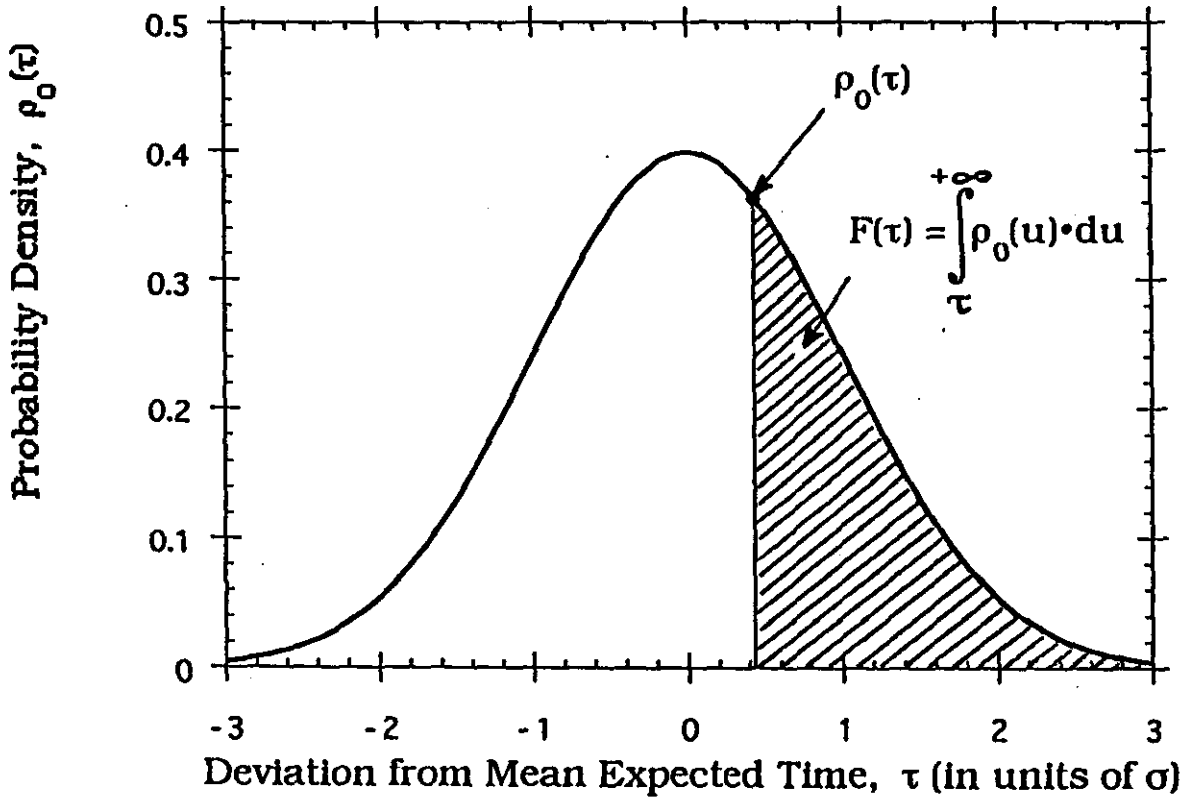
0.5
0.4
0.3
0.2
0.1
0
Probability Density, $\rho_0(\tau)$
$\rho_0(\tau)$
$F(\tau) = \int_{\tau}^{+\infty} \rho_0(u) \cdot du$
-3
-2
-1
0
1
2
3
Deviation from Mean Expected Time, τ (in units of σ)

Fig. 1.

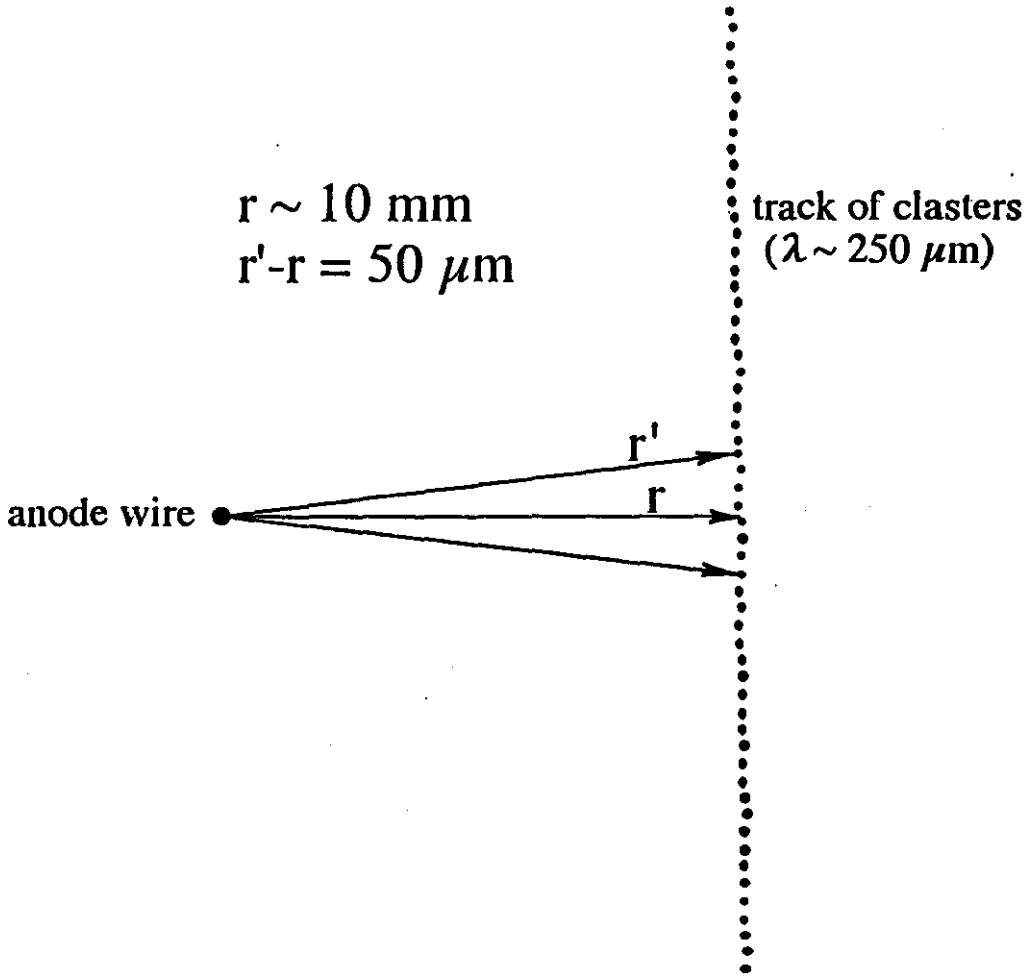

Fig. 2.

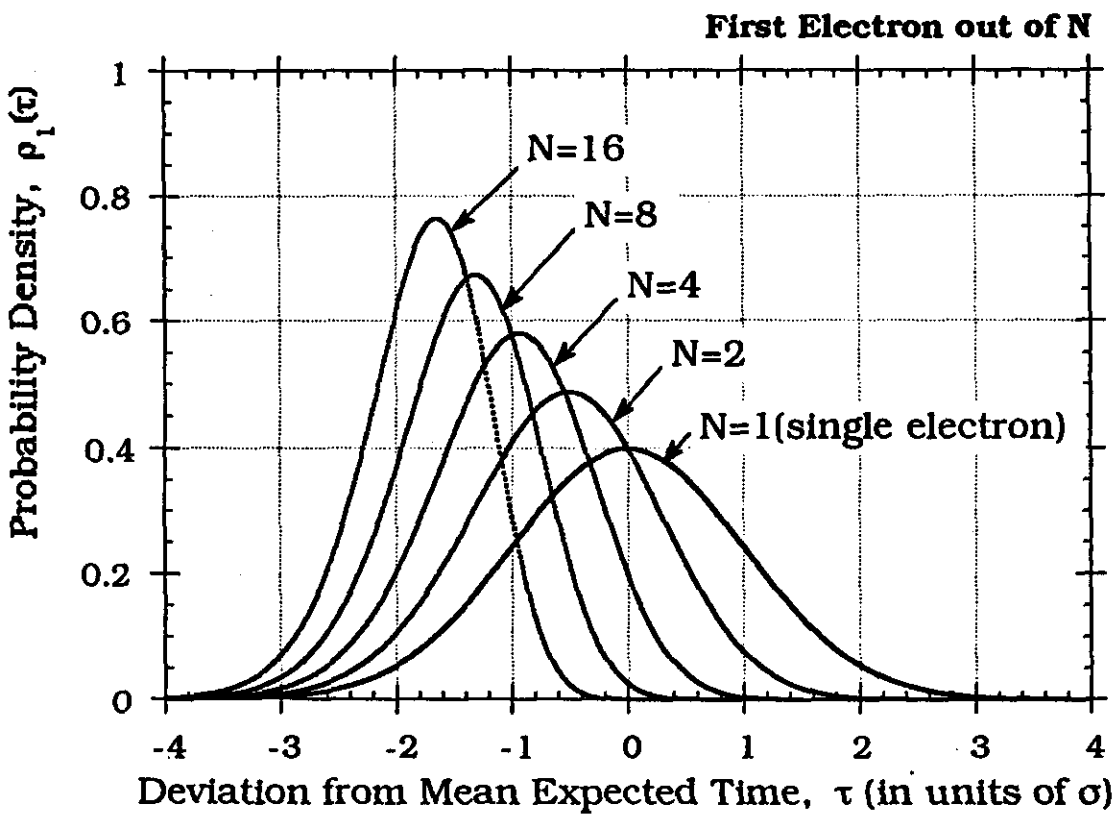
First Electron out of N
Probability Density, $\rho_1(\tau)$
1
0.8
0.6
0.4
0.2
0
N=16
N=8
N=4
N=2
N=1(single electron)
-4
-3
-2
-1
0
1
2
3
4
Deviation from Mean Expected Time, τ (in units of σ)

Fig.3.

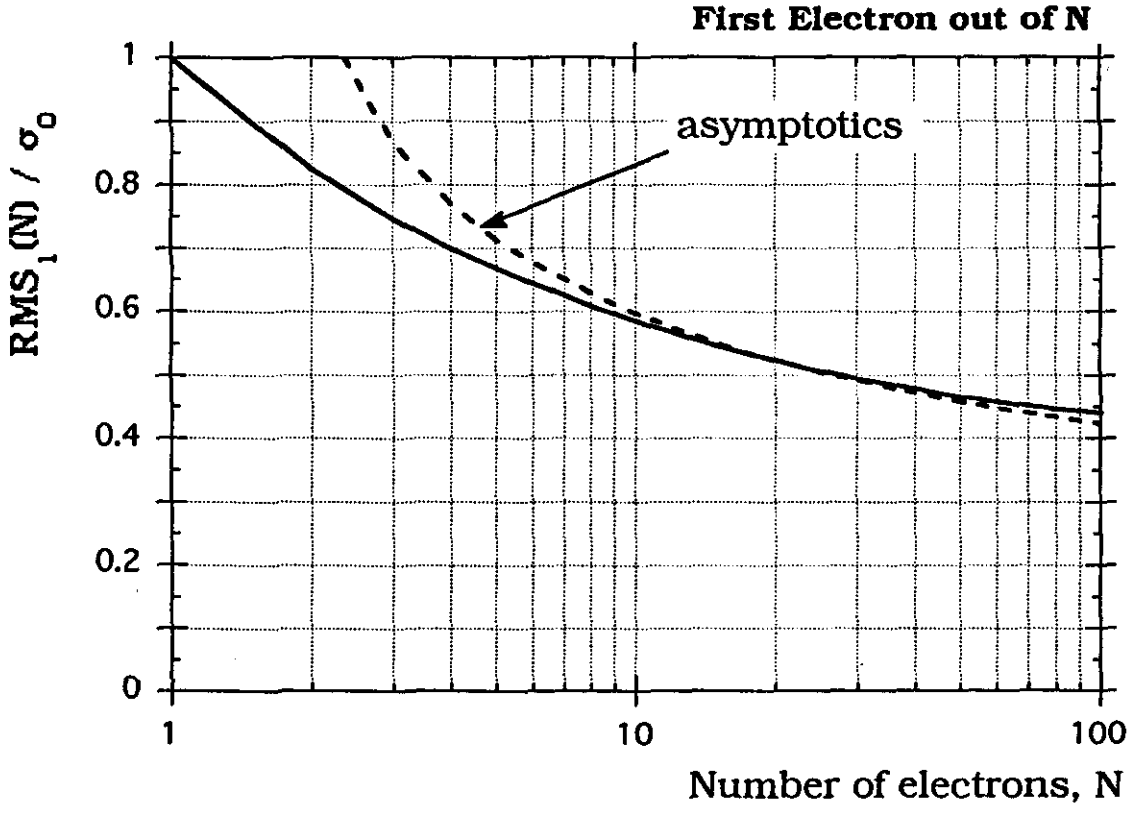
First Electron out of N
asymptotics
RMS$_1$(N) / σ_0
1
0.8
0.6
0.4
0.2
0
1
10
100
Number of electrons, N

Fig. 4.

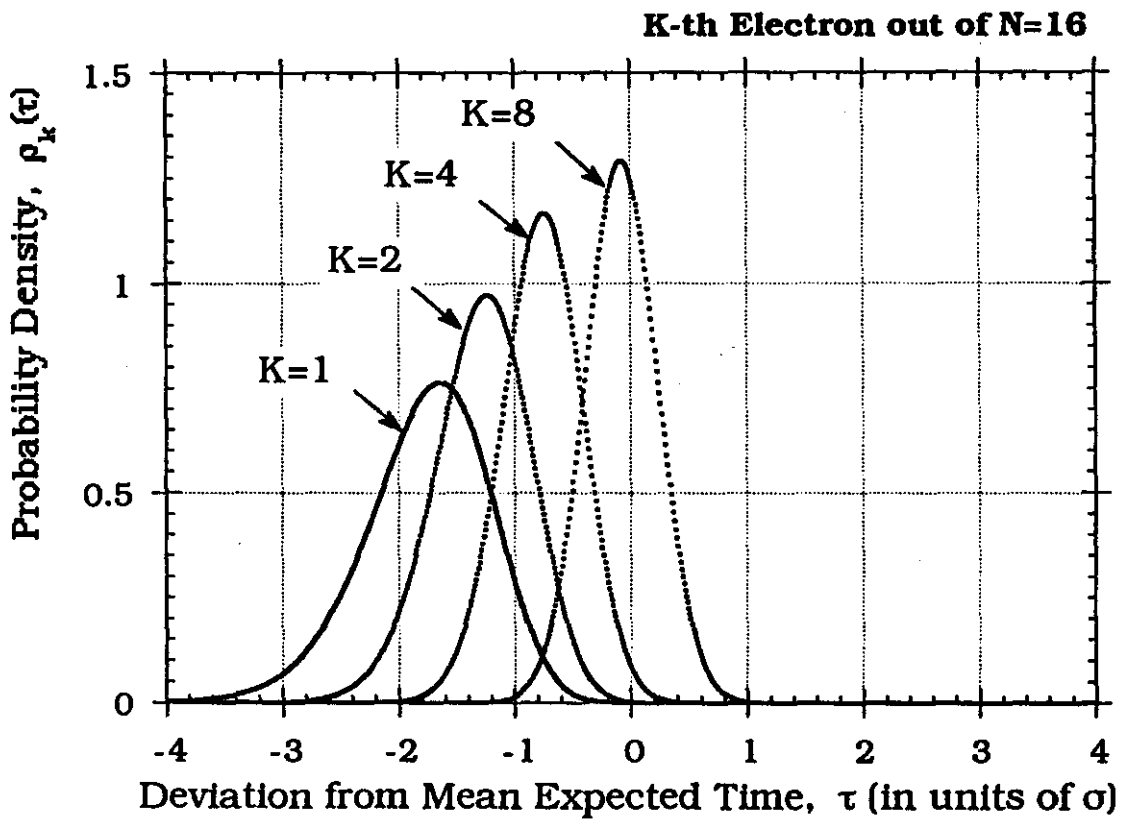
K-th Electron out of N=16
Probability Density, $\rho_k(\tau)$
1.5
1
0.5
0
K=8
K=4
K=2
K=1
-4
-3
-2
-1
0
1
2
3
4
Deviation from Mean Expected Time, τ (in units of σ)

Fig. 5.

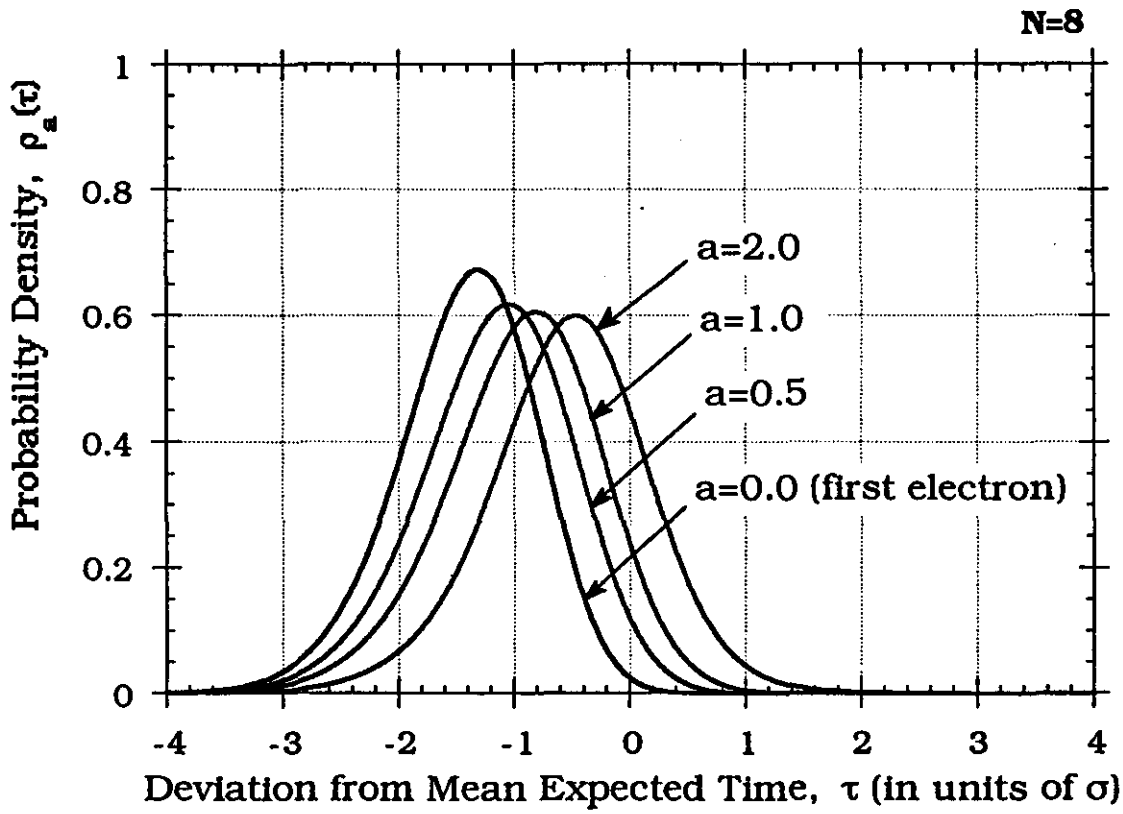
N=8
Probability Density, $\rho_a(\tau)$
1
0.8
0.6
0.4
0.2
0
-4
-3
-2
-1
0
1
2
3
4
Deviation from Mean Expected Time, τ (in units of σ)
a=2.0
a=1.0
a=0.5
a=0.0 (first electron)

Fig. 6(a)

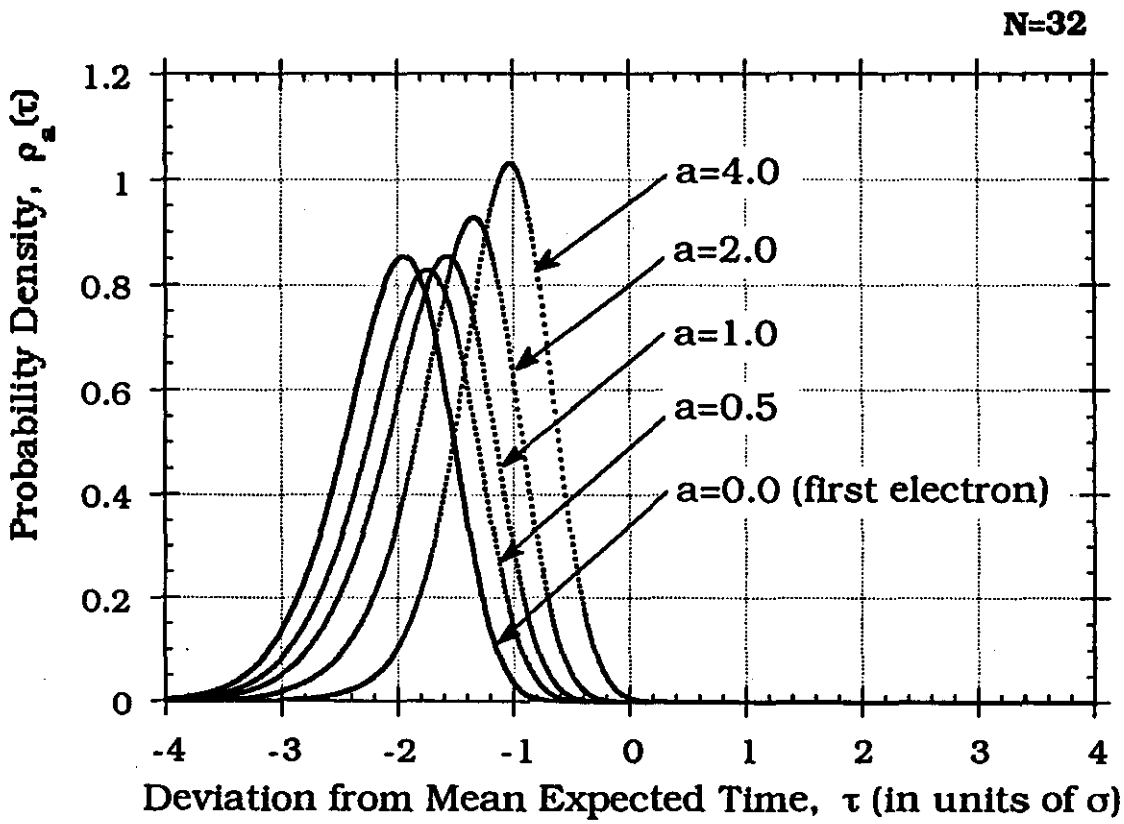
N=32
Probability Density, $\rho_a(\tau)$
1.2
1
0.8
0.6
0.4
0.2
0
-4
-3
-2
-1
0
1
2
3
4
Deviation from Mean Expected Time, τ (in units of σ)
a=4.0
a=2.0
a=1.0
a=0.5
a=0.0 (first electron)

Fig 6(b)

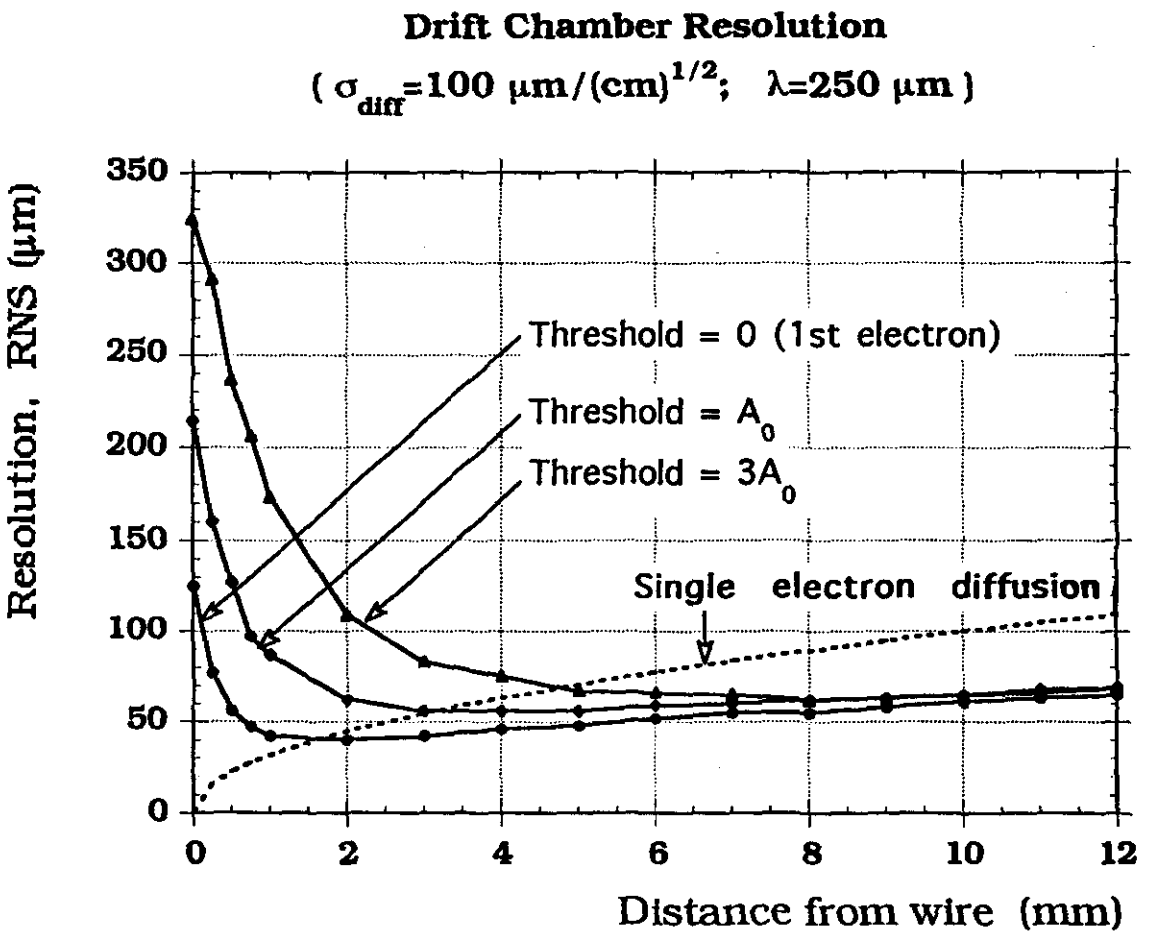
Drift Chamber Resolution
(σ_{diff}=100 μm/(cm)$^{1/2}$; λ=250 μm)
Resolution, RNS (μm)
350
300
250
200
150
100
50
0
0
2
4
6
8
10
12
Distance from wire (mm)
Threshold = 0 (1st electron)
Threshold = A_0
Threshold = $3A_0$
Single electron diffusion

Fig. 7.

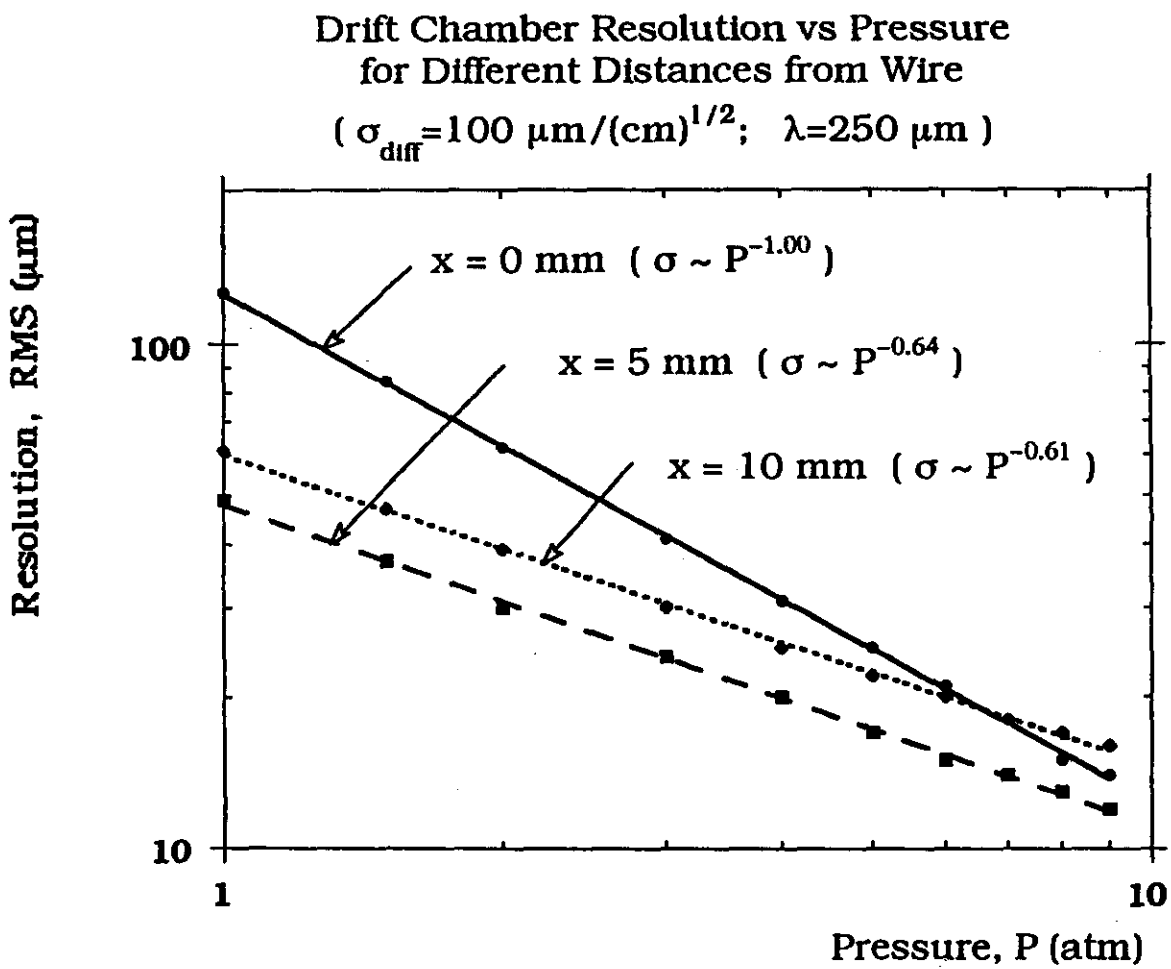
Drift Chamber Resolution vs Pressure
for Different Distances from Wire
(σ_{diff}=100 μm/(cm)$^{1/2}$; λ=250 μm)
x = 0 mm (σ ~ $P^{-1.00}$)
x = 5 mm (σ ~ $P^{-0.64}$)
x = 10 mm (σ ~ $P^{-0.61}$)
Resolution, RMS (μm)
100
10
1
10
Pressure, P (atm)

Fig. 8.

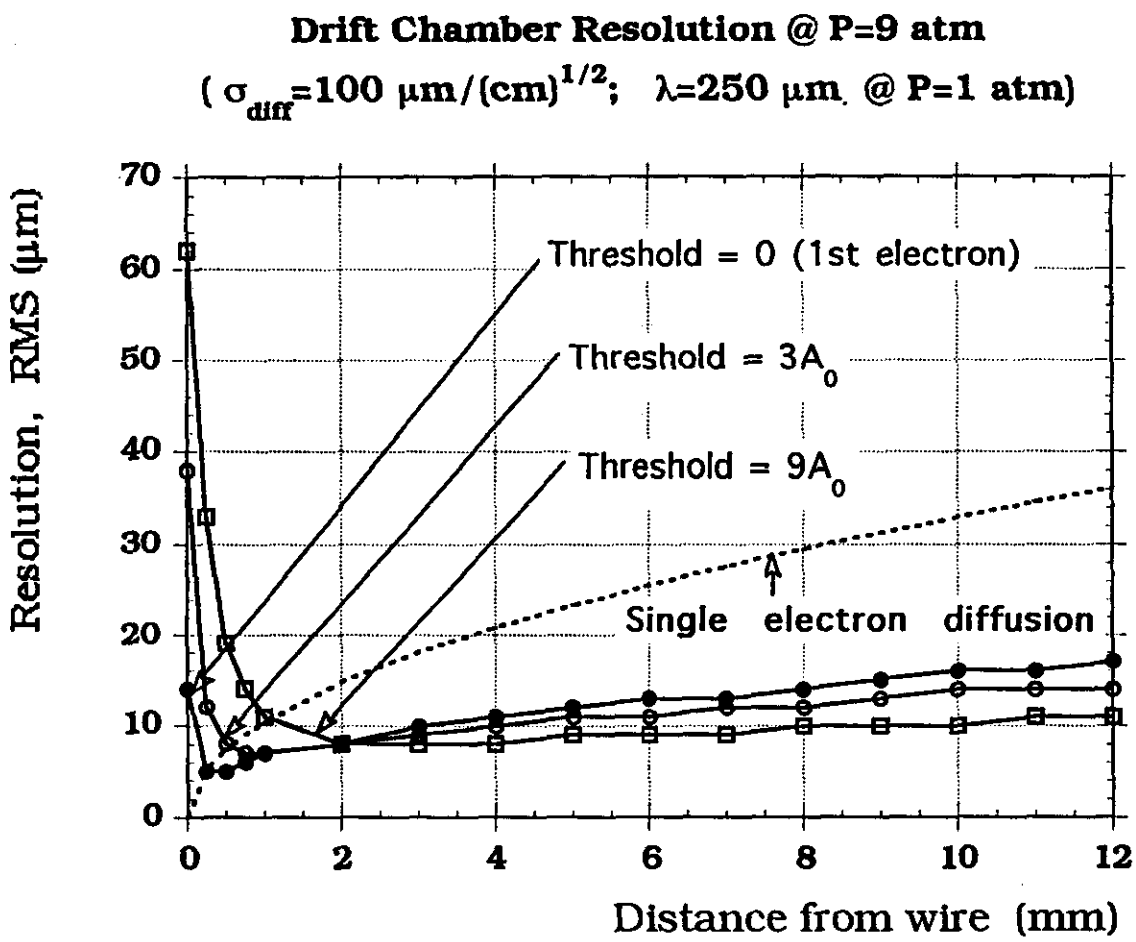
Drift Chamber Resolution @ P=9 atm
(σdiff=100 μm/(cm)1/2; λ=250 μm @ P=1 atm)
Threshold = 0 (1st electron)
Threshold = 3A0
Threshold = 9A0
Single electron diffusion
Resolution, RMS (μm)
Distance from wire (mm)
0
10
20
30
40
50
60
70
0
2
4
6
8
10
12

Fig. 9.

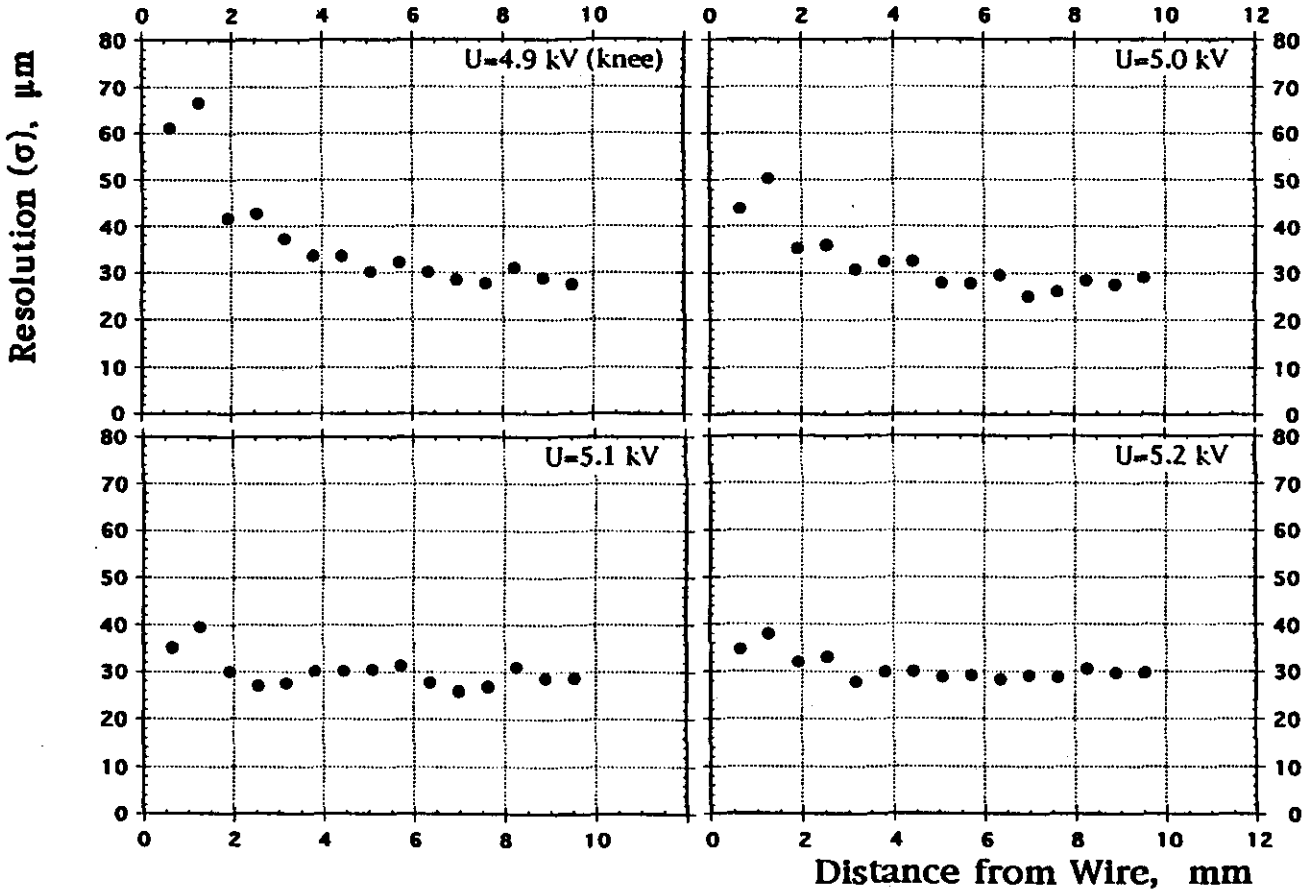
Resolution (σ), μm
U=4.9 kV (knee)
U=5.0 kV
U=5.1 kV
U=5.2 kV
Distance from Wire, mm

Fig. 1b

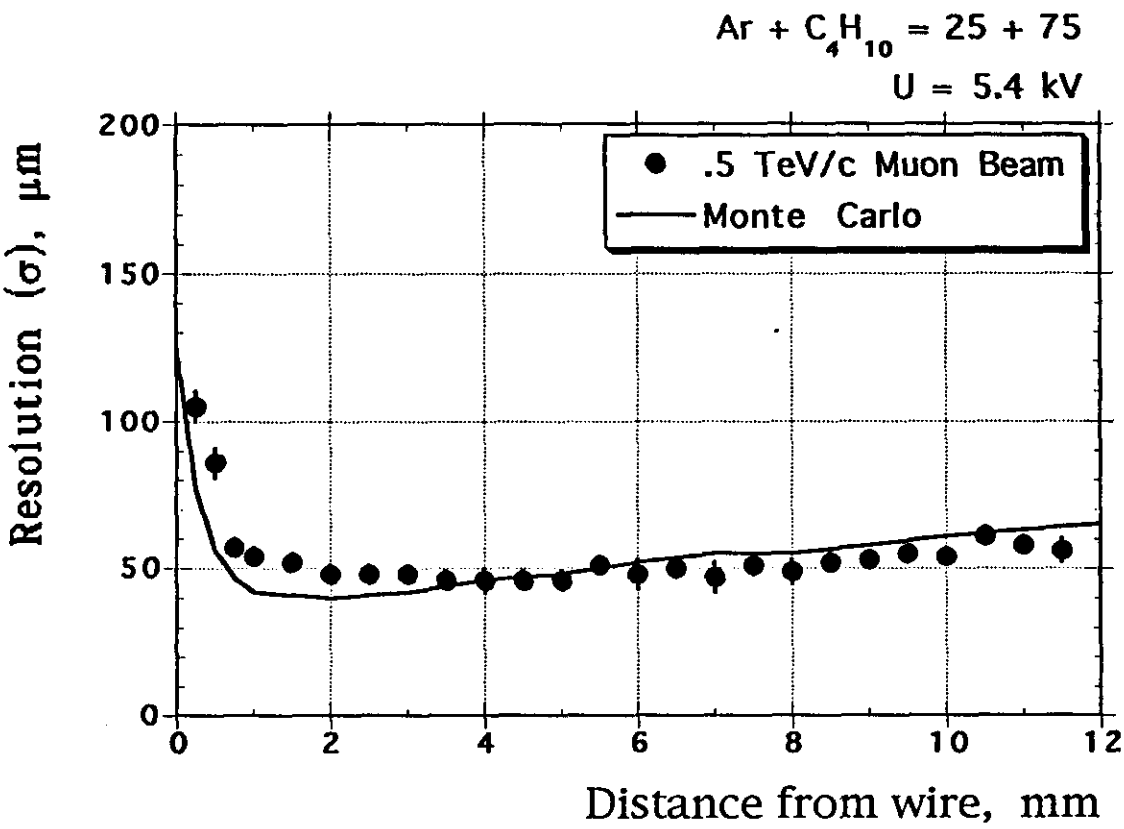
Ar + C4H10 = 25 + 75
U = 5.4 kV
.5 TeV/c Muon Beam
Monte Carlo
Resolution (σ), μm
200
150
100
50
0
0
2
4
6
8
10
12
Distance from wire, mm

Fig. 11