



Light Attenuation Length of Barium Fluoride Crystals

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This report presents a deduction of a formula which can be used to calculate the light attenuation length of BaF₂ crystals based on the transmittance (or absorbance) data measured by a spectrophotometer.

LIGHT ATTENUATION LENGTH OF BARIUM FLUORIDE CRYSTALS¹

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Abstract

This report presents a deduction of a formula which can be used to calculate the light attenuation length of BaF₂ crystals based on the transmittance (or absorbance) data measured by a spectrophotometer.

1 Introduction

The light attenuation length (λ) is an important parameter in evaluating quality of BaF₂ crystals [1]. An approximated formula, Equation (5) of reference [1], was suggested to calculate the light attenuation length of BaF₂ crystals based on measured transmittance (T) and ideal theoretical transmittances (T_0) of a BaF₂ crystal with infinite light attenuation length:

$$\lambda = \frac{\ell}{\ln(T_0/T)} \quad (1)$$

where ℓ is the path length, i.e. crystal length.

However, this equation suffers two approximations:

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1. the ideal theoretical transmittances (T_s) was calculated according to an extrapolation of measured refractive index [2], while the original measurement was carried out for wavelengths of longer than 260 nm only; and
2. the equation does not take into account multiple bounces between two end surfaces of the crystal.

The consequence of these two approximations thus is

1. at wavelength of BaF₂'s fast component (220 nm), the ideal theoretical transmittances (T_s) is not accurate; and
2. the light attenuation length calculated by using Equation 1 is not accurate.

In this note, we present a deduction of an accurate formula which can be used to calculate the light attenuation length of BaF₂ crystals based on the transmittance (or absorbance) data measured by a spectrophotometer.

2 Determination of T_s

As defined above, T_s is an ideal transmittance of a BaF₂ crystal with infinite light attenuation length and two parallel end surface. The loss of light in each surface can be calculated by using Fresnel's law:

$$R = \frac{(n - 1)^2}{(n + 1)^2} \quad (2)$$

where n is the refractive index which is a function of the wavelength.

Taking into account multiple bounces between two end surfaces, the ideal transmittance (T_s) can be written as

$$\begin{aligned} T_s &= (1 - R)^2 + R^2(1 - R)^2 + \dots \\ &= (1 - R)/(1 + R) \end{aligned} \quad (3)$$

Assuming the crystal has a light attenuation length of λ which is also a function of wave length, the light intensity after traversing a path length of ℓ is:

$$L = \exp(-\ell/\lambda) \quad (4)$$

Taking into account both multiple bounces between two end surfaces and light attenuation loss (L), the real transmittance (T) can be written as

$$\begin{aligned} T &= (1 - R)^2 L + R^2(1 - R)^2 L^3 + \dots \\ &= L(1 - R)^2 / (1 - L^2 R^2) \end{aligned} \quad (5)$$

For a thin crystal with good quality (typical light attenuation length of >200 cm), the ℓ/λ is very small. Equation 4 thus can be written as

$$L \approx 1 - \ell/\lambda \quad (6)$$

Equation 5 can be written as

$$T \approx T_s(1 - \ell/\lambda) \quad (7)$$

For a 2 mm crystal with light attenuation length of >200 cm and two parallel, well polished surfaces, the $\ell/\lambda < 0.001$. We thus can use spectrophotometer to measure its transmittance and take this measured value as T_s . A crystal with a dimension of $40 \times 40 \times 2$ mm³ was provided by Shanghai Institute of Ceramics for this purpose. The crystal was polished at Lawrence Livermore National Laboratory by using diamond pitch wheel method. It is shown that this polishing technique provides perfect optical surface [3].

Figure 1 shows the measured transmittance and the refractive index calculated by using Equations 2 and 3. Also shown in Figure 1 is the measured n from reference [2]. It is clear that the agreement between refractive indexes measured in this work and in reference [2] is reasonably good. The T_s value at 220 nm (for the fast component of BaF₂) is 90.06%, instead of 91.1% used in reference [1].

3 Determination of λ

Equation 5 can be written as

$$TR^2L^2 + (1 - R)^2L - T = 0 \quad (8)$$

which can be solved with a solution of

$$L = \frac{\sqrt{4T_s^4 + T^2(1 - T_s^2)^2} - 2T_s^2}{T(1 - T_s)^2} \quad (9)$$

where Equation 3 is used.

Using Equation 4, we thus have an accurate formula which can be used to calculate the light attenuation length (λ) by using measured transmittance (T):

$$\lambda = \frac{\ell}{\ln\{[T(1 - T_s)^2]/[\sqrt{4T_s^4 + T^2(1 - T_s^2)^2} - 2T_s^2]\}} \quad (10)$$

Note, in the case T_s is close to 1, the Equation 10 can be approximated to Equation 1. Figure 2 shows light attenuation length at 220 nm, as a function of the transmittance, calculated by using Equation 10 for a 25 cm long BaF₂ crystal. For a BaF₂ crystal with length of ℓ cm, the corresponding light attenuation length should be multiplied by 0.04 ℓ .

Theoretically, absorbance (A), or optical density (D), is more adequate to describe quality of a crystal, since they are linearly related to the color center density, or the cross section of light absorption. They are defined as:

$$A \text{ or } D = \log(1/T) = -\log T \quad (11)$$

By using approximate Equation 1, one has

$$\lambda = 0.43\ell/[A - \log(1/T_s)] \quad (12)$$

Figure 3 shows light attenuation length at 220 nm, as a function of absorbance, calculated by using Equation 10 and 11 for a 25 cm long BaF₂ crystal. For a BaF₂ crystal with length of ℓ cm, the corresponding light attenuation length should be multiplied by 0.04 ℓ .

4 Acknowledgement

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References

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- [2] I. H. Malitson, J. of Optical Society of America Vol 54 No. 5 (1964) 628.
- [3] C. Wuest and B. Fuchs, GEM TN-92-163, July 1992.

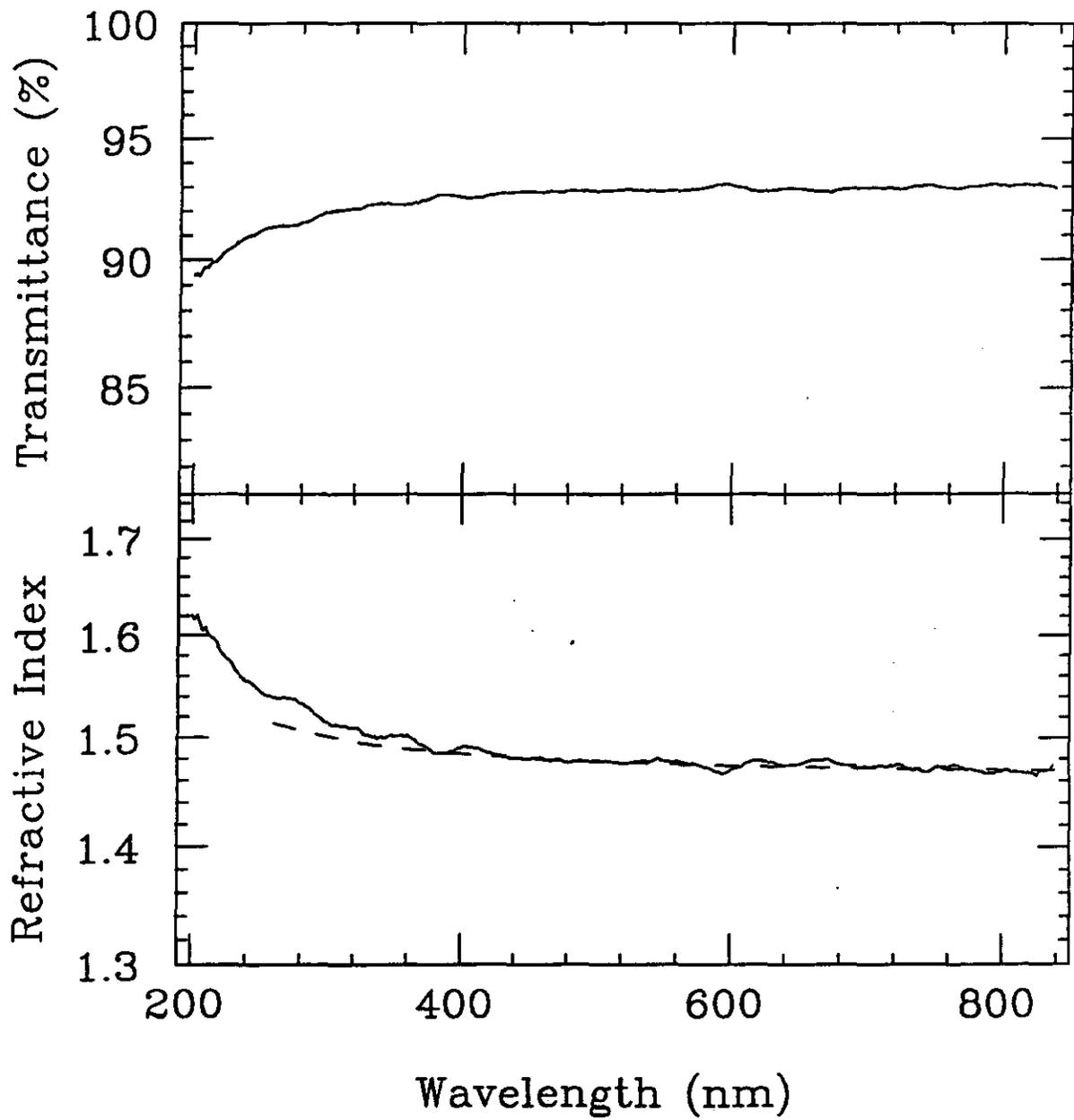


Figure 1: Ideal transmittance (a) and refractive index (b) of a BaF_2 crystal are plotted as a function of wavelength. The refractive index from reference [2] is shown as a dashed line for a comparison.

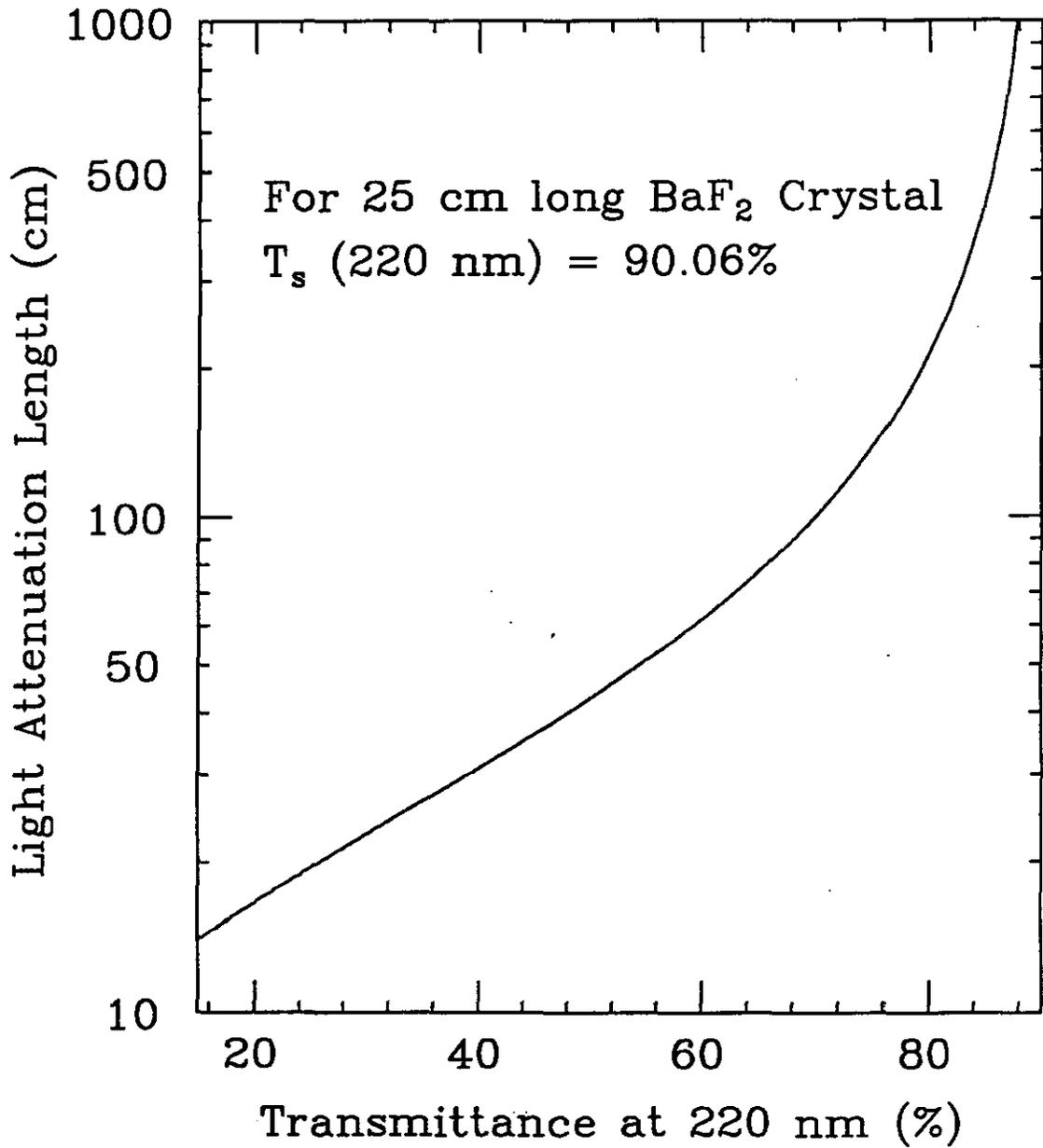


Figure 2: Light attenuation length at 220 nm as a function of transmittance measured for a 25 cm long BaF₂ crystal.

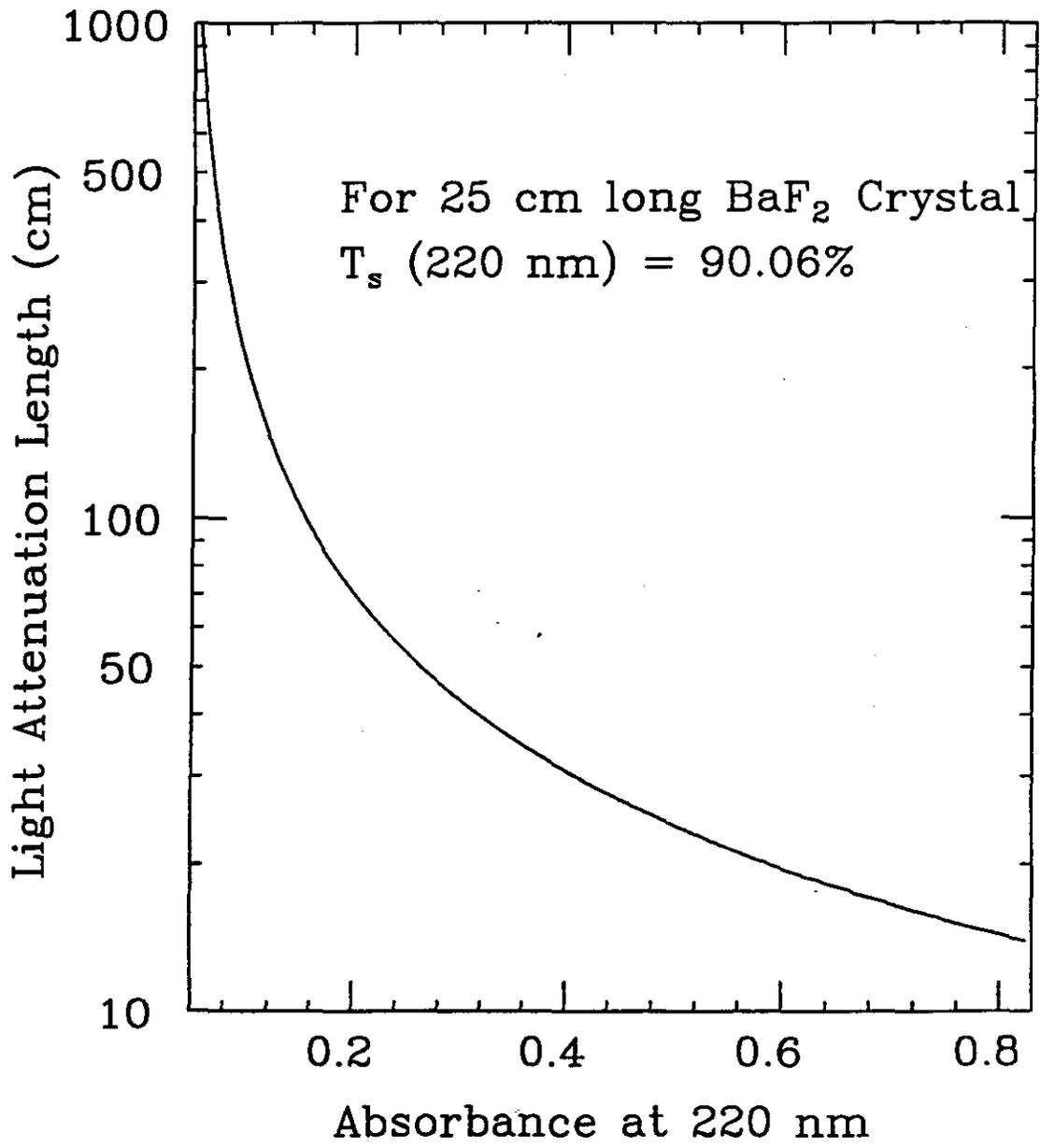


Figure 3: Light attenuation length at 220 nm as a function of absorbance measured for a 25 cm long BaF₂ crystal.