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Calculation Method for GEM Vessel Random Vibration Study

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Abstract:

Computer analysis of the GEM vacuum vessel subject to random vibration from the experimental hall floor has been accomplished. This paper documents the method used. The results were reported in a separated document entitled "Vessel Random Vibration Analysis", GEM-TN-92-219.

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To: R.H. Sawicki

From: F.A. House

Subject: Calculation method for GEM vessel random vibration study

Computer analysis of the GEM vacuum vessel subject to random vibration from the experimental hall floor has been accomplished. This paper documents the method used. Complete results will be reported separately. The procedure is as follows: A finite element computer model was constructed of the vacuum vessel; a modal analysis was performed; a dynamic frequency response (DFR) was performed; power spectral density (PSD) curves were approximated for the experimental hall floor; these PSD curves were multiplied by the DFR to obtain spectral responses; this result was integrated to get mean square displacement and acceleration, and rms displacement and acceleration. This procedure is outlined in Figure 1.

Flowchart

```
                                FEA Model
                                   |
                              Modal analysis
                                   |
                          Dynamic frequency response
                                   |
Input PSD ------------------> Calculate spectral response -------> Spectral output
                                   |
                            Convolution integral
                                   |
                        rms displacement and acceleration
```

Figure 1

Details of the procedure follow.

FEA model

A finite element model of the vessel was created using the Applied Structure code from RASNA Corporation. This model was based on modeling done for structural analysis of the vessel.

Modal analysis

A modal analysis was performed with the vessel model and the first nine modes were recovered. These modes are described in Table 1

mode 1	2.7 hz	lateral motion in x-axis
mode 2	4.5 hz	longitudinal motion in z-axis
mode 3	6.5 hz	vertical motion in y-axis
mode 4	7.0 hz	rotation about vertical y-axis
mode 5	7.8 hz	
mode 6	11.5 hz	
mode 7	11.6 hz	
mode 8	14.1 hz	
mode 9	14.9 hz	

Table 1

Dynamic frequency response

The modal analysis was used in dynamic frequency response analyses where the structure was excited in each of the global axes with a sinusoidal acceleration of 386.4 inches/second/second. The DFR response is recovered at points corresponding to muon space frame mounting locations.

Input PSD

The input PSD was calculated based on bandwidth limited white noise at 0.001mg level from 1 to 5 hz with a 6db/octave roll-off to 10 hz. The amplitude was calculated as follows:

$$a = \sqrt{A * (f2 - f1) + A / (f3 - f2)}$$

where

a = rms acceleration
f1 = starting frequency
f2 = beginning of roll-off frequency
f3 = ending frequency
A = PSD amplitude

The PSD curve is shown in Figure 2.

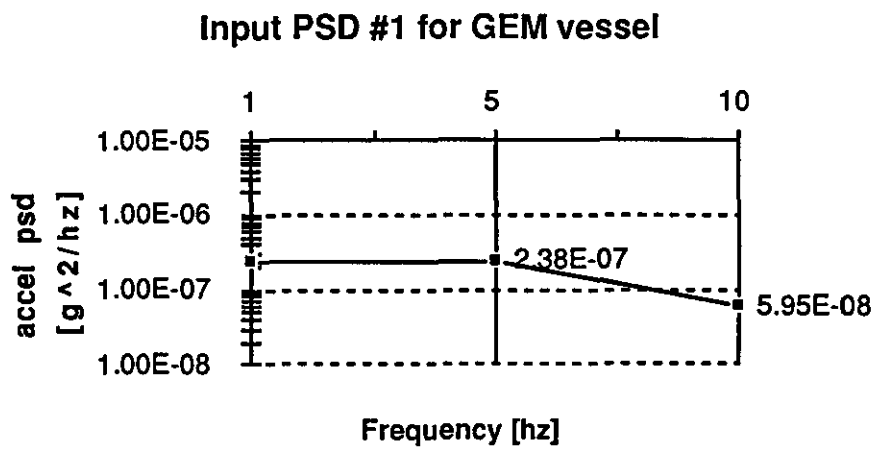


Figure 2

The bandwidth of the PSD can be increased by increasing f2 since f3 is a function of f2.

Spectral response

The spectral responses were then calculated according to the following formulas

displacement spectra = PSD(freq) * DFR(freq)**2
acceleration spectra = PSD * DFR**2 * (2PI*freq)**4

These output spectra can be used to excite other structures attached to the vessel structure.

Convolution integral

To get mean square displacement and acceleration values the previous spectra can be integrated over the frequency range of the input PSD using equation 1.

$$\overline{y^2} = \int_{f_{min}}^{f_{max}} PSD(f)DFR^2(f)df \qquad \text{Eqn 1}$$

Simpson's Rule is used to perform this numerical integration. The result of the integration is the mean square of the displacement or acceleration of the point in question. Rms values are achieved by taking the square root of these values.

Analytical verification

Consider a single degree-of-freedom system subject to random base excitation and shown in Figure 3

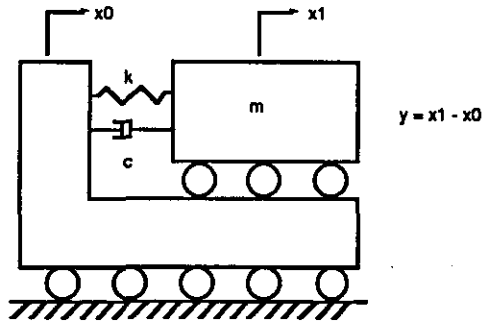


Figure 3

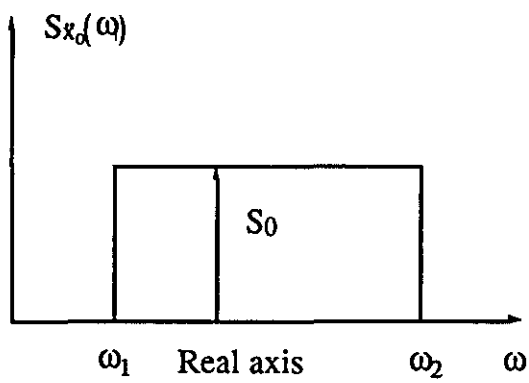
where

$m\ddot{y} + c\dot{y} + ky = -m\ddot{x}_0$

Response y(t) will be a stationary random process whose spectrum is

$$S_y(w) = \begin{cases} \dfrac{S_0}{(\omega_n^2 - \omega^2)^2 + 4\zeta^2\omega_n^2\omega^2} & \omega_1 < \omega < \omega_2 \\ 0 & \text{elsewhere} \end{cases}$$

where



Let k = 800 lb/in, c = 2%, m = 7.5e-4 lbs^2/in
Let the input for the DFR be a 1g sinusoidal load from 50 - 200 hz

$$\omega_n = \sqrt{\frac{800\ lb/in}{7.5e\text{-}4\ lbs^2/in}}$$
$= 1033\ rad/s^4$
$= 164.4\ hz$

Let S_0 represent 0.001 g_{rms} from 50-200 hz

The PSD magnitude can be determined using the following equation

$g_{rms} = \sqrt{(f_2 - f_1)A}$

where A is the PSD amplitude. So for $f_2 = 200$ hz, $f_1 = 50$ hz, and $g_{rms} = 0.001$, A = 6.66e-9 g^2/hz. The frequency range was chosen to surround ω_n as shown in Figure 5

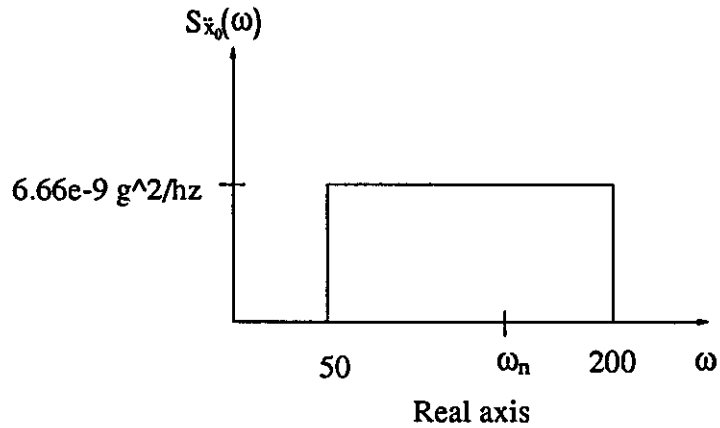


Figure 5

For $50 < \omega < 200$

$$S_y = \frac{6.66e\text{-}9\ g^2/hz}{((1033 rad/s)^2 - \omega^2)^2 + 4(0.02)^2(1033\ rad/s)^2\omega^2}$$

Substituting $\omega = 50$ hz = 314 rad/s yields

S_y (314) = 7.099e-21 g^2s^4/hz

The same system modeled in Applied Structure with an input acceleration of 1g yields 7.1067e-21 g^2s^4/hz.If the DFR input is 386.4 in/in/s^2 the result is 1.06e-15 in^2/hz. Or taking a frequency closer to resonance say 163.64 hz (this frequency was one of the computer program's frequencies)

S_y (1028) = 3.490e-18 g^2s^4/hz

Applied Structure is again very close at 3.514e-18g^2s^4/hz. This is within 1%. Performing the integration and taking the square root is a slightly time consuming but trivial task.

Distribution

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