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Previous Analysis of Tracks Data in Solenoid

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Without tracking; determination and separation of primary vertices; preparation of arrays for reconstruction of single track; division of the tracks data onto kinematical subregions; determination of jets.

JOINT INSTITUTE FOR NUCLEAR RESEARCH

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The possibility of previous analysis of the tracks data of the high multiplicity experiments in the solenoidal magnetic field is considered. Such analysis provides the realization of next functions:

1) the determination of the primary (intensive) vertices;
2) the creation of arrays containing measurment coordinats for the reconstruction of single track;
3) the cleaning(removal) of the vertex for the following search for next vertices;
4) to divide whole tracks data onto "kinematical" subregions that allows one to reduce the CPU time of the event reconstruction;
5) the determination of "jets" (hard components).

The typical number of operations is proportional to M^2 ("M" is the multiplicity or the number of tracks). This ideology is based on the "integral mathematical model of the track pattern" and does not contain a procedure of the reconstrunction of trajectories.

I. MATHEMATICAL FORMALISM

a) The system of coordinates and designations: the (XYZ) or (RϕZ) system will be used, magnetic field has the Z-component ("H"); there are "N" detectors along the Z axis, each detector(Z_n;n=1,2,..,N) registers "M_n" space points $\{(R_{mn},\phi_{mn},Z_n),m=1,2,...,M_n\}$ at the accuracy "σ_{mn}".

b) Analytical view of the trajectory:

$$x(t)=X_o + r_o\sin\alpha + r_o\sin(\omega t-\alpha)(\equiv R\cos\phi) \qquad (I.1a)$$

$$y(t)=Y_o - r_o\cos\alpha + r_o\cos(\omega t-\alpha)(\equiv R\sin\phi) \qquad (I.1b)$$

$$z(t)=Z_o + tv_o\cos\beta \qquad (I.1c)$$

where: X_o,Y_o,Z_o are the vertex position, "β" is the (pZ) angle and "α" is the (XY) angle of momentum 'p',

$$\omega = \frac{ecH}{E}, \quad E=\frac{p\,c^2}{v_o}, \quad r_o= \frac{v_o\sin\beta}{\omega} \qquad (I.2)$$

$$A = \frac{1}{2}\,\frac{\omega}{v_o\cos\beta} \equiv \frac{e\,H}{2cp\,\cos\beta} \qquad (I.3)$$

c) The vertex function (abbr.VF). In the accordance of the rule of the VF creation [1,p.424] it is necessary to redefine parameters "r_o, ω, A" through X_o,Y_o,Z_o and two formal detector counts

at Z_ν and Z_k. Simple transformations of expressions (I.a-c) allow one to get next relations

$$\alpha + 2A(Z_o - Z_k - Z_\nu) = \mathrm{Arctg}\frac{y_{lk} - y_{\mu\nu}}{x_{lk} - x_{\mu\nu}} \quad (\equiv P_{lk\mu\nu}) \qquad (I.6)$$

$$\alpha - A(Z_k - Z_o) = \mathrm{Arctg}\,\frac{y_{lk} - Y_o}{x_{lk} - X_o} \quad (\equiv F_{lk}(X_o, Y_o)) \qquad (I.7)$$

$$\alpha - A(Z_\nu - Z_o) = \mathrm{Arctg}\,\frac{y_{\mu\nu} - Y_o}{x_{\mu\nu} - X_o} \quad (\equiv F_{\mu\nu}(X_o, Y_o)) \qquad (I.8)$$

These expressions redefine parameters "α,A"

$$A = -\frac{F_{lk} - F_{\mu\nu}}{Z_k - Z_\nu} \qquad (I.9)$$

$$\alpha = P_{lk\mu\nu} - A(I.9)\ (2Z_o - Z_k - Z_\nu) \qquad (I.10)$$

The (I.8) expression can be used for the definition of the vertex function

$$V(X_o, Y_o, Z_o) = \sum_{l=1}^{M_k}\sum_{\mu=1}^{M_\nu}\sum_{n=1}^{N}\sum_{m=1}^{M_n} G\{\alpha - A(Z_n - Z_o) - F_{mn}; \sigma_{mn}\}\sigma_{mn} \qquad (I.11)$$

where parameters "α,A" are used as (I.9,10).

G(x;s) is the coordinates precision (s) function that can be used in some forms (views):

$$G(x;s)=\exp(-\frac{x^2}{2s})/s\sqrt{2\pi} \qquad (I.12a)$$

or

$$G(x;s)=\begin{cases}(2s)^{-1} & \text{at } |x|\leq s\\ 0, & |x|>s\end{cases} \qquad (I.12b)$$

or

$$G(x;s)=\begin{cases}\frac{3}{s4\sqrt{5}}(1-x^2/5s^2) & \text{at } |x|\leq s\sqrt{5}\\ 0, & |x|>s\sqrt{5}\end{cases} \qquad (I.12c)$$

These interpretations of the G(x;s) reflect not only various types of detectors (for instance, (I.12b) is typical for proportional chambers), but simplify also the analysis of VF.

This function (I.11) requires about M^3 number of operations (the abbrivation NOP will be used in the following) and reflects itself the track reconstruction by the exhaustive search.

However one can exclude the (μ) summation out of this definition or to change "ν ->n" and new expression of this VF

$$V(X_o,Y_o,Z_o)=\sum_{l=1}^{M_k}\sum_{n=1}^{N}\sum_{m=1}^{M_n} G\{\frac{F_{lk}-F_{mn}}{Z_k-Z_n}(Z_o-Z_k)-F_{mn}+P_{lkmn};\sigma_{mn}\}\sigma_{mn} \qquad (I.11a)$$

looks more attractive in the order of NOP ($\sim M^2$). It can be mentioned that this definition of VF does not contain a possibility to do simple step to the reconstruction of trajectories (as it was for straight tracks) and this VF(I.11a) is "independent" description of the track pattern.

II. SEARCH FOR PRIMARY VERTEX

To be solving this task one can neglect of the transversal sizes of colliding beams which are small enough (15mkm at LHC, <1mm for SSC) in the comparision with counts of detectors (it means that minimal coordinate of a detector is more biger than the beam radius). It can be mentioned also that the problem of the z-separation of primary vertices will be very seriously [2].

So, the $V(Z_o)$ gets the view:

$$V(Z_o)=\sum_{l=1}^{M_k}\sum_{n=1}^{N}\sum_{m=1}^{M_n} G\{\frac{\phi_{lk}-\phi_{mn}+(i-j)\pi}{Z_k-Z_n}(Z_o-Z_k)-\phi_{mn}+P_{lkmn}-j\pi\ ;\sigma_{mn}\}\sigma_{mn} \qquad (II.1)$$

$i,j=\ldots-2,-1,0,1,2,\ldots$ these parameters should be determined for concrete experiment.

In the following the notation of the $V(Z_o)$ will be used also (for the simplicity):

$$V(Z_o)=\sum_{l=1}^{M_k}\sum_{n=1}^{N}\sum_{m=1}^{M_n} G\{\ A(Z_o-Z_k)-C\ ;\sigma_{mn}\}\sigma_{mn} \qquad (II.1a)$$

This function has multi extrema character, especially for the multi vertices events and the determination of the vertex position Z_o forsees two stages:

1) The localization of the subregion of global maximum of $V(Z_o)$. In the same way as it was done for the straight tracks, the economic method [1,p.426] can be used. The main task at this step is the analitycal integration:

$$I_\mu = \sum_{l=1}^{M_k} \sum_{n=1}^{N} \sum_{m=1}^{M_n} J^\mu_{lkmn} \quad (\mu=0,1,\ldots,4) \tag{II.2}$$

where

$$J^\mu_{lkmn} \equiv \int_{Z_L}^{Z_U} dZ_o (Z_o - \bar{Z}_o)^\mu \; G\{ A(Z_o - Z_k) - C \; ; \sigma_{mn} \} \sigma_{mn} \tag{II.2a}$$

This problem is very simple if the G(x;s) is used in discrete form (I.12b). It is necessary to solve simple system of inequalities:

$$\begin{cases} |A(Z_o - Z_k) - C| \leq \sigma_{mn} \\ Z_L \leq Z_o \leq Z_U \end{cases} \tag{II.3}$$

If this system has no solutions, it means that for the "base" point (R_{lk}, Φ_{lk}, Z_k) there are no the "track continuation" into the point (R_{mn}, Φ_{mn}, Z_n), or $J^\mu_{lkmn} \equiv 0$. In opposite case

$$J^\mu_{lkmn} \equiv \int_{Z_1}^{Z_2} dZ_o (Z_o - \bar{Z}_o)^\mu \equiv \frac{1}{\mu+1} (Z_o - \bar{Z}_o)^{\mu+1} \Big|_{Z_1}^{Z_2} \tag{II.4}$$

where:

$$Z_1 = \max \left(Z_L \; , \; Z_k + \frac{C}{A} - \frac{\sigma}{|A|} \right)$$

$$Z_2 = \min \left(Z_U \; , \; Z_k + \frac{C}{A} + \frac{\sigma}{|A|} \right) \tag{II.5}$$

Z_U, Z_L are upper and lower limits of the searching region (it can be comparale with while experimental instalation). Approximative position of vertex is determined by the solution of cubic equation that has coefficients determined by integrals I_μ.

2)Precise determination of the vertex. On this stage the parabolic fit[1,p.427] can be used. The paraolic interpretation (I.12c) of the function of the accuracy is useful too.

Final result has exact value of Zo and one can get the dispersion of the $V(Z_o)$ - $S(Z_o)$.

Results of very simple test of this vertex determination are shown in Fig.1-3.

III. THE REMOVAL OF THE VERTEX AND PREPARATION OF ARRAYS FOR THE TRACK RECONSTRUCTION

This possibility is obviously enough if one looks back to the integration (II.2-5). The width of the $V(Z_o)$ peak is narrow in the comparision with initial big limits. Coordinates of trajectories emitted from finding vertex correspond to this peak and while the integration is performed over the peak width, one can prepare the array $\{R_{mn}, \phi_{mn}, Z_n\}_k$ of the "track continuation" for the point (R_{1k}, ϕ_{1k}, Z_k). This array,of course, can has false points too (a noise, counts of other vertices,trajectories),this "overloading" is determined mainly by the detector accuracy. However the task of the reconstruction of single trajectory over this limited array looks more simple than the analysis of all track data.

Similary,one can exclude all coordinates, belonging to the vertex, out of original data or,in other words, to do the "cleaning" (removal) of current vertex for the following search for other vertices.

By these methods (ch.II+III) one can exclude all points of all primary intensive vertices and thereby get more simple track data in the remainder for the following analysis (let it be the search of rare decays).

Fig.4 illustrates how this method is working for very simple simulated 3-vertices event.

IV.SEARCH FOR SECONDARY VERTICES

This problem is related to mentioned "rare decays" and,of course, means the analysis of general definition of VF (I.11a). From point of view of "mathematical technology", the task of the localization of a region of gloal maximum for $V(X_o,Y_o,Z_o)$ looks more complicated,than the same analysis of $V(Z_o)$. This problem is under consideration now.

Nevertheless, one can give the recipe of the solution for the case,when secondary vertex is closed to the Z-axis,i.e.at the condition that $R_o=\mathrm{sqrt}(X_o^2+Y_o^2) \ll R_{min}$ (minimal coordinate of a detector is more biger than the vertex position). The principal key to the solution is the method of the "defocusing of an image" [3], that can be explaned as artificialy deterioration of the detector

accuracy. It should be noted,that the "integral tracks model" has very high sencibility to the detector accuracy (σ) in the comparision with the "chi-squared" (χ^2),where this parameter plays very passive role. When the accuracy is deteriorating (i.e. instead of real "σ",the parameter $\sigma'=3*\sigma$,for instance, is used),then the multi extrema character of a function is simplified (all functions become more smooth). It allows one to use traditional ("local") methods for the determination of the vertex position.

This method has been used already in the software for the vertex detrmination (also in the $V(Z_o)$ analysis) and the problem of the specification of transversal position of the vertex will be realised soon.

V. DIVISION OF TRACKS DATA ONTO KINEMATICAL DIAPASONES

The "integral model" allows one to get not only the vertex functions,but also to construct functions of other parameters. For the solenoid one can create the function F(A),that has "kinematical" parameter "A"(I.3) as the argument. This parameter connects the charge of particles and the value of longitudinal momentum. Looking back again to equations (I.7,8) one can exclude parameters α,Z_o and get:

$$F(A)=\sum_{l=1}^{M_k}\sum_{n=1}^{N}\sum_{m=1}^{M_n} G\{A - A^*_{lkmn} ;\sigma_{mn}\}\sigma_{mn} \qquad (V.1)$$

where

$$A^{*}_{lkmn} \equiv - \frac{\Phi_{lk} - \Phi_{mn} + (i-j)\pi}{Z_k - Z_n} \qquad (V.1a)$$

Real trajectories manifest themselves in this functions as high enough peaks (amplitude is about of ~N) at combinatorial background. Simple illustrations, which corroborate this affirmation will be shown in following text.

For the high multiplicity event this F(A) will have very complicated view and analytical search for trajectories does not seem expedient one. However, this function is extreme simple for the integration like (II.2-5) and, respectively, for the extraction of the limited track arrays, as it was done in ch.III. This F(A) can be used in the next way:

One can determine boundaries of the parameter "A" for concrete expirement. This region is divided onto "Q" subregions, each one contains "K=M/Q" tracks (here "M" is total quantity of tracks). And then, for various methods of the track reconstruction with typical value of NOP biger than "K" (for instance,equals "K^2"),one can has considerable profit in the nubmer of operations ("N_t") in comparision with the reconstruction over whole array (NOP is about of M^2):

a) at sequential reconstruction in subregions the NOP is in "Q" times less

$$N_t = Q*(M/Q)^2 = M^2/Q \qquad (V.2)$$

b) at parallel, it is in "Q^2" times faster

$$N_t = (M/Q)^2 = M^2/Q^2 \qquad (V.3)$$

It should be noted, that such division is not a "geometrical division",which leads to the distortion of the track picture.

VI.SEARH FOR JETS

As it follows out of results of the paper[4], devoted ,in particular, to the Monte-Carlo simulation of jets at Tev region of the energy, hard component of the jet (the momentum value > 1 Tev) is concentraed in narrow cone with typical angle size about of 50 mrad. In the following the 'hard component of the jet' will be implied as the jet.

Looking back at kinematical function (V.1),one can suppose that, peaks, which are corresponding to the jet, will be concentrated in narrow region of the "A". The positive-charged part of the jet is concentrated close to the point "A_o" (for instance,it is positive), and the negative-charged part of the jet is near at symmetrical point "$-A_o$". Perhaps,it is most significant manifestation of the jet in terms of kinematical variale "A". This manifestation can be increased by the very simple method: in the argument of the "G" function (V.1a) one must replace $A^*_{lkmn} \Rightarrow |A^*_{lkmn}|$ (see definition (V.1a) and get the 'jet function'

$$J(A)=\sum_{l=1}^{M_k}\sum_{n=1}^{N}\sum_{m=1}^{M_n} G\{A-|A^*_{lkmn}|;\sigma_{mn}\}\sigma_{mn} \qquad \text{(VI.1)}$$

Jointed by this way, total peak of the jet will look as the prominent one at the background of "usual" particles. "Mathematical simplicity" of the J(A) allows one to use analytical method in the search for these 'prominent' peaks (jets),as it is used in the vertex determination. Moreover, for the most amplification of the jet peak, one can engage the coordinate-amplitude information of calorimeters to the construction (VI.1).

Corresponding software will be created at nearest future.

Simple track pattern (fig.5) was simulated for the illustration of the F(A). The view of the F(A) is shown in fig.6-6b. The jet function J(A) for track picture of fig.5 is shown in fig.7.

CONCLUSION

The analysis of vertex function allows one to separate vertices and to prepare of arrays for the reconstruction of single track.

The use of kinematical division of tracks data substantiates simple way for parallel reconstruction of trajectories and has the profit in the number of operations.

Analytical search for jets can be used as 'software trigger' in some experiments.

This ideology can be used at UNK,LHC,SSC;nuclear accelerators at Los-Alamos,Darmstadt etc.

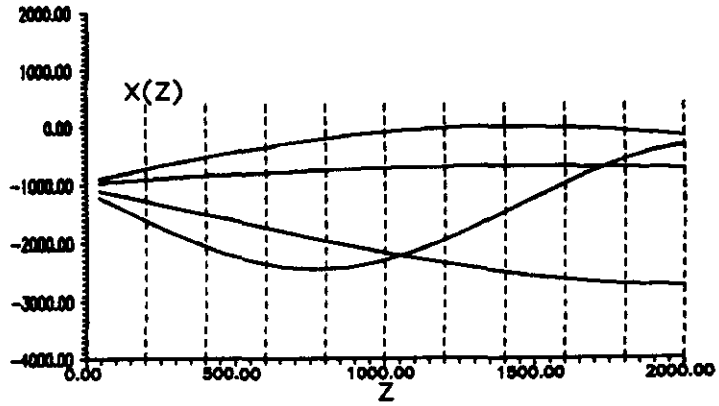


Fig.1 The simplest simulation of 4-tracks pattern. The X(Z) plane. Detectors are placed at z=200,400,600,... mm .

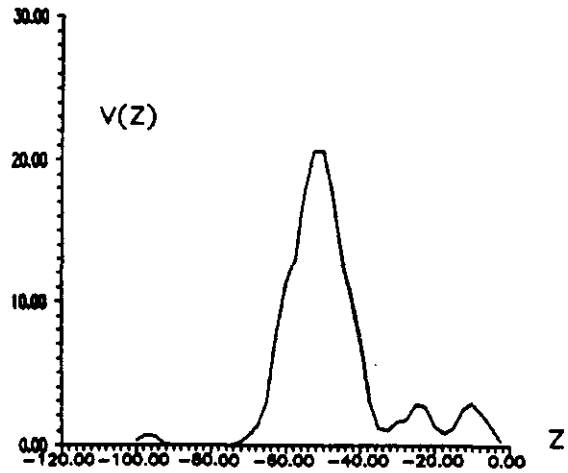


Fig.2 The view of vertex function V(Z) for track pattern shown in Fig.1.

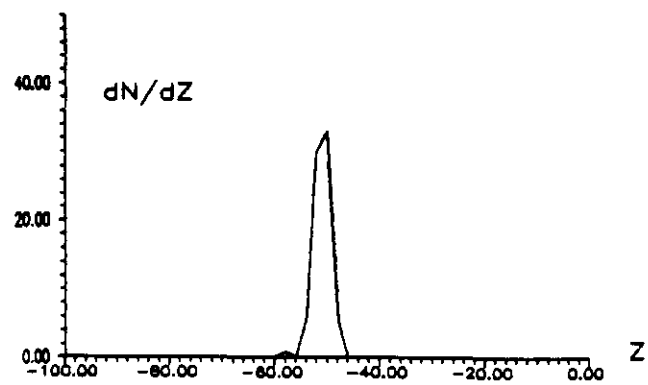
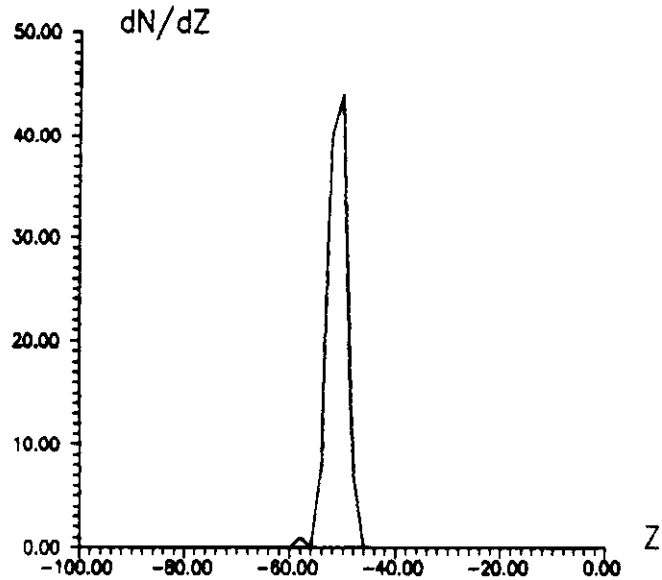


Fig.3 Statistical distribution of the vertex determination for 100 simulated events like shown in Fig.1.



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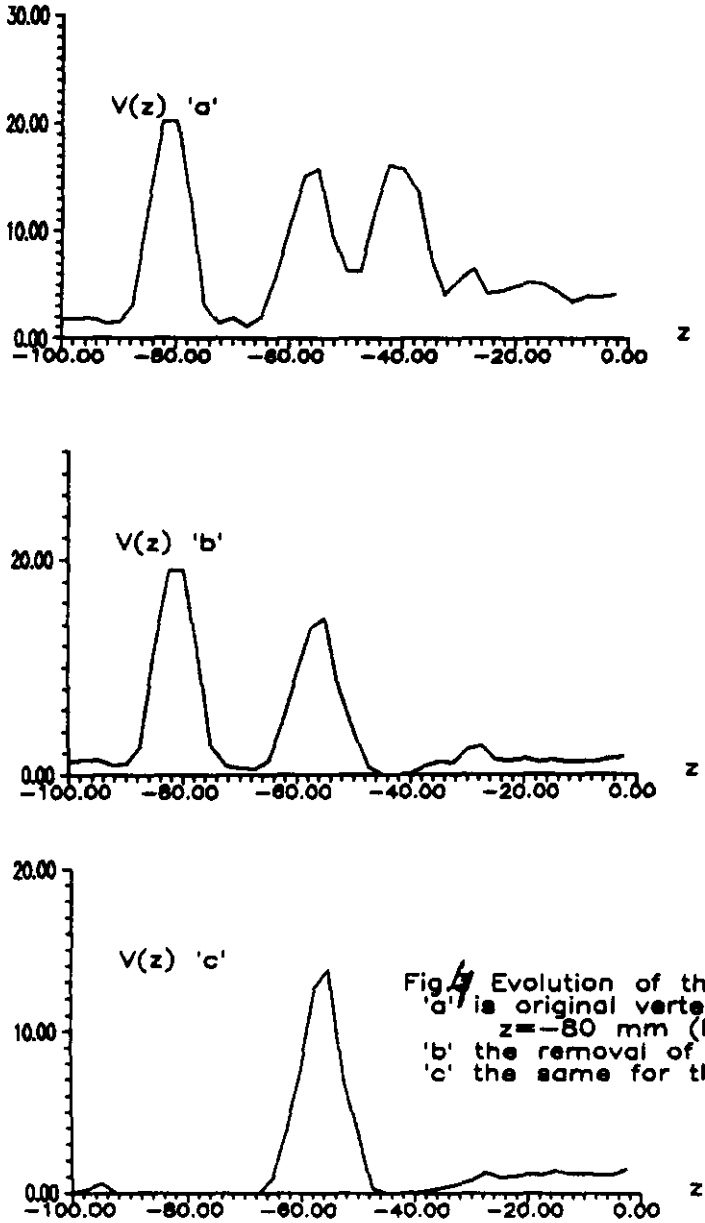

Fig.4 Evolution of the V(z) at the 'cleaning':
'a' is original vertex function for 3 vertices at z=−80 mm (Mult.=4),−55mm(M=3),−40mm(M=3);
'b' the removal of the vertex (z=−40) was done;
'c' the same for the vertex at z=−80.

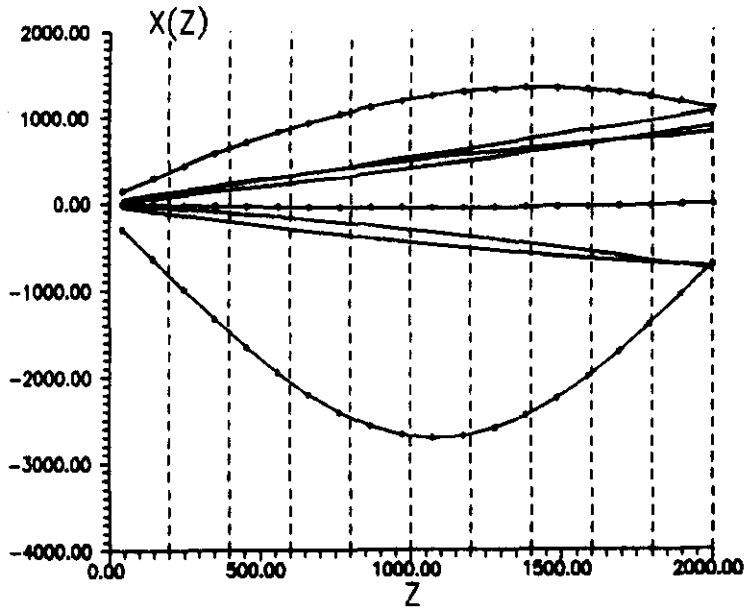

Fig.5 The view of X(Z) for 3 'usual' particles (marked by dots) and 5 'jet' particles.

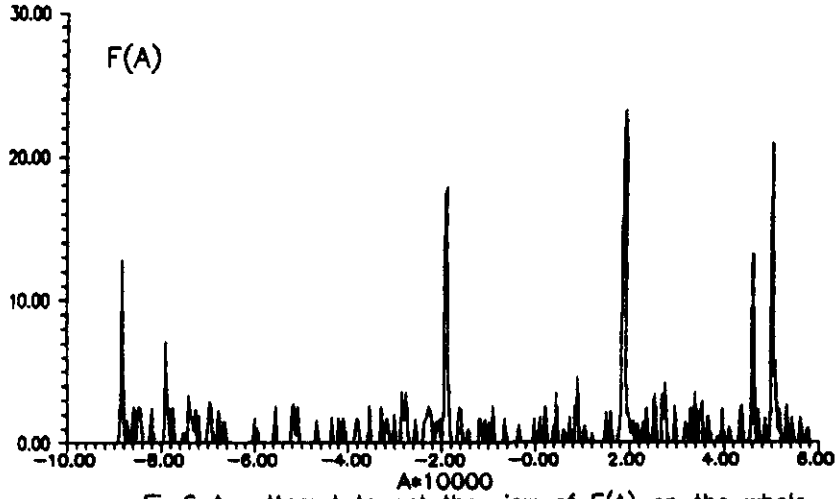

Fig.6 An attempt to get the view of F(A) on the whole. The resolution of graph does not separate single peaks (particles)~~ell~~.

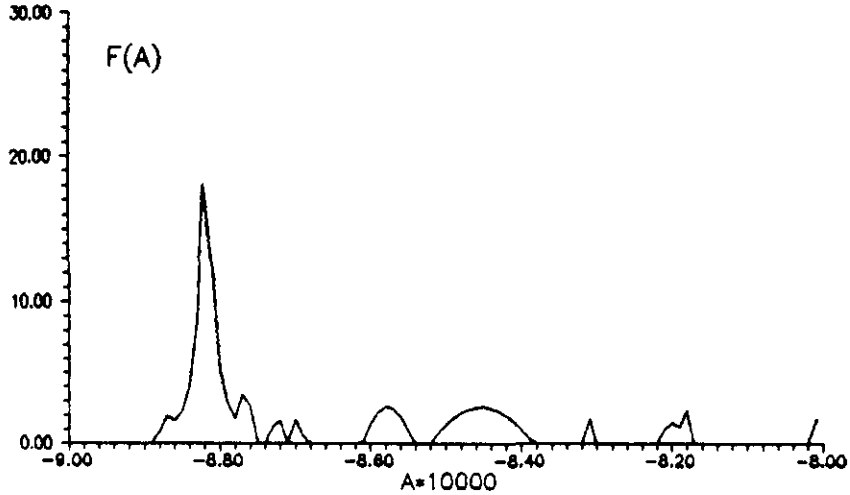

Fig.6a Same view of the F(A) for one particle at A=−8.8E−4.

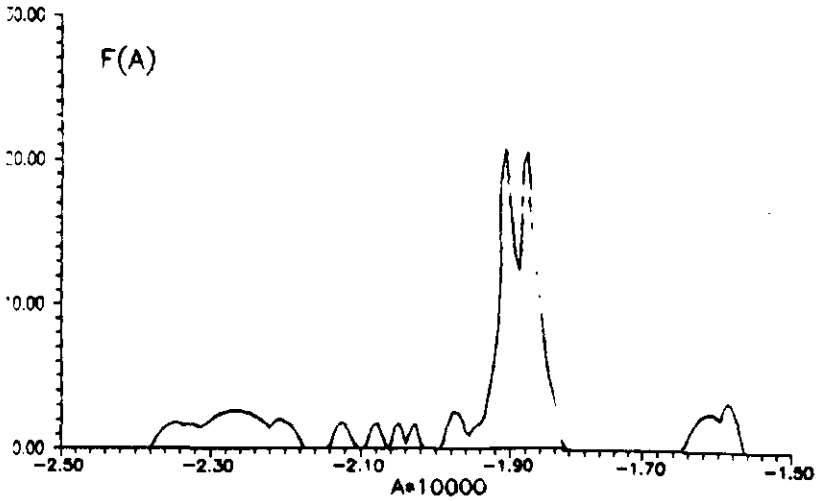

Fig.6b The piec of F(A) for 2 negati 'jet' particles.

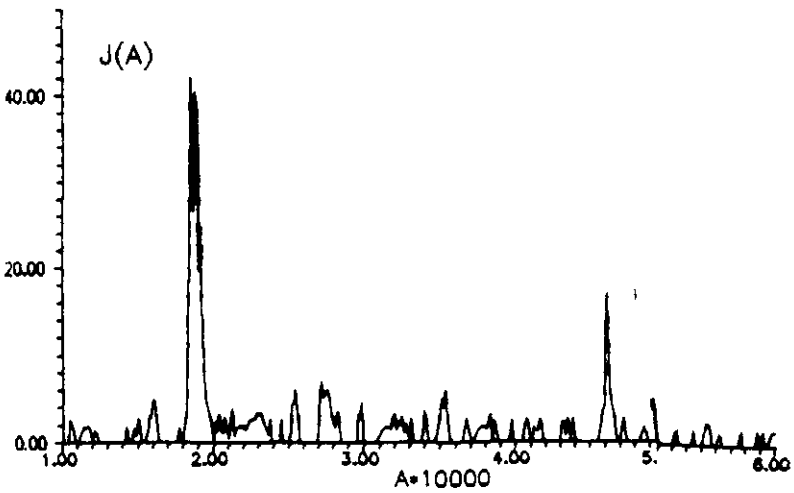

Fig.7 The 'jet' function J(A). The 'jet' looks as prominent peak at A=2.E-4 .