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Moment Method for Analysis of Induced Charge Distribution

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Abstract:

Moment method of random distribution analysis is proposed as a two-hits resolution procedure and as a method distinguishing from pick-up noise.

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1 Introduction

The problem of two-hit resolution is serious for the GEM central tracker and muon system.

Both subdetectors use chambers (IPC for the central tracker and CSC for the muon system [1]) with the identical principle of coordinate measurement: analysis of induced charge distribution.

The method, which could be used for very fast (maybe at hardware level) resolution of two nearby hits, is proposed.

This method is founded on the feature of random distribution to be characterized by its moments.

Taking into account such features of IPC and CSC,that the single-hit distributions of induced charge are symmetrical and have stable dispersion, two-hit distribution of induced charge could be completely described by their first three moments.

Later the detailed proof of this procedure is adduced.

2 Three-moments method for two-hit resolution

To consider application of moment method for two-hit resolution we define functions:

$f_1(x)$ - distribution of induced charge for the first track;
$f_2(x)$ - distribution of induced charge for the second track;
$f(x) = f_1(x) + f_2(x)$ - resulting distribution of induced charge.

We can calculate values of first four moments for the function $f(x)$:

$$M_0 = \int f \ = \int f_1 \ + \int f_2 \ \equiv F_1 + F_2; \tag{1}$$

$$M_1 = \int xf \ = \int xf_1 \ + \int xf_2 \ = x_1F_1 + x_2F_2; \tag{2}$$

To calculate the second moment we use the definition of RMS:

$$\int (x - x_i)^2 f^i \ = \sigma_i^2 F_i; i = 1, 2.$$

Then:

$$\int x^2 f_i \ - 2x_i \int xf_i \ + x_i^2 \int f_i \ = \int x^2 f_i \ - x_i^2 \int f_i \ = \sigma_i^2 F_i,$$

or

$$\int x^2 f_i \ = (\sigma_i^2 + x_i^2) F_i.$$

So, the second moment is equal to:

$$M_2 = \int x^2 f \ = \int x^2 f_1 \ + \int x^2 f_2 \ = (\sigma_1^2 + x_1^2)F_1 + (\sigma_2^2 + x_2^2)F_2 \tag{3}$$

Analogous for the third moment (we assume symmetrisity of distributions):

$$\int (x - x_i)^3 f_i \ = 0; i = 1, 2$$

So:

$$\int (x^3 - 3x^2 x_i + 3x x_i^2 - x_i^3) f_i \ = \int x^3 f_i - 3x_i \int x^2 f_i + 3x_i^2 \int x f_i - x_i^3 \int f_i \ = \int x^3 f_i - (3x_i \sigma_i^2 + x_i^3) F_i = 0.$$

And

$$M_3 = \int x^3 f_1 \ + \int x^3 f_2 \ = (3x_1\sigma_1^2 + x_1^3)F_1 + (3x_2\sigma_2^2 + x_2^3)F_2 \tag{4}$$

So, we have the system of equations:

$$M_0 = F_1 + F_2 \tag{5}$$

$$M_1 = x_1 F_1 + x_2 F_2 \tag{6}$$

$$M_2 = (\sigma_1^2 + x_1^2)F_1 + (\sigma_2^2 + x_2^2)F_2 \tag{7}$$

$$M_3 = (3x_1\sigma_1^2 + x_1^3)F_1 + (3x_2\sigma_2^2 + x_2^3)F_2 \tag{8}$$

If we assume that $\sigma_1 = \sigma_2 = \sigma$, we will have four equations for four unknown quantities: x_1, x_2, F_1, F_2.

To solve this system we define

$$a \equiv \frac{F_1}{F_1 + F_2}; (1 - a) = \frac{F_2}{F_1 + F_2},$$

and divide Eqs. 6, 7, and 8 by Eq. 5:

$$m_1 = M_1/M_0 = x_1 a + x_2(1 - a) \tag{9}$$

$$m_2 = M_2/M_0 = (\sigma^2 + x_1^2)a + (\sigma^2 + x_2^2)(1 - a) \tag{10}$$

$$m_3 = M_3/M_0 = (3x_1\sigma^2 + x_1^3)a + (3x_2\sigma^2 + x_2^3)(1 - a) \tag{11}$$

And finally:

$$x_1 a + x_2(1 - a) = m_1 \tag{12}$$

$$x_1^2 a + x_2^2(1 - a) = m_2 - \sigma^2 \tag{13}$$

$$x_1^3 a + x_2^3(1 - a) = m_3 - 3m_1\sigma^2 \tag{14}$$

Decision of this system of equations has very elegant and clear form. First of all we introduce central moments of the total distribution $f(x)$:

$$\mu_n = \frac{\int (x - m_1)^n f}{\int f}; n = 2, 3.$$

The central moments have the following connections with the moments:

$$\mu_2 = m_2 - m_1^2;$$

$$\mu_3 = m_3 - 3m_1m_2 + 2m_1^3$$

If we define value

$$\Delta \equiv \frac{1}{2}\frac{\mu_3}{\mu_2 - \sigma^2} \tag{15}$$

then the middle point between two tracks has the coordinate:

$$\frac{x_1 + x_2}{2} = m_1 + \Delta \tag{16}$$

and separation between two tracks is equal to

$$\mid x_1 - x_2 \mid = 2\sqrt{\mu_2 - \sigma^2 + \Delta^2}. \tag{17}$$

We see that if the resulting distribution is symmetric and $\mu_3 = 0$, then coordinate of the middle point is equal to the value of the first moment (center-of-gravity of total distribution), and the separation between tracks is a function of the second moment. The value Δ is a correction due to asymmetry of the distribution.

3 Procedure for signal distinguishing from pick-up noise

The resulting distribution of induced charge is equal to sum of the signal distribution and the distribution of pick-up noise:

$$f_{tot} = f_s(x) + f_p(x).$$

We introduce the function

$$r(x) = f'_{tot} = f'_s + f'_p,$$

and, if we assume that $f_p = const$, then $r(x) = f'_s$.

We can calculate moments of distribution f_s, using the value of the function $r(x)$:

$$M_1 = \int xr = \int xf'_s = -\int f_s; \tag{18}$$

$$M_2 = \int x^2 r = \int x^2 f'_s = -2\int xf_s. \tag{19}$$

Center of gravity for the signal distribution is equal to

$$m_1 = \frac{\int xf_s}{\int f_s} = \frac{M_2}{2M_1}. \tag{20}$$

4 Conclusions

This method was tested with MonteCarlo program by Cherniatin and Chikanian, simulating CSC response. Promising results were observed. Now we are going to use this method for two-track resolution analysis of experimental data, obtained at ITEP test beam for small CSC prototype.

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References

[1] GEM Letter of Intent, SSCL-SR-1184, GEM TN-92-49.