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Comparison of Three - and Four - Point Momentum Measurement in a Uniform Magnetic Field

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Abstract:

When multiple scattering is considered, the classical 1:2:1 distribution of measurements is not necessarily optimum for measuring momentum in a uniform magnetic field. This note discusses 1:1:1 distribution with the spacing of the two middle measurements as a parameter. Such a four-point measurement can give better momentum resolution at low momentum (e.g., below 200 GeV/c) and may have advantages for pattern recognition.

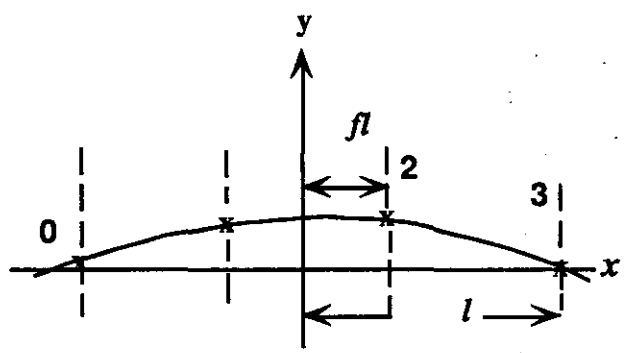
Introduction

We discuss the case of measuring momentum in a uniform magnetic field in the case where each measurement introduces multiple scattering. In the standard optimization, a fixed number of measurements is distributed in a 1:2:1 fashion, with one-quarter of the measurements being made at each end of the arc and half at the center. While this is optimum if measurement error dominates, it is not if multiple scattering is important.

In a sample GEM configuration, momentum resolution can be improved below some quite high momentum (e.g., 200 GeV/c) by using a four-point measurement.

Approach

The geometry is shown in figure 1. Consider a rectangular system where the measurement planes are at four x-coordinates of $-l$, $-fl$, fl and l, i.e. over an arc of length $2l$. In practice, several measurements are made at each plane. We collapse them into one. In the classic optimization, we would set f=0, so that twice as many measurements are made in the center as at each end.



We then consider the error in measurement of the y-coordinate at each point, and calculate the uncertainty introduced in the coefficient of the second-order term in a least-square fit for a parabola. For the region of interest, this is an adequate approximation.

We define the following quantities:

σ = The standard deviation of one measurement due to measurement error only.

θ = The multiple scattering angle associated with that measurement.

For convenience, we look from the center of the arc out, and remove the constant curvature due to momentum, calculating only the uncertainties. We define the *uncertainties* in the y-coordinate as follows:

$$y_0 = \sigma_0 \oplus (1-f)\, l\, \theta_1$$

$$y_1 = \sigma_1$$

$$y_2 = \sigma_2$$

$$y_3 = \sigma_3 \oplus (1-f)\, l\, \theta_2$$

The measurement errors (σ_0, σ_1, σ_2, σ_3) are uncorrelated (but equal in magnitude to σ), as are θ_1 and θ_2. The symbol $\oplus$ means that the two terms are to be combined in quadrature. The significance of the subscripts is that values are to be combined in quadrature wherever they appear.

We perform a least-squares fit to the equation $y = a_0 + a_1 x + a_2 x^2$ to the points $(x_0, y_0) \ldots (x_3, y_3)$ so as to examine the effect of varying f on the term a_2. (a_2 is related to the radius of curvature ρ by $a_2 = \frac{1}{2\rho}$.)

Doing this we find that the uncertainty in a sagitta measurement is given by:

$$a_2 l^2 = \frac{\sqrt{\sigma^2 + (1-f)^2\, l^2\, \theta^2/2}}{(1-f^2)} .$$

This is minimized as a function of f when:

$$\frac{(1-f)^3}{f} = 4\, \sigma^2/l^2\theta^2 .$$

For a specific momentum p (GeV/c), magnetic field B (Tesla) and radius R(m), we can set p = 0.3BR, and

$$\theta \approx \frac{0.014}{p}\sqrt{L/X_o} ,$$

where the number of radiation lengths corresponding to each measurement is L/X_o.

At very high momentum, a_2 will depend only on measurement error; at very low, only on multiple scattering.

The sagitta s is then given by $s = \frac{l^2}{2R}$ and the uncertainty in momentum Δp by

$$\frac{\Delta p}{p} = \frac{a_2 l^2}{s} = 2Ra_2 = \frac{2a_2 p}{0.3B}$$

A useful way to look at these results is to compare the result for a_2 with that using the classical distribution (obtained by setting $f = 0$). The ratio is then:

$$r = \frac{\sqrt{\sigma^2 + (1-f)^2\, l^2\theta^2/2}}{(1-f^2)\quad \sqrt{\sigma^2 + l^2\theta^2/2}}$$

We then can define a low momentum asymptote (for $l^2\theta^2/2 >> \sigma^2$)

$$r_{low} = \frac{1}{1+f};$$

a high momentum asymptote

$$r_{high} = \frac{1}{1-f^2};$$

and a cross-over momentum, given by

$$\frac{2\sigma^2}{l^2\theta^2} = \frac{2-3f+f^3}{2f+f^3},$$

or

$$p = \frac{0.014\sqrt{L/X_0}\ l}{\sqrt{2}\ \sigma} \cdot \sqrt{\frac{2-3f+f^3}{2f+f^3}}$$

In table I, we tabulate these results for a specific case ($\sigma = 50$ μm, $L/X_0 = 0.07$, l = 1.7m). Figure 2 shows the resolution as a function of momentum (for $\sigma = 50$ μm, L/X_0 = 0.1, l = 1.7m).

Table I - Low and high momentum ratios and cross-over momentum

f	Cross-over Momentum (GeV/c)	r low	r high
0.05	383	0.95	1.003
0.1	259	0.91	1.01
0.2	165	0.83	1.04
0.3	119	0.77	1.10
$\frac{1}{3}$	108	0.75	1.13
0.4	89.1	0.71	1.19

Conclusion

Low-momentum resolution can be improved by making measurements in four positions rather than three, at little cost to the high momentum resolution. Further the four-point measurement gives added constraints on possible tracks, enhancing pattern recognition.

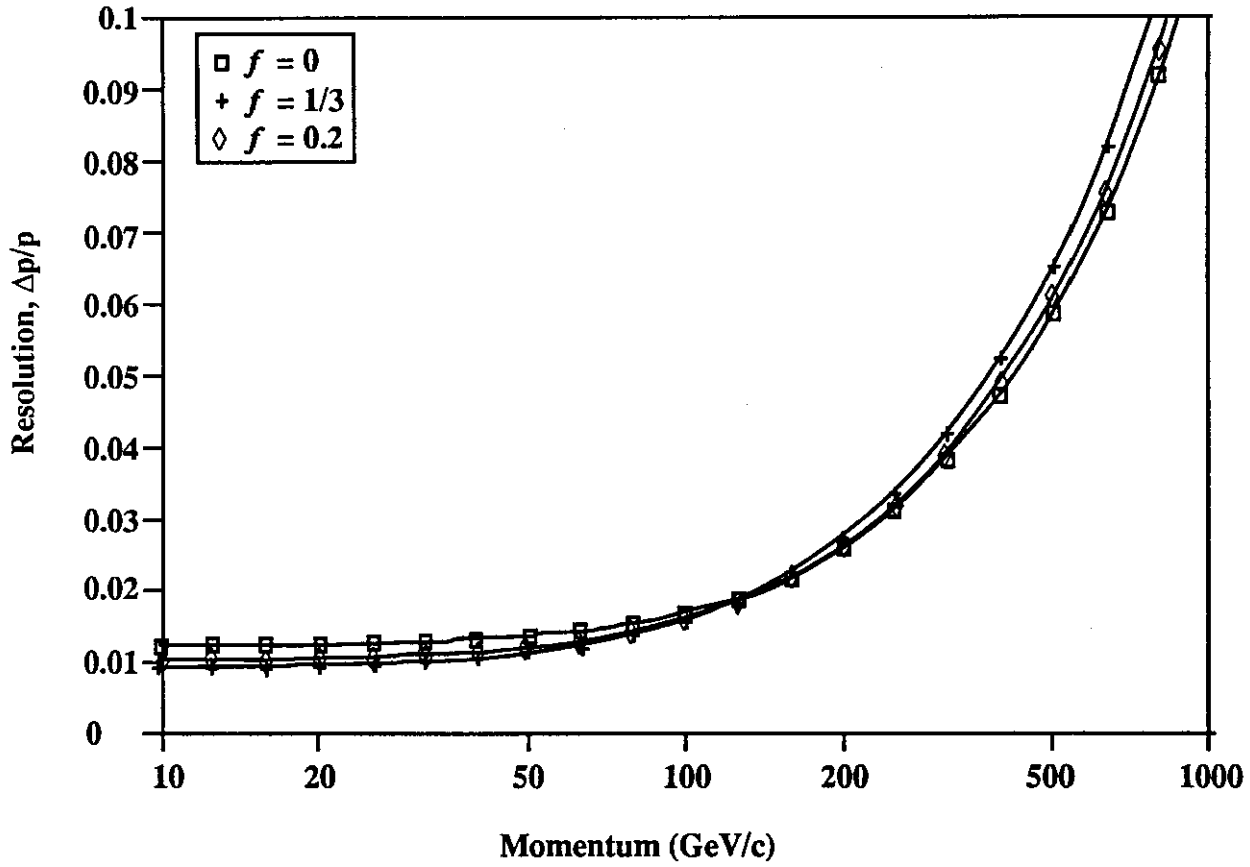


Figure 2 Momentum resolution as a function of momentum for various values of f (σ = 50 μm, L/X_0 = 0.1, B = 1.0 T, l = 1.7m)