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Ion Loading in Liquid Ionization Calorimeters

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1. INTRODUCTION

In the high rate environments anticipated for the next round of hadron colliders such as the Superconducting Super Collider (SSC) in the US and the Large Hadron Collider (LHC) at CERN, electromagnetic and hadronic calorimeters for measuring particle energies will play a prominent role and face difficult challenges. One major concern is radiation damage. Sampling calorimeters using conventional scintillators will suffer gain variations and light attenuation from darkening of the plastic by massive doses of radiation. On the other hand, liquid Argon calorimeters can readily be made rad hard if the readout preamps are located remotely. The argon, absorber plates, charge collection pads, and cables can all be selected to stand up to the largest predicted doses. While the signal from an ionization calorimeter is slower than other technologies, it can be shaped to times not much longer than the hadronic shower development time. For this reason it is thought that liquid argon calorimeters are a very attractive option for forward calorimetry at luminosities of 10^{33} cm^{-2} $sec^{-1}$and sure to perform well as a 4π calorimeter at much higher luminosities.

While the electrons liberated from the argon atoms by passage of charged particles through the liquid argon gap drift quickly in the electric field, the positive ions move much more slowly. In the high rate environment of the SSC or LHC, there is a large rate of low energy particles constantly hitting the calorimeter. In a given unit area of calorimeter, particularly near electromagnetic shower maximum, the rate of ionization is large enough that positive ions accumulate in the gap and distort the electric field. When a high energy particle hits the calorimeter the large shower liberates excess electrons (and ions) on top of the background production. The distorted electric field degrades the signal from these drifting electrons. In this paper we calculate the effect of the build-up of positive ions on the electric field and on the signal in a single gap of an ionization calorimeter.

Fig. 1 is a schematic outline of a forward calorimeter covering the range $3 < |\eta| < 5.5$. (The calorimeter extends to $|\eta|$ above 5.5 in order to contain the EM portion of the hadronic jets at $|\eta| = 5.5$.

For specific rate calculations we will consider a sampling gap located near electromagnetic shower maximum within the circle in Fig. 1. We will find that without due care liquid ionizing calorimeters will fail to operate adequately not only at $|\eta| = 5.5$ but at significantly lower values of $|\eta|$ as well.

2. PRINCIPLE OF AN IONIZATION CALORIMETER

[Source of ionization. 23.6 ev/ion pair. Induced currents, a single electron drifting in a gap. Refer to Radeka here. Perhaps talk about columnar and/or initial recombination.]

3. PLAN OF ATTACK

[Need to develop analytic expressions to explore parameter space. Computer model is used to verify approximations made to get analytic expressions.]

4. ELECTRON AND ION DRIFT VELOCITIES

[Review the experimental data on electron and positive ion drift velocities. Discuss the different possible mechanisms of positive ion drift, i.e. ion motion or hole tunneling. In the case of ion motion is it one atomic ion or molecular ions or several species? And discuss problems with the measurements of positive ion drift velocity measurements. Is bulk flow set up in some of the measurements? Are the indirect measurements sensitive to other unknown parameters such as recombination rates? Stress the large range of speeds that have been measured by different groups.]

[Outline the approximations that will be made for the analytic model. And the definitions such as (v_+/v_-) and (μ_+/μ_-). Refer to Fig. 2.]

5. LINEAR MODEL AND SIMPLIFYING ASSUMPTIONS

The mathematics of the above approach quickly becomes sufficiently complicated such as to obscure some of the simpler aspects of the problem. For this reason we develop here a one-dimensional model which allows simple calculations but which must be used with care.

When the gap width a is small compared to the transverse dimensions then assuming that the plates are infinitely large is a good approximation. The one dimensional model assumes that charges are deposited in the gap in planes parallel to the plates of uniform surface charge density σ. Here *uniform* means independent of x and y. This is equivalent, in some sense, to an electrostatics problem defined along a line, e.g. along the z axis in

our case.

For a single ionizing particle traversing a gap at a small angle to the z axis this is a poor approximation. But for electromagnetic showers near shower maximum with Moliere radius of several centimeters the ionization density doesn't change in x and y so rapidly on the scale of the gap width a of 2 mm. So a linearized model, while not an excellent description, makes more sense for showers.

Fig. 2 shows the geometry of the gap we will consider throughout this paper. The electric field is aligned along the positive z direction. So the electrode at $z = 0$ is at potential V_o and the electrode at $z = a$ is at ground. A very schematic readout is indicated by an ammeter. We will assume a zero impedance readout so that the potential across the gap is always at V_o.

5.1 Example 1

The surface charge density on the plate at $z = 0$ is $\sigma(0) = \epsilon E(0)$ and on the plate at $z = a$, $\sigma(a) = -\epsilon E(a)$. With no charges in the gap, $E(0) = E(a) = V_o/a$ and $\sigma(a) = -\sigma(0)$ so the plate at $z = 0$ is positively charged and the plate at $z = a$ is negatively charged. Now we introduce in the gap a sheet of surface charge density σ_+ ($\ll \epsilon V_o/a$) at $z = z_+$. Gauss' Law for this geometry gives

$$\epsilon \frac{\partial E}{\partial z} = \rho(z) \tag{5.1}$$

and setting $\rho(z) = \sigma_+ \delta(z - z_+)$ we get

$$E = E_o + \theta(z - z_+)\sigma_+/\epsilon \tag{5.2}$$

where E_o is chosen so that $\int_0^a E dz = V_o$. We find $aE_o + (a - z_+)\sigma_+/\epsilon = V_o$ or

$$E = \begin{cases} V_o/a - (1 - z_+/a)\sigma_+/\epsilon & 0 < z < z_+ \\ V_o/a + (z_+/a)\sigma_+/\epsilon & z_+ < z < a. \end{cases} \tag{5.3}$$

Now suppose the charge sheet σ_+ moves to the right with velocity v_+, then the charge density on the plates changes as

$$\frac{d\sigma(0)}{dt} = -\frac{d\sigma(a)}{dt} = \epsilon \frac{dE(0)}{dt} = \epsilon \frac{dE(0)}{dz_+}\frac{dz_+}{dt} = \epsilon \frac{dE(0)}{dz_+} v_+ = \frac{v_+ \sigma_+}{a}. \tag{5.4}$$

The changing surface charge density on the plates requires that a current flow through the external circuit. Per unit area of the plates, this current is given by

$$J = v_+ \sigma_+ / a \tag{5.5}$$

which is equivalent to the average current density of the moving charge sheet within the gap. That is, a current is *induced* in the external circuit by the moving charges within the

gap. If 1 C/cm^2 moves through the gap from the plate at $z = 0$ to the plate at $z = a$, then 1 C/cm^2 of electrical charge will flow from one plate to the other through the external circuit as well.

Fig. 4a shows the electric field within the gap as a function of z. Fig. 4b shows $E(a)$ and $E(0)$, the electric field at the surface of the two plates, as a function of z_+, the location of the charge sheet. Fig. 4c shows $E(a)$ and $E(0)$ as a function of time assuming the charge sheet is initially placed at $z_+ = 0$ and then drifts at uniform velocity v_+ towards the plate at $z = a$. Fig. 4d shows the induced current due to this drifting charge. Note that when the charge sheet hits the plate at $z = a$, i.e. when $z_+ = a$, then $E(a)$ changes abruptly so that the induced current exhibits a negative delta function of area σ_+. This is exactly balanced by the charge current due to the charge sheet reaching the plate at $z = a$ so that the total current, induced plus charge, has no negative delta function.

5.2 Example 2

Suppose next that between the same two plates we introduce in the gap a sheet of positive surface charge density of positive ions $\sigma_+ = \sigma_o$ at $z = z_+$ *and* a sheet of negative charge density of electrons $\sigma_- = -\sigma_+ = -\sigma_o$ also at $z = z_+$. In this example the electric fields at the plates are V_o/a as they were with no charges in the gap. This is a somewhat more realistic example corresponding, in the 3-D case, to an ion pair created within the gap by, say, photo-ionization.

For simplicity, and as a realistic approximation, suppose that the drift velocity of the electrons is v_- and that the positive ions are immobile. Then at some later time the electron charge sheet will have drifted to the left to position $z = z_-$ and the positive ion charge sheet will remain at $z = z_+$. It is easy to show that the electric field in the gap (always requiring $\int_0^a dzE = V_o$) will be

$$E = \begin{cases} V_o/a + (z_+/a - z_-/a)\sigma_o/\epsilon & 0 < z < z_- \\ V_o/a + (z_+/a - z_-/a - 1)\sigma_o/\epsilon & z_- < z < z_+ \\ V_o/a + (z_+/a - z_-/a)\sigma_o/\epsilon & z_+ < z < a \end{cases} \tag{5.6}$$

and that the change in the surface charge density at each of the plates is

$$\frac{d\sigma(0)}{dt} = -\frac{d\sigma(a)}{dt} = \epsilon\frac{dE(0)}{dt} = \sigma_o v_-/a \tag{5.7}$$

and so the induced current per unit area of plate in the external circuit is again $J = \sigma_o v_-/a$.

Fig. 4e shows the electric field within the gap as a function of z. Fig. 4f shows $E(a)$ and $E(0)$, the electric field at the surface of the two plates, as a function of $z_+ - z_-$, the distance between the separating charge sheets. Fig. 4g shows $E(a)$ and $E(0)$ as a function of time assuming that $z_+ = a$, i.e. that the two charge sheets started at $z = a$ (or just a small amount less than $z = a$). Fig. 4h shows the induced current due to the separating charge sheets.

Generalizing this example to the case where the positive and negative surface charge densities aren't necessarily equal and both charge carriers can drift we will find

$$J = \sigma_+ v_+/a + |\sigma_-|v_-/a \tag{5.8}$$

When $\sigma_+ = -\sigma_-$ and $v_+ = v_-$, twice the current will be observed in the external circuit compared to the case above with immobile positive ions. But usually $v_+ \ll v_-$.

5.3 Example 3

For our next example, assume that the whole gap is uniformly filled with immobile positive ions of charge density $\rho_+ = \rho_o$ ($\rho_+ \ll V_o\epsilon/a^2$) and mobile electrons of charge density $\rho_- = -\rho_o$ such that the total charge density within the gap is zero and the electric field is V_o/a everywhere as it was with no charge in the gap at all. This case is the linearized analog of the ionization deposited by a high energy charged particle traversing the gap in the z direction leaving uniform ionization. And, as we argued earlier, it is an approximation to an electromagnetic shower near shower maximum.

After some time the electrons will drift to the left (and some will collect on the plate at $z = 0$) leaving a region of the gap from z_o to a free of electrons. (We're neglecting diffusion.) Using Gauss' Law we find the electric fields in the gap at this time are

$$E = \begin{cases} V_o/a - (1 - z_o/a)^2\rho_o a/2\epsilon & 0 < z < z_o \\ V_o/a - (1 - z_o/a)^2\rho_o a/2\epsilon + (z - z_o)\rho_o/\epsilon & z_o < z < a \end{cases} \tag{5.9}$$

In this case we find at the plates ($z_o = a - v_-t$)

$$\begin{aligned} \epsilon\frac{dE(0)}{dt} &= \frac{d\sigma(0)}{dt} = -\rho_o v_-(1 - z_o/a) = -\rho_o v_-^2 t/a \\ \epsilon\frac{dE(a)}{dt} &= \frac{-d\sigma(a)}{dt} = +\rho_o v_-(z_o/a) = (\rho_o v_-/a)(a - v_-t) \end{aligned} \tag{5.10}$$

The induced currents aren't equal. But electrons are being deposited on the plate at $a = 0$ at a rate of $\rho_o v_-$ per unit area so the induced current plus the charge current at the plate at $z = 0$ equals the induced current at the plate at $z = a$. The current in the external circuit per unit area of the plates is

$$J = (\rho_o v_-/a)(a - v_-t) \quad 0 < t < a/v_- \tag{5.11}$$

Fig. 4i shows the electric field within the gap as a function of z at time $t = (a - z_o)/v_-$. Fig. 4j shows $E(a)$ and $E(0)$ as a function of z_o. Fig. 4k shows $E(a)$ and $E(0)$ as a function of time and Fig. 4l shows the induced current at $z = 0$ and $z = a$.

The total charge (per unit cross section of collection gap) flowing through the ammeter of Fig. 3 is obtained by integrating equation 5.11. We get

$$Q = \frac{a\rho_o}{2}. \tag{5.12}$$

Note that only half the electrons produced in the gap via ionization traverse the external circuit. For an electron created via ionization at z, only z/a of an electron traverses the external circuit as this drifting electron moves from z to 0.

For the case where the positive ions are also mobile but $v_+ \ll v_-$,

$$J = J_+ + J_- \tag{5.13}$$

$$J_- = \begin{cases} (\rho_o v_-/a)(a - v_- t) & 0 < t < a/v_- \\ 0 & \text{otherwise} \end{cases} \tag{5.14}$$

$$J_+ = \begin{cases} (\rho_o v_+/a)(a - v_+ t) & 0 < t < a/v_+ \\ 0 & \text{otherwise} \end{cases} \tag{5.15}$$

All of the above assumes that ionization is deposited in the gap at one instant of time $t = 0$.

5.4 Example 4

We will now consider an artificial example that will be instructive later. We assume the whole gap is uniformly filled with immobile positive ions of charge density $\rho_+ = \rho_o$ ($\rho_- \ll V_o\epsilon/a^2$) and mobile electrons of charge density $\rho_- = -\rho_o$ such that the total charge density within the gap is zero. This is the same as in example 3. But here we assume that the electrons to the right of z_r (where $0 < z_r < a$) drift to the left with drift velocity v_- but the electrons to the left of z_r remain stationary. So with time, charge builds up at $z = z_r$. The situation at time $t = (a - z_o)/v_-$ (where $z_o > z_r$) is shown in Fig. 5. The charge sheet at z_r has charge per unit area

$$\sigma = -\rho_o(a - z_o). \tag{5.16}$$

The electric field in the gap is found from Gauss' Law and by requiring the potential across the gap to remain fixed at V_o. When evaluated at $z = a$ we find

$$E(a) = \frac{V_o}{a} - \frac{\sigma}{\epsilon a}(a - z_r) - \frac{\rho_o}{2\epsilon a}(a - z_o)^2 + \frac{\sigma}{\epsilon} + \frac{\rho_o}{\epsilon}(a - z_o). \tag{5.17}$$

When $t = (a - z_r)/v_-$, i.e. when $z_o = z_r$, then

$$E(a) = \frac{V_o}{a} + \frac{\rho_o}{2\epsilon a}(a - z_r)^2. \tag{5.18}$$

The total charge (per unit cross section of collection gap) which flows through the ammeter of Fig. 3 is

$$Q = \frac{a\rho_o}{2}\left(1 - \frac{z_r}{a}\right)^2. \tag{5.19}$$

which is less than in example 3. (See equation 5.12.)

5.5 Example 5

One last artificial example will also be useful later. This case is the same as in the previous subsection but now the drifting charge does not collect at z_r. Imagine that a grid is placed at z_r which collects all the charge as it drifts from the right and which is held at V_o so that the electric field in the region $0 < z < z_r$ is zero. This case is quite easy to solve and corresponds to a gap of width $a - z_r$ rather than the gap of width a which we have considered throughout. The charge collected will be

$$Q = \frac{a\rho_o}{2}\left(1 - \frac{z_r}{a}\right). \tag{5.20}$$

This is smaller than example 3 but larger than example 4.

6. CHARGE BUILD-UP IN A LIQUID IONIZATION GAP

6.1 The Scale of the Problem

Let us consider, for the moment, a worst case condition that might actually be realized some years after turn-on at the SSC or the LHC. At a luminosity of 10^{34} cm^{-2} sec^{-1}there are of order 16 minimum bias interactions per 16 nsec beam crossing. A forward calorimeter might be located 12 m from the interaction point and have coverage out to $|\eta| = 5.5$. At this value of $|\eta|$ there will be 0.1 photons from π^o decay per cm^2 with average energy 40 GeV incident on the surface of the calorimeter. At a depth within the calorimeter near electromagnetic shower maximum the electromagnetic shower size is of order 2 cm in radius so that it covers an area of order 10 cm^2. So at a given point within a gap near electromagnetic shower maximum and near $|\eta| = 5.5$ there will be one electromagnetic shower per beam crossing. For the typical 2 mm gap width of liquid argon the time to drift out the electrons is of order 400 nsec so we can view the deposition of ionization as approximately constant in time. In section 10 we will discuss fluctuations. The positive ions drift far more slowly and can build up to appreciable levels within the gap.

The net creation rate of ion pairs per unit volume within the gap will be given by $D(z)$. We will consider only two contributions, ionization and recombination. As an approximation we assume that all charged particles traversing the gap lose only a small fraction of their energy so that the ionization is independent of z. We also assume the ionization is uniform in x and y as argued above. Also as an approximation we consider only one form of recombination. As a result we take the recombination rate per unit volume to be proportional to the product of electron density and positive ion density. Thus

$$D(z) = D_i - D_r = D_i - R_r n_+ n_-. \tag{6.1}$$

D_i is the rate per unit volume of production of ion pairs via ionization while D_r is the rate per unit volume of recombination of ion pairs. R_r is the recombination rate per unit

volume for unit density. So we are neglecting columnar recombination. This might be justified by arguing that many of the charged particle tracks near electromagnetic shower maximum are sufficiently oblique to the electric field direction due to multiple scatter. We also neglect for now the fact that initial recombination takes place when the electric field is small enough. There will be instances where this neglect is not warranted.

At what rate of deposition of ionization does the positive charge accumulation significantly distort the electric fields and modify the signal? Let us start with a very qualitative argument to set the scale before we begin a more precise analysis. Consider (again?) the gap in Fig. 3 with the plate at the left at electrical potential V_o and the plate at the right at ground. Then the electric field within the gap of width a is $E = V_o/a$ and is directed to the right. The surface charge density on each plate is $|\sigma| = \epsilon|E| = \epsilon V_o/a$. We will say that the electric field within the gap is ***significantly*** altered when the number of positive ions distributed across the gap equals the number of charges on the two plates. We will call this a critical condition. Thus

$$\rho_c = en_c = 2|\sigma|/a = 2V_o\epsilon/a^2 \tag{6.2}$$

where ρ_c is the critical charge density and n_c is the corresponding critical number density where we assume that the positive ions are singly charged. If the positive ions drift at the speed $v_+ = \mu_+ V_o/a$ and, to be removed from the gap, a positive ion must drift a distance $a/2$ on average, then the characteristic time to clear the gap is $\tau = a/2v_+ = a^2/2V_o\mu_+$. We will define the critical ionization rate, D_c, as that rate of ion pair creation per unit volume equal to the rate at which the critical density of positive ions is removed from the gap, i.e.

$$D_c \equiv \frac{n_c}{\tau} = \frac{4V_o^2\epsilon\mu_+}{ea^4}. \tag{6.3}$$

This corresponds to a critical current density of positive ions at the plate on the right of

$$J_c = eaD_c = \frac{4V_o^2\epsilon\mu_+}{a^3} = \frac{2V_o\mu_+}{a}\rho_c = 2v_+\rho_c. \tag{6.4}$$

We can also define a critical recombination rate per unit volume for unit density as

$$(R_r)_c = D_c/n_c^2 = \frac{e}{\epsilon}\mu_+ \tag{6.5}$$

and a relative recombination rate per unit volume for unit density as

$$\mathcal{R} = \frac{R_r}{(R_r)_c} \tag{6.6}$$

so that

$$D(z) = D_i - \mathcal{R}(D_c/n_c^2)n_+n_-. \tag{6.7}$$

Finally we define the relative rate per unit volume of ionization as

$$r \equiv \frac{D_i}{D_c} \tag{6.8}$$

so that

$$D(z) = D_c \left[r - \mathcal{R} \left(\frac{n_+}{n_c} \right) \left(\frac{n_-}{n_c} \right) \right]. \tag{6.9}$$

6.2 The Defining Equations and Constraints

Referring again to Fig. 3 with the electric field directed to the right, the electrons drift to the left and the positive ions, much more slowly, to the right. An imaginary volume dV of width dz and area dA is shown in the figure. It is easy to show that

$$\partial_\mu j_+^\mu = -\partial_\mu j_-^\mu = eD(z)$$

where $j_\pm^i = \mathrm{v}_\pm \rho_\pm$ and $j_\pm^o = \rho_\pm$. In our case $j_\pm^x = j_\pm^y = 0$ and we use $j_\pm^z \equiv J_\pm(z)$. For $D(z)$ constant in time a steady state condition will eventually be reached so that

$$\frac{\partial \rho_\pm}{\partial t} = 0 \quad \text{and} \quad \frac{\partial J_\pm}{\partial z} = \pm eD(z). \tag{6.10}$$

Since electrons do not emanate from the plate at the right nor do positive ions from the plate at the left we have the boundary conditions

$$J_+(0) = 0 \quad \text{and} \quad J_-(a) = 0 \tag{6.11}$$

which gives

$$J_+(z) = e \int_0^z dz' D(z') = \mathrm{v}_+ \rho_+ = (\mu_+ E)\rho_+ \tag{6.12}$$

$$J_-(z) = e \int_z^a dz' D(z') = \mathrm{v}_- \rho_- = \begin{cases} v_- |\rho_-|; & E > E_o \\ (\mu_- E)|\rho_-|; & E < E_o \end{cases} \tag{6.13}$$

and

$$J = J_+(z) + J_-(z) = e \int_0^a dz' D(z') \tag{6.14}$$

which is constant independent of z as required by charge conservation.

Gauss' Law for our geometry gives

$$\epsilon \frac{dE}{dz} = \rho = \rho_+ + \rho_- \tag{6.15}$$

so that

$$E = \frac{1}{\epsilon} \int_0^z dz' \rho(z') + c \tag{6.16}$$

and the constant of integration must be adjusted to give

$$\int_0^a dz E = V_o. \tag{6.17}$$

The surface charge density on the plate on the left is given by

$$\sigma(0) = \epsilon E(0) \tag{6.18}$$

while the surface charge density on the plate at the right is

$$\sigma(a) = -\epsilon E(a). \tag{6.19}$$

The electric field is zero to the left of the plane at $z = 0$ and to the right of the plane at $z = a$ so the net charge contained between these two planes must be zero. This condition is already ensured by the above set of equations, i.e. it is easy to show that

$$\sigma(0) + \sigma(a) + \int_0^a dz\rho(z) = 0 \tag{6.20}$$

for all times.

6.3 Solutions to the Steady State Equations

In this section we will find steady state solutions which satisfy equations 6.7, 6.12, 6.13, 6.16, and 6.17 for various ranges of r where r is given by equation X.8. These form a set of non-linear coupled differential equations whose solutions must be found by trial and error.

The Case $r = 0$

When $r = 0$ there is no production of ionization in the gap, i.e. $D(z) = 0$. Thus $J_+(z) = J_-(z) = 0$. Also $\rho_+(z) = \rho_-(z) = 0$ and $E(z) = V_o/a$.

The Case $0 < r \ll 1$

Here the rate of ionization in the gap is very small so there is very little charge build-up. Therefore the recombination rate can be completely neglected, i.e. $D(z) = D_i$. Thus

$$J_+(z) = ezD_i \quad \text{and} \quad J_-(z) = e(a - z)D_i. \tag{6.21}$$

Using definitions 6.3, 6.4, and 6.8 we can rewrite the above as

$$J_+/J_c = (z/a)r \quad \text{and} \quad J_-/J_c = (1 - z/a)r. \tag{6.22}$$

From equations 6.12 and 6.13 we see that $|\rho_-| \ll \rho_+$ because $v_- \gg \mu_+ E$ so we will neglect ρ_-. Equation 6.12 can be written

$$J_+/J_c = (z/a)r = \frac{1}{2}\left(\frac{Ea}{V_o}\right)\frac{\rho_+}{\rho_c} \tag{6.23}$$

As a first order approximation we take $E = V_o/a$ which gives

$$\rho_+ = 2r(z/a)\rho_c. \tag{6.24}$$

Now we use equations 6.16 and 6.17 to give a second order approximation for E.

$$E = \frac{V_o}{a}\{1 + 2r[(z/a)^2 - 1/3]\}. \tag{6.25}$$

The series of plots in Fig. 6a depict this case.

The Case $0 < r < 1$

While the positive ions do build up to significant levels for r close to one, the electrons do not. Then it is still safe to neglect the recombination rate compared to the ionization rate so we continue to assume $D(z) = D_i$. Thus equations 6.21 and 6.22 apply here as well. And we also neglect ρ_- relative to ρ_+. A solution is

$$\frac{\rho_+}{\rho_c} = \frac{2r(z/a)}{\sqrt{4r(z/a)^2 + k}} \tag{6.26}$$

$$E = \frac{V_o}{a}\sqrt{4r(z/a)^2 + k} \tag{6.27}$$

where

$$k = \frac{4}{[\sqrt{1+y} + \sqrt{\frac{1}{y}}\ln(\sqrt{y} + \sqrt{1+y})]^2}; \quad y \equiv \frac{4r}{k}, \tag{6.28}$$

a transcendental equation whose solution is plotted in Fig. 7. Note that when

$$r = 0; \quad k = 1 \quad \text{and when}$$
$$r = 1; \quad k = 0.$$

An example of this case can be seen in the plots of Fig. 6b. Now we can check the validity of our approximations. Using equations 6.13, 6.22, and 6.4 we have

$$|\rho_-(z)| = \frac{J_-(z)}{v_-} = 2\frac{v_+}{v_-}(1 - z/a)r\rho_c \tag{6.29}$$

if $E > E_o$ which is true as long as r is not too close to unity. Since $v_+/v_- \ll 1$, the neglect of ρ_- relative to ρ_+ is valid except for very small z where ρ_+ goes to zero. As r approaches unity, the electric field at small z falls below E_o so that the region of small z where ρ_- cannot be neglected relative to ρ_+ grows a bit.

The recombination rate is negligible if

$$r \gg \mathcal{R}\left(\frac{\rho_+}{\rho_c}\right)\left(\frac{-\rho_-}{\rho_c}\right) = \mathcal{R}\frac{4r^2(z/a)(1 - z/a)}{\sqrt{4r(z/a)^2 + k}}\left(\frac{v_+}{v_-}\right) \tag{6.30}$$

or

$$\mathcal{R} \ll \frac{\sqrt{4r(z/a)^2 + k}}{4r(z/a)(1 - z/a)}(\frac{v_-}{v_+}). \tag{6.31}$$

The right hand side is the smallest when $z = z_w$ where

$$z_w = \frac{a}{2}\left(\frac{k}{r}\right)^{\frac{1}{3}}\left[\left\{1 + \sqrt{1 + \frac{8}{27}\frac{k}{r}}\right\}^{\frac{1}{3}} + \left\{1 - \sqrt{1 + \frac{8}{27}\frac{k}{r}}\right\}^{\frac{1}{3}}\right] \tag{6.32}$$

$$z_w = \begin{cases} a/2 & \text{when} \quad r \to 0 \\ 0 & \text{when} \quad r \to 1. \end{cases}$$

At this worst case value of z, the expression becomes

$$\mathcal{R} \ll F(r) \tag{6.33}$$

where $F(r)/(v_-/v_+)$ is plotted in Fig. 8.

The Case $r = 1$

Continuing the solutions of the previous subsection, equations 6.26 and 6.27, to $r = 1$ gives

$$\frac{\rho_+}{\rho_c} = 1 \tag{6.34}$$

$$E = \frac{V_o}{a}\frac{2z}{a} \tag{6.35}$$

$$\frac{\rho_-}{\rho_c} = 2\frac{v_+}{v_-}(1 - z/a) \tag{6.36}$$

Again this is the approximate solution, shown in Fig. 6c, which is valid for $|\rho_-| \ll \rho_+$. Further we must assume

$$\mathcal{R} \ll \frac{1}{2}\left(\frac{v_-}{v_+}\right) \tag{6.37}$$

This solution is clearly not correct very near $z = 0$.

The Case $r > 1$

The first order solution for this case is shown in Fig. 6d where

$$z_r = a\left(1 - \frac{1}{r^{\frac{1}{4}}}\right) \tag{6.38}$$

The most striking feature is that for $z < z_r$ the electric field in the gap is (nearly) zero. We will say that this part of the gap has "closed down". In this region the positive and

negative number densities are equal, i.e. $n_+ = n_-$, so the net charge density, ρ, is zero. Rather than drifting out of the gap, the ions in this region are recombining at the same rate they are produced. The net ion creation rate, $D(z)$, is zero in this region. Nevertheless there are free ions in this region. In fact there is a large reservoir of highly mobile electrons.

The rest of the gap behaves very much like the case $r = 1$ but scaled by the smaller effective gap size. Note also that the maximum value of the electric field occurs at $z = a$ and has a value

$$E(a) = 2r^{\frac{1}{4}}\frac{V_o}{a} \tag{6.39}$$

7. HIGHER ORDER CORRECTIONS

The solutions obtained so far are approximate. We explained in section 4 how we would treat the drift velocities in order to obtain analytic solutions. For the cases $r < 1$ and $r = 1$ we have, in addition, assumed that we could neglect the charge density due to electrons, ρ_-, in comparison to ρ_+, i.e. we assumed $\rho \simeq \rho_+$. [Refer to Fig. 9.]

8. DEGRADATION OF THE SIGNAL IN THE CHARGE BUILD-UP ENVIRONMENT

The source of ionization in sections 6 and 7 is from the approximately steady showering of π^o decay photons from minimum bias events. These photons have $\langle p_T \rangle \sim 0.3$ GeV/c which is very small on the scale of interest. We now want to investigate what happens when a high energy event comes in on top of this steady state background. Imagine, for instance, that a jet of $p_T = 100$ GeV strikes the calorimeter. Ionization in excess of the steady state value will be produced in the gap. The electric field will cause this excess charge to drift out of the gap. So later the ionization will return to the steady state value. The current through the external circuit will increase above its steady state value while the excess ionization drifts out of the gap. It is this current in excess of the steady state value that constitutes the signal we wish to measure.

In section 5 we discussed several examples where different patterns of ionization produced current waveforms in an external circuit when the ions drifted in the gap. Only example 3 was apparently relevant. The others were for pedagogic purposes. The last two examples will be seen to be of interest to us now. In the examples of section 5 there were no charges in the gap before the event that caused the ionization. But we now wish to consider a small transient signal on top of a large steady state current.

But first let us be more specific about what we mean by the signal from a shower in the calorimeter gap of interest. To keep the discussion simple we will assume that the signal is the amount of charge which flows through the ammeter of Fig. 3 during the time that electrons flow through the gap. We will completely ignore the small contribution from

the much slower-moving positive ions. That is we will accumulate charge for the time that it takes electrons to move out of the gap but we will not wait for any significant number of positive ions to drift out of the gap. Further we will take only the charge from the high energy shower, i.e. we will subtract off the steady state current.

To get a first approximation we will assume that the electron drift velocity is independent of electric field for all but the smallest values of electric field. I.e. referring to Fig. 2 we assume that $E_o \ll V_o/a$. Now consider the cases $r < 1$ of section 6. The electrons from the instantaneous ionization deposited by a high energy shower on top of the steady state ionization from minimum bias events will drift out of the gap in exactly the same way as in example 3 of section 5.3. While the electric field in the gap is quite different from the case considered in section 5.3 (and displayed in Fig. 4i) the change in the electric field (from that shown in Fig. 6a or 6b) due to the instantaneous deposition of ionization is the same as the change in Fig. 4i from V_o/a. So the signal is the same as in section 5.3. The accumulated charge per unit area of collection gap (the signal as defined above) is (see equation 5.12)

$$Q = \frac{a\rho_o}{2}. \tag{8.1}$$

We will take this as the standard signal and refer all other signals to it. For $r \geq 1$ the signal is less than this standard signal. We develop an approximate expression for the relative signal here.

As outlined above, we assume that at $t = 0$ there is some instantaneous ionization deposited on top of the steady-state charge density ρ (see Fig. 6d) leaving excess charge density $\delta\rho_+ = -\delta\rho_- = \rho_o$ such that $\delta\rho = 0$ at time $t = 0$ and $\rho_o \ll \rho_c$. We will assume that the positive ions remain fixed on the time scales of interest to us here. In the region $z > z_r$, the electrons drift to the left at velocity v_- independent of any changes in the electric field because $E > E_o$. In the region $z < z_r$ the electrons drift so slowly that, on the time scale of interest, no charges drift a significant distance from $z = z_r$. So a charge sheet will build up at $z = z_r$ of magnitude σ. At some time t after the instantaneous ionization, the charge density in the gap, in excess of the steady-state values, is as shown in the top three plots of Fig. 5 except that the size of the delta function charge build-up at $z = z_r$ will not, in general, be the same. As in section 5, we have $z_o = a - v_-t$. Due to this excess charge density the electric fields in the gap are modified as follows:

$$\delta E = \begin{cases} E_1 & 0 < z < z_r \\ E_1 + \sigma/\epsilon & z_r < z < z_o \\ E_1 + \sigma/\epsilon + (z - z_o)\rho_o/\epsilon & z_o < z < a \end{cases} \tag{8.2}$$

where E_1 and σ are to be determined. We require as always that the potential across the gap be V_o. In this case where the steady-state charge distribution already establishes the electrical potential we have $\int_0^a \delta E dz = 0$. Solving for E_1 we get

$$E_1 = -\frac{\sigma}{\epsilon a}(a - z_r) - \frac{\rho_o}{2\epsilon a}(a - z_o)^2 \tag{8.3}$$

which gives

$$\delta E(z < z_r) = E_1. \tag{8.4}$$

The charge build-up at $z = z_r$ comes from two contributions, 1) the electrons in the region $z < z_r$ drifting slowly to the left at velocity $\mu_- E$ leaving behind positive charge at z_r and 2) electrons drifting from the region $z > z_r$ at velocity v_- piling up at z_r. We now solve for this charge build-up $\sigma(t)$.

$$\begin{aligned} d\sigma &= [\mu_- \left(E(z_r) + \delta E(z_r^-)\right) |\rho_-(z_r^-)| - v_- |\rho_-(z_r^+)|]dt \\ &= \left[\mu_- \sqrt{\frac{r}{\mathcal{R}}} \rho_c \left(E(z_r) + \delta E(z_r^-)\right) - v_- \left(2r^{3/4} \left(\frac{v_+}{v_-}\right) \rho_c + \rho_o\right)\right] dt \\ &= \left[\mu_- \sqrt{\frac{r}{\mathcal{R}}} \rho_c E_1 - v_- \rho_o\right] dt \end{aligned} \tag{8.5}$$

In the last step above the steady state parts have cancelled as they must when $\rho_o = 0$. We must solve the differential equation

$$\frac{d\sigma}{dt} = -\left[\mu_- \sqrt{\frac{r}{\mathcal{R}}} \rho_c \frac{a - z_r}{\epsilon a}\right] \sigma - \left[\mu_- \sqrt{\frac{r}{\mathcal{R}}} \rho_c \rho_o \frac{v_-^2}{2\epsilon a}\right] t^2 - v_- \rho_o \tag{8.6}$$

with boundary condition $\sigma(0) = 0$. We then evaluate the solution for $\sigma(t)$ at time

$$t_o = \frac{a - z_r}{v_-} = \frac{a}{r^{1/4} v_-} \tag{8.7}$$

i.e. when $z_o = z_r$ which is the time that the signal current goes to zero and the signal charge has been collected. We then evaluate $\delta E(a)$ to give us the signal.

$$\sigma(t_o) = -\frac{a\rho_o}{2r^{1/4}} [1 + f(w)] \tag{8.8}$$

$$E_1 = \frac{a\rho_o}{2\epsilon r^{1/2}} f(w) \tag{8.9}$$

$$\begin{aligned} \delta E(a) &= \frac{\sigma}{\epsilon} \frac{z_r}{a} + \frac{a - z_r}{\epsilon} \rho_o - \frac{(a - z_r)^2}{2\epsilon a} \rho_o \\ &= \frac{a\rho_o}{2\epsilon r^{1/4}} \left[1 - \frac{z_r}{a} f(w)\right] \end{aligned} \tag{8.10}$$

where

$$w \equiv \frac{\sqrt{\mathcal{R}}}{2R_-} \tag{8.11}$$

$$R_- \equiv \frac{\mu_-(V_o/a)}{v_-} \equiv \frac{u_-}{v_-} \tag{8.12}$$

(see Fig. 2 for an interpretation of u_-) and

$$f(w) = 2w \left\{w - (w + 1)e^{-1/w}\right\} \tag{8.13}$$

The extreme values of $f(w)$ are

$$\lim_{w \to 0} f(w) = 2w^2 \tag{8.14}$$

and

$$\lim_{w \to \infty} f(w) = 1. \tag{8.15}$$

The charge collected during the signal integration time is $Q = \epsilon\delta E(a)$. The collected charge for $r > 1$ relative to that collected at $r \ll 1$ (see equation 8.1), i.e. the "relative signal" is

$$S = \frac{1}{r^{1/4}} \left[1 - \frac{z_r}{a} f(w)\right]. \tag{8.16}$$

which, in the limits noted above, is

$$\lim_{w \to 0} S = \frac{1}{r^{1/4}} \tag{8.17}$$

and

$$\lim_{w \to \infty} S = \frac{1}{r^{1/2}}. \tag{8.18}$$

The limit $w \to 0$ occurs when the relative recombination rate $\mathcal{R}$ is small so the reservoir of ions and electrons in the region $0 < z < z_r$ is large. In this case the charge sheet, σ, accumulated at z_r is just the right amount to keep E_1, the incremental field in the region $0 < z < z_r$ near zero. Nevertheless it is large enough to speed up the reservoir of electrons a small amount leaving behind at z_r a small positive charge. This small positive charge at z_r is more than cancelled by the electrons that drift from the region $z > z_r$ and collect there. The net charge $\sigma(t_o)$ is half the electron charge originally deposited by the instantaneous ionization of density $-\rho_o$ in the region $z_r < z < a$. This limit is similar to the artificial example in section 5.5.

The limit $w \to \infty$ occurs when the relative recombination rate $\mathcal{R}$ is large so the reservoir of ions and electrons in the region $0 < z < r_r$ is small. While E_1 is now positive there are an insignificant number of electrons in this region to move away and expose the remaining positive ions. So all the electrons drifting from the region $z_r < z < a$ build up at z_r. This limit is similar to the artificial example in section 5.4.

[Discuss the result for reasonable values of the parameters and compare with the computer model to be discussed in the next section.]

Fig. 10 shows the relative signal as a function of r. In the simple approximation we have made, the relative signal is constant for $r < 1$. But for $r > 1$ the signal drops off steeply. The degree to which it drops off depends on $\mathcal{R}$. The full range of possibilities falls between the two curves.

9. DETAILS OF THE COMPUTER MODEL

10. FLUCTUATIONS IN THE IONIZATION RATE

[Borrow from work done with Andy Milder. Maybe put in something on "scaling moments".]

11. LIMITS IN THE OPERATION OF LIQUID ARGON CALORIMETERS

To translate all of this to the SSC or the LHC we will make the assumption that the calorimeter will not yield acceptable operation above $r = 1$. Further assume that the calorimeter has gap thickness $a = 2$ mm, potential across the gap of $V_0 = 2000$ V, that the face of the forward calorimeter is 12 m from the interaction point, and that $\mu_+ = 0.06$ mm^2/Vs. This last assumption is very uncertain and makes the following conclusions very tentative. (See section 4.) Fig. 11 shows the limits of operation of such a calorimeter at the nominal luminosity of the SSC and at a factor 10 above this nominal where the LHC, or the SSC some years after turn-on, might operate. The calorimeter beyond $|\eta| = 4.8$ will be in the region $r > 1$ and so, by our definition, will not work at luminosities of 10^{33} cm^{-2} sec^{-1}. At luminosities of 10^{34} cm^{-2} sec^{-1}the calorimeter will cease to work above $|\eta| = 4.0$. If the chosen value of μ_+ is low by a factor 2, then the two lines in Fig. 11 will move down by a factor 2 extending the range where the calorimeter will operate by $\Delta\eta = 0.2$. Raising V_o to 4000 V (doubling the average electric field) will lower the lines in Fig. 11 by a factor 4. This extends the operating range by $\Delta\eta = 0.4$. But near $r = 1$ the maximum electric field in the gap will be 4 KV/mm (see Fig. 6c) which may be difficult to achieve in this harsh environment. One could also decrease the gap width to 1 mm. This would also double the average electric field and the maximum electric field would again be 4 KV/mm but will lower the lines in Fig. 11 by a factor 16 increasing the range of operation of the calorimeter by $\Delta\eta = 0.8$. So with considerable care (and perhaps with some luck if the actual value of μ_+ is much larger than assumed here) a liquid ionization calorimeter might be made to work in the forward direction.

The choice $r = 1$ is clearly an arbitrary cutoff and depends on what performance is required. But near $r = 1$ the response of the calorimeter will be very unstable and is certainly a region that will cause great difficulties. One might argue that stable operation is all that is required and that while the relative signal for $r = 16$ is degraded to between 25% and 50%, the calorimeter still operates adequately for the larger energies where fluctuations in the minimum bias events are relatively small. Special electronics might be built to deal with the very large d.c. currents drawn from the high voltage power supplies.

12. ACKNOWLEDGEMENTS

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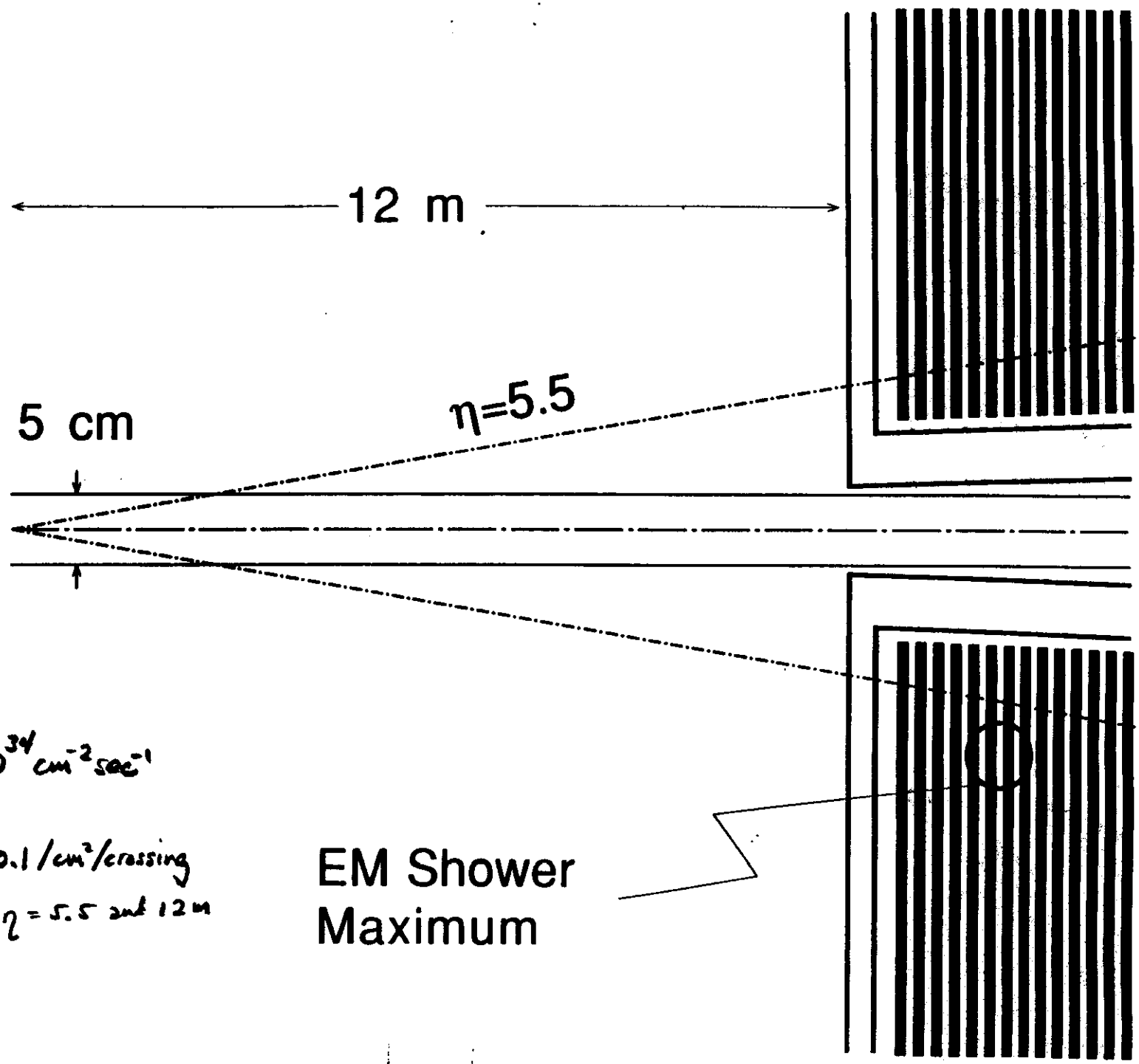
[Include references to Kadyk, Wentzel ...]



$\mathcal{L} = 10^{34}\,cm^{-2}\,sec^{-1}$

$\frac{dN_\gamma}{dA} \approx 0.1/cm^2/crossing$

at $\eta = 5.5$ and 12 m

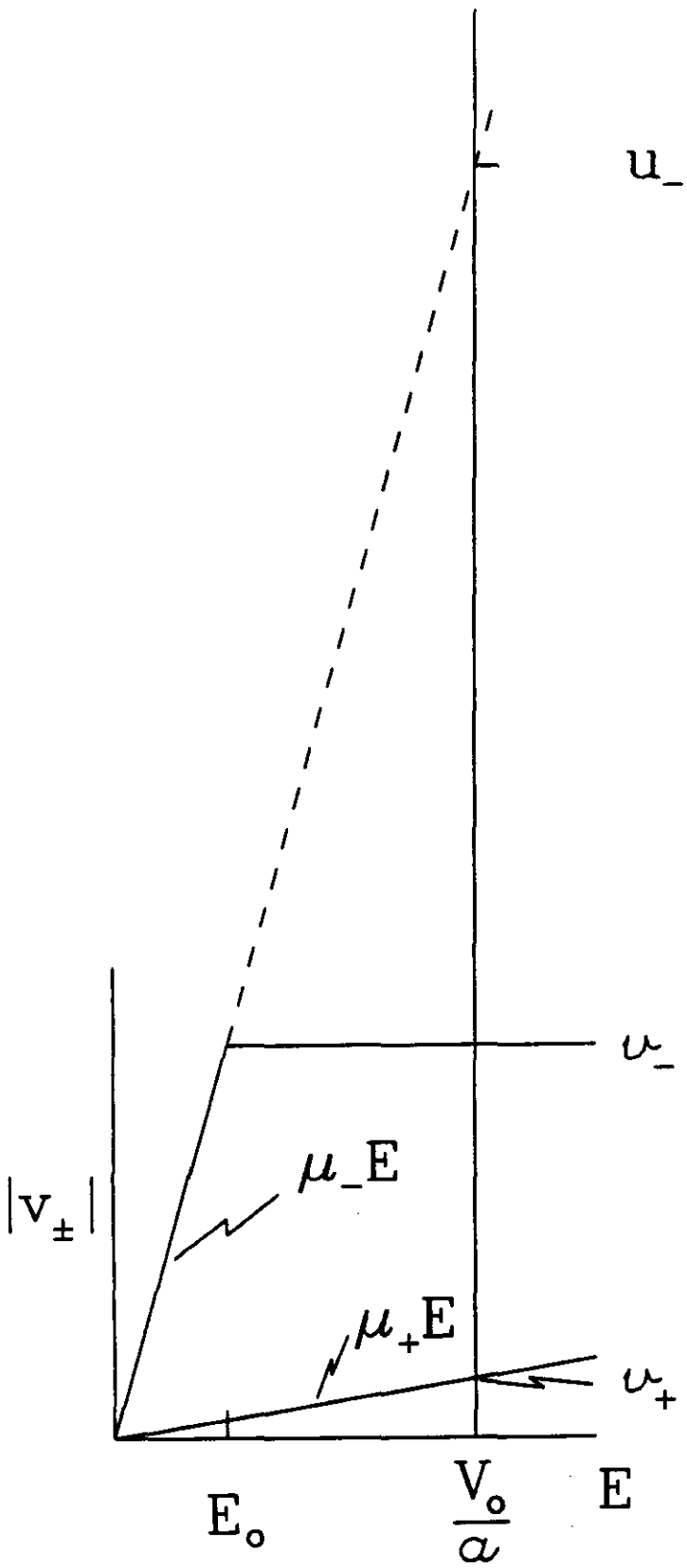
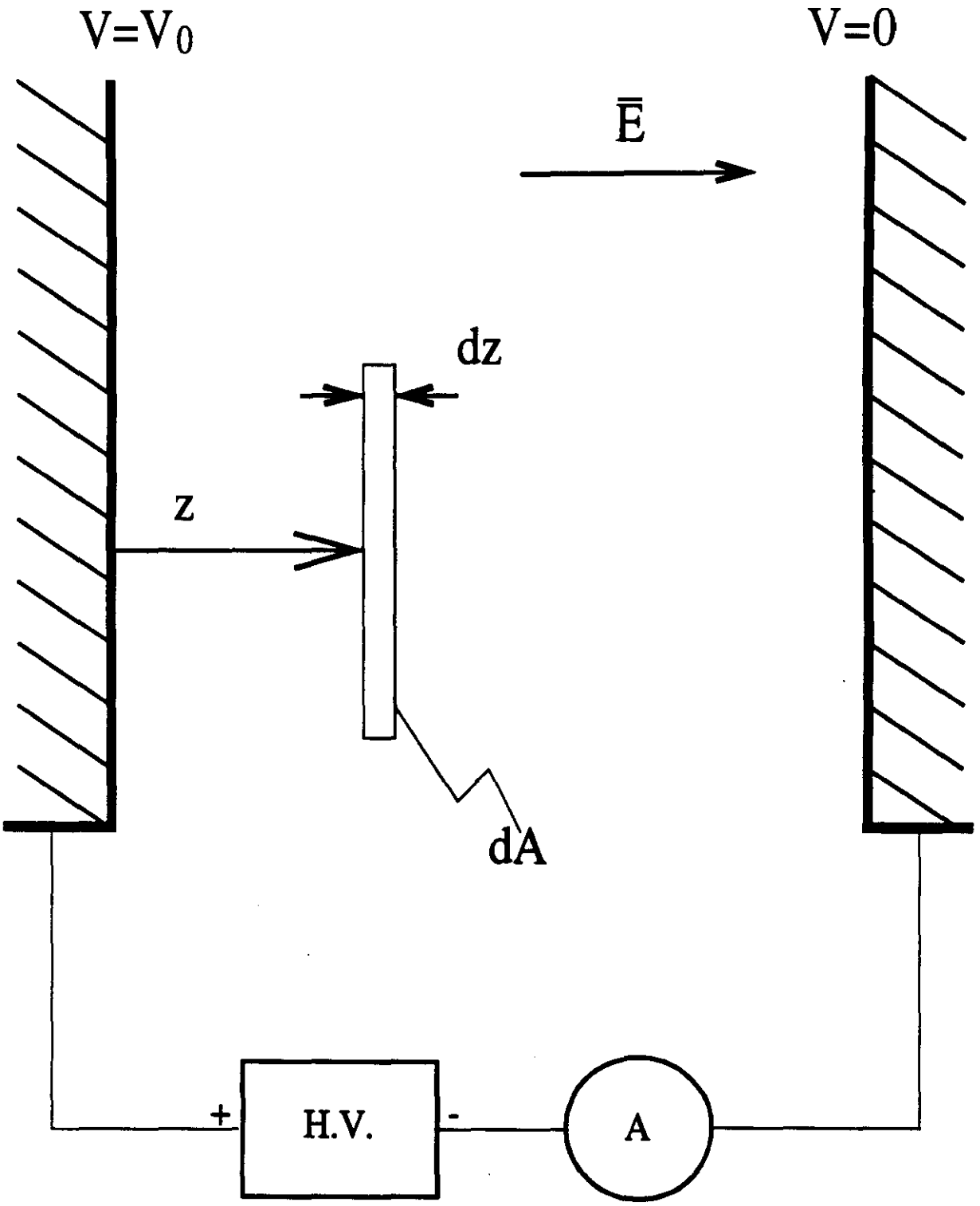


Fig. 5


V=V0
V=0
Ē
dz
z
dA
+
H.V.
-
A

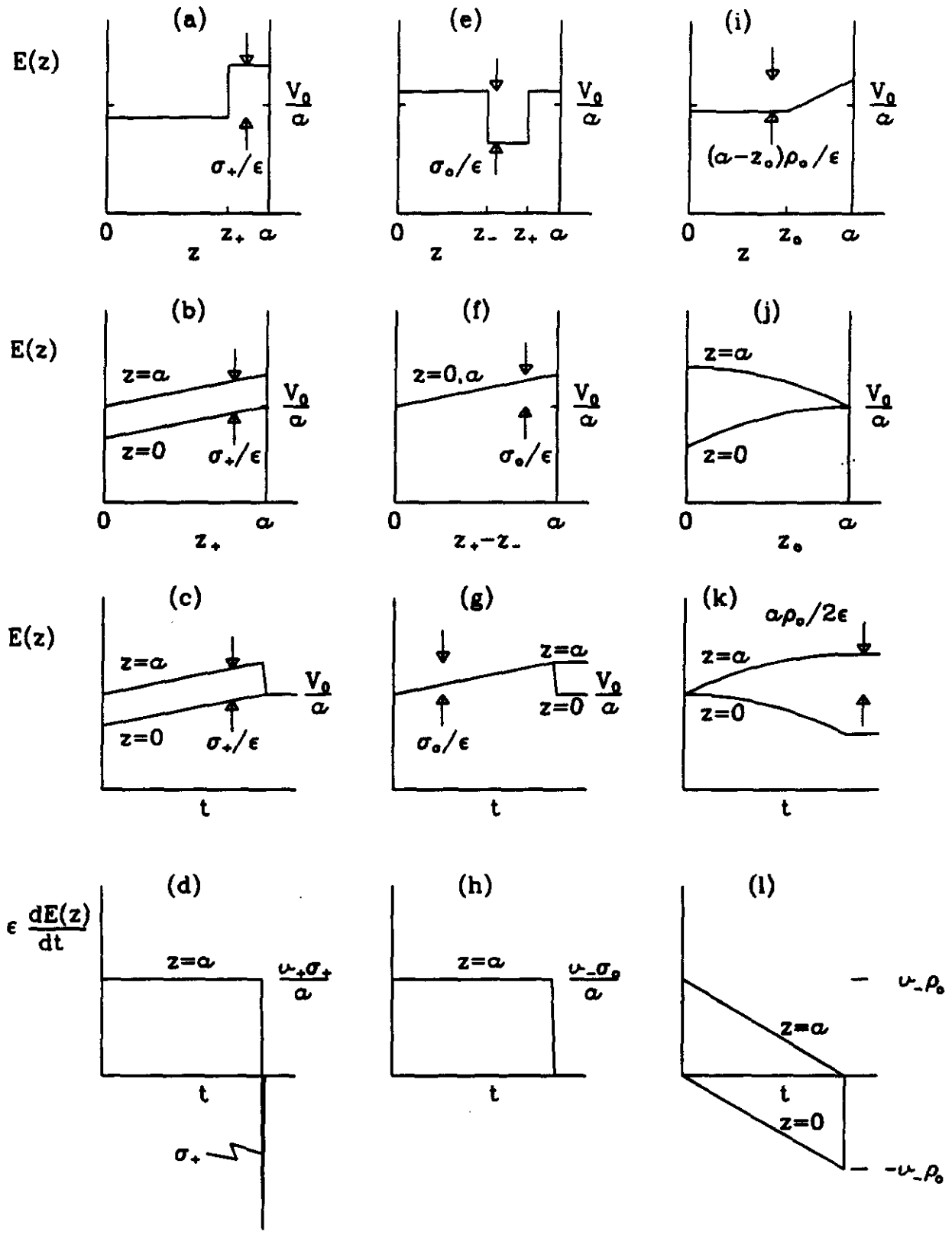

(a)
E(z)
V_0/a
σ_+/ϵ
0
z_+
a
z
(e)
V_0/a
σ_0/ϵ
0
z_-
z_+
a
z
(i)
V_0/a
$(a-z_0)\rho_0/\epsilon$
0
z_0
a
z
(b)
E(z)
z=a
z=0
V_0/a
σ_+/ϵ
0
a
z_+
(f)
z=0,a
V_0/a
σ_0/ϵ
0
a
z_+-z_-
(j)
z=a
z=0
V_0/a
0
a
z_0
(c)
E(z)
z=a
z=0
V_0/a
σ_+/ϵ
t
(g)
z=a
z=0
V_0/a
σ_0/ϵ
t
(k)
$a\rho_0/2\epsilon$
z=a
z=0
t
(d)
$\epsilon \frac{dE(z)}{dt}$
z=a
$\frac{v_+\sigma_+}{a}$
t
σ_+
(h)
z=a
$\frac{v_-\sigma_0}{a}$
t
(l)
$v_-\rho_0$
z=a
t
z=0
$-v_-\rho_0$

Fig. 4

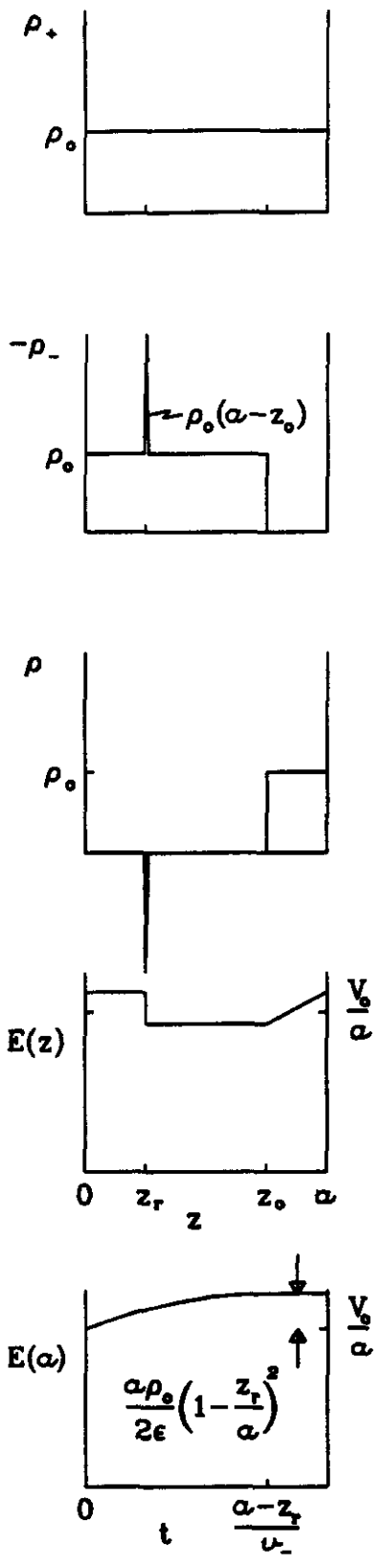
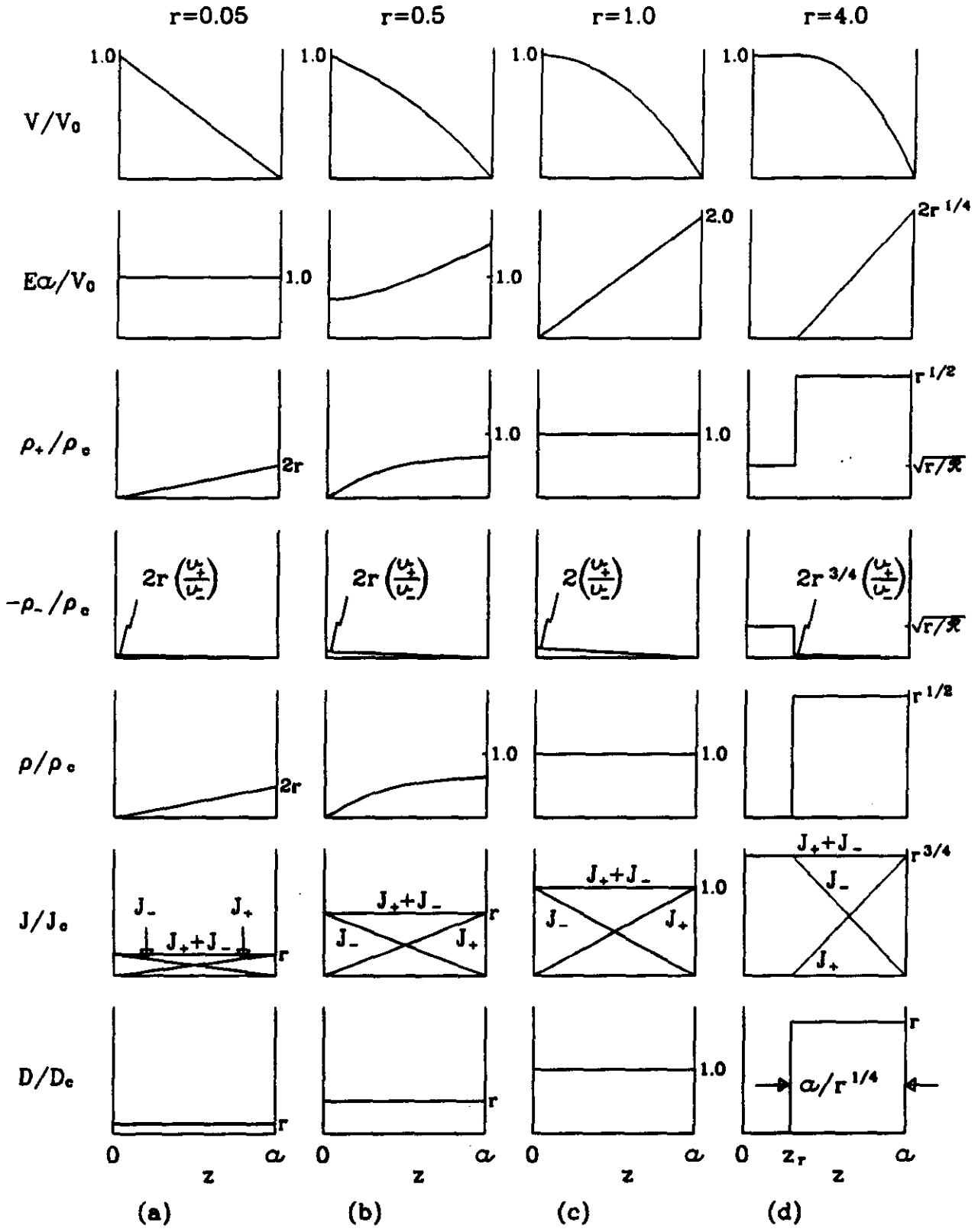
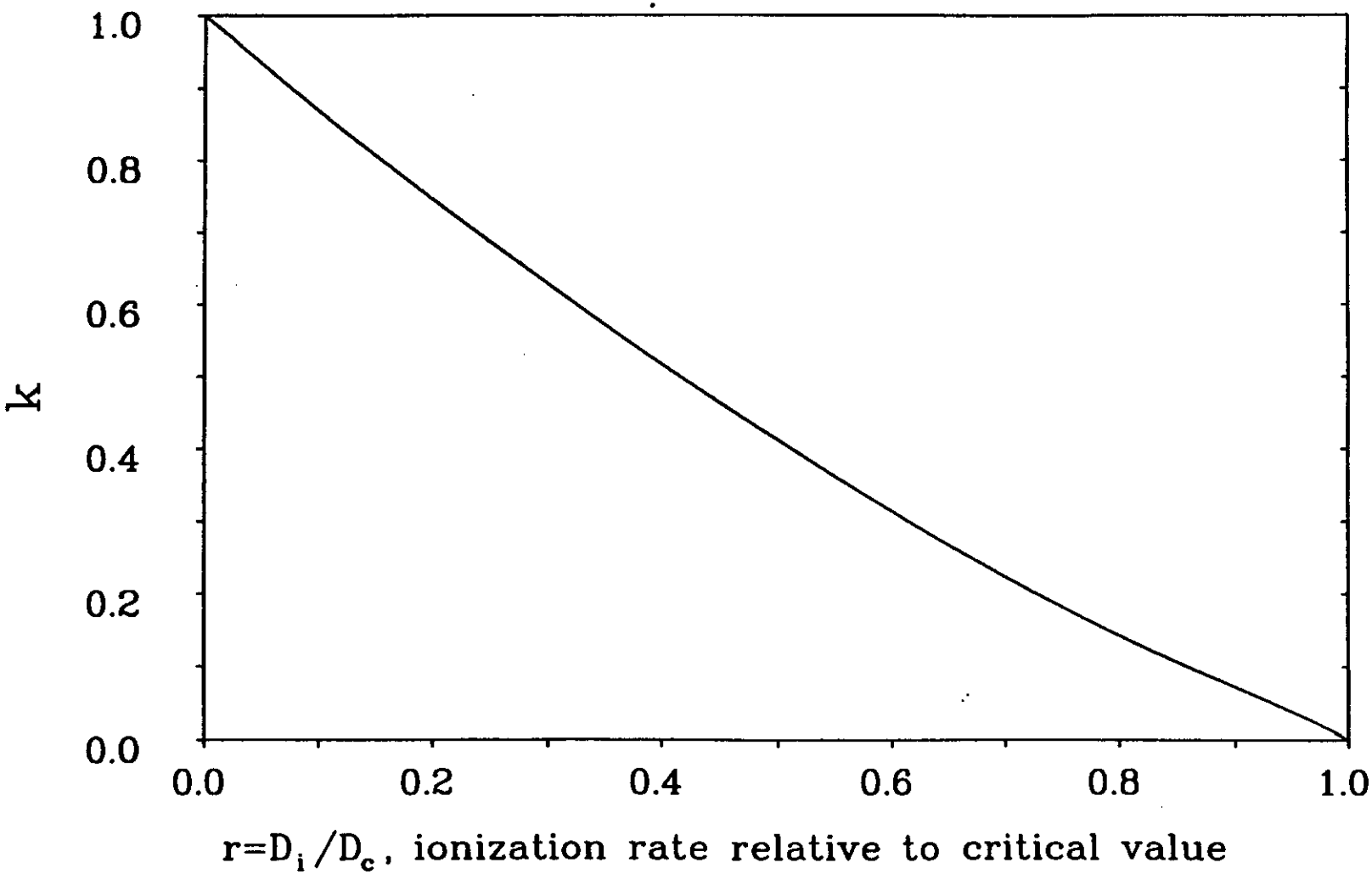
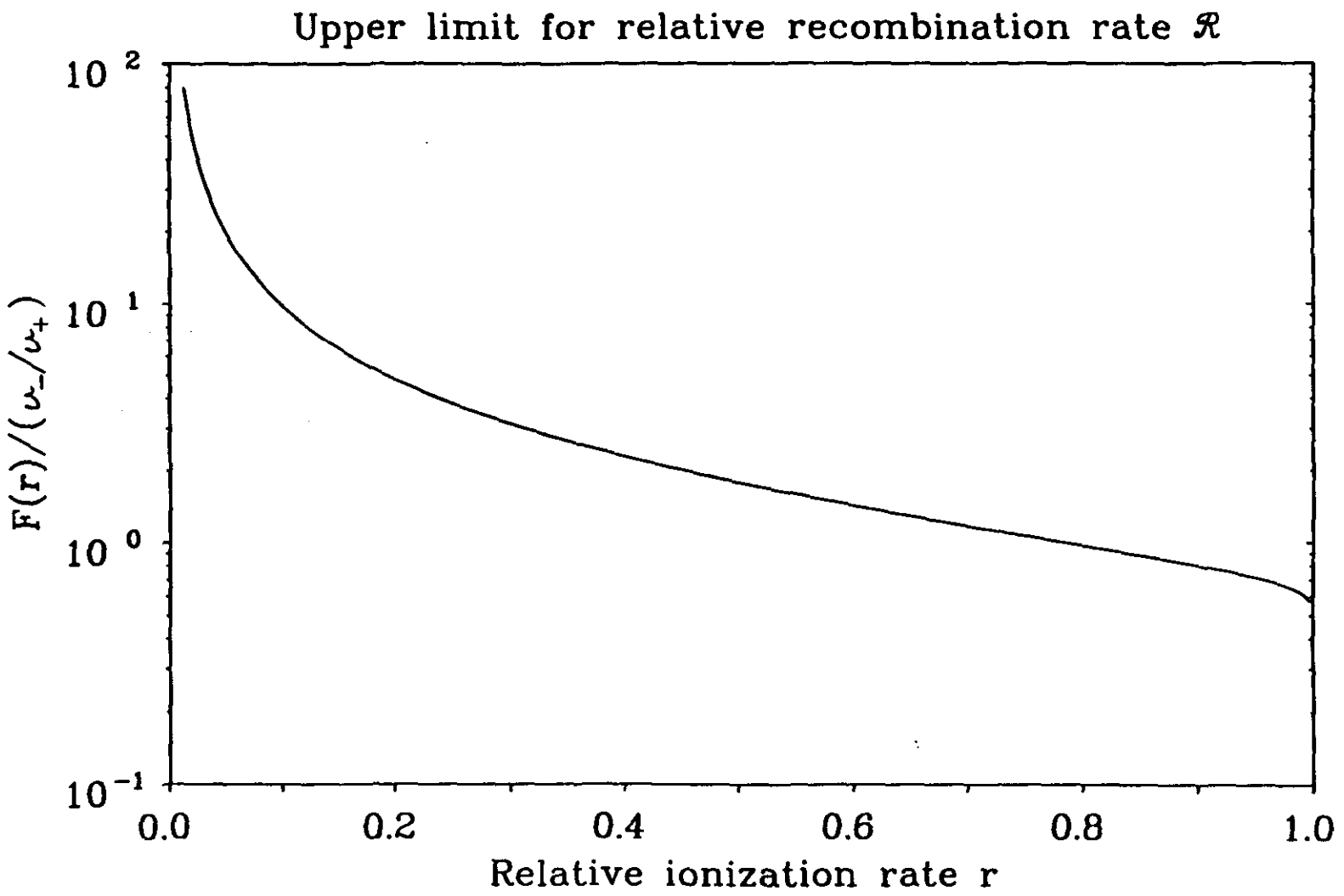

ρ_+
ρ_0
$-\rho_-$
$-\rho_0(a-z_0)$
ρ_0
ρ
ρ_0
E(z)
$\frac{V_0}{a}$
0
z_r
z
z_0
a
E(a)
$\frac{V_0}{a}$
$\frac{a\rho_0}{2\epsilon}\left(1-\frac{z_r}{a}\right)^2$
0
t
$\frac{a-z_r}{v_-}$

Fig. 5



r=0.05
r=0.5
r=1.0
r=4.0
V/V₀
Eα/V₀
ρ₊/ρ_c
−ρ₋/ρ_c
ρ/ρ_c
J/J_c
D/D_c
1.0
2.0
2r
2r^{1/4}
r^{1/2}
√r/ℜ
2r(υ₊/υ₋)
2r^{3/4}(υ₊/υ₋)
J₋
J₊
J₊+J₋
r^{3/4}
α/r^{1/4}
0
z
α
z_r
(a)
(b)
(c)
(d)


1.0
0.8
0.6
0.4
0.2
0.0
k
0.0
0.2
0.4
0.6
0.8
1.0
r=D_i/D_c, ionization rate relative to critical value


Upper limit for relative recombination rate $\mathcal{R}$
10^{2}
10^{1}
10^{0}
10^{-1}
$F(r)/(\nu_-/\nu_+)$
0.0
0.2
0.4
0.6
0.8
1.0
Relative ionization rate r

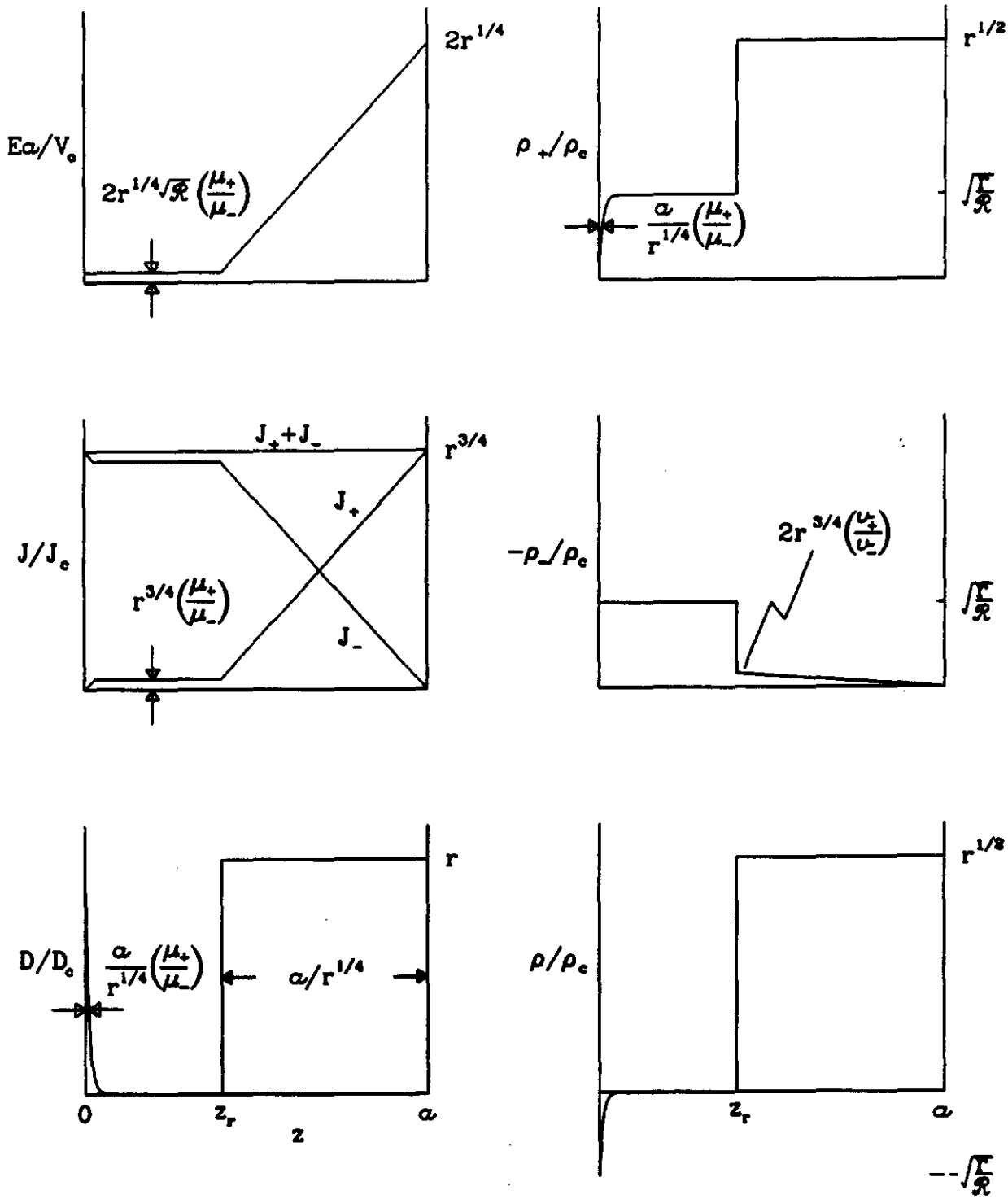
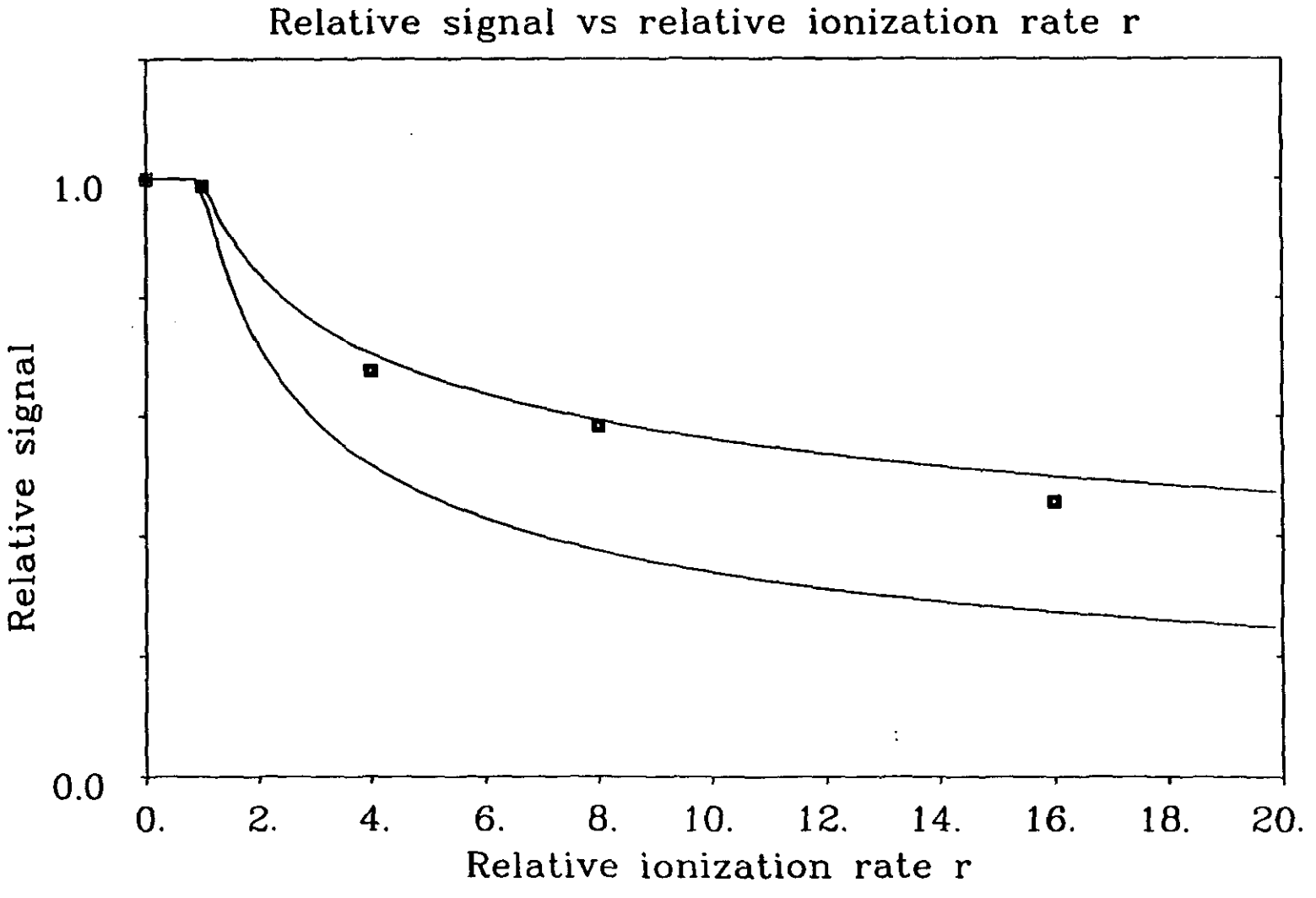
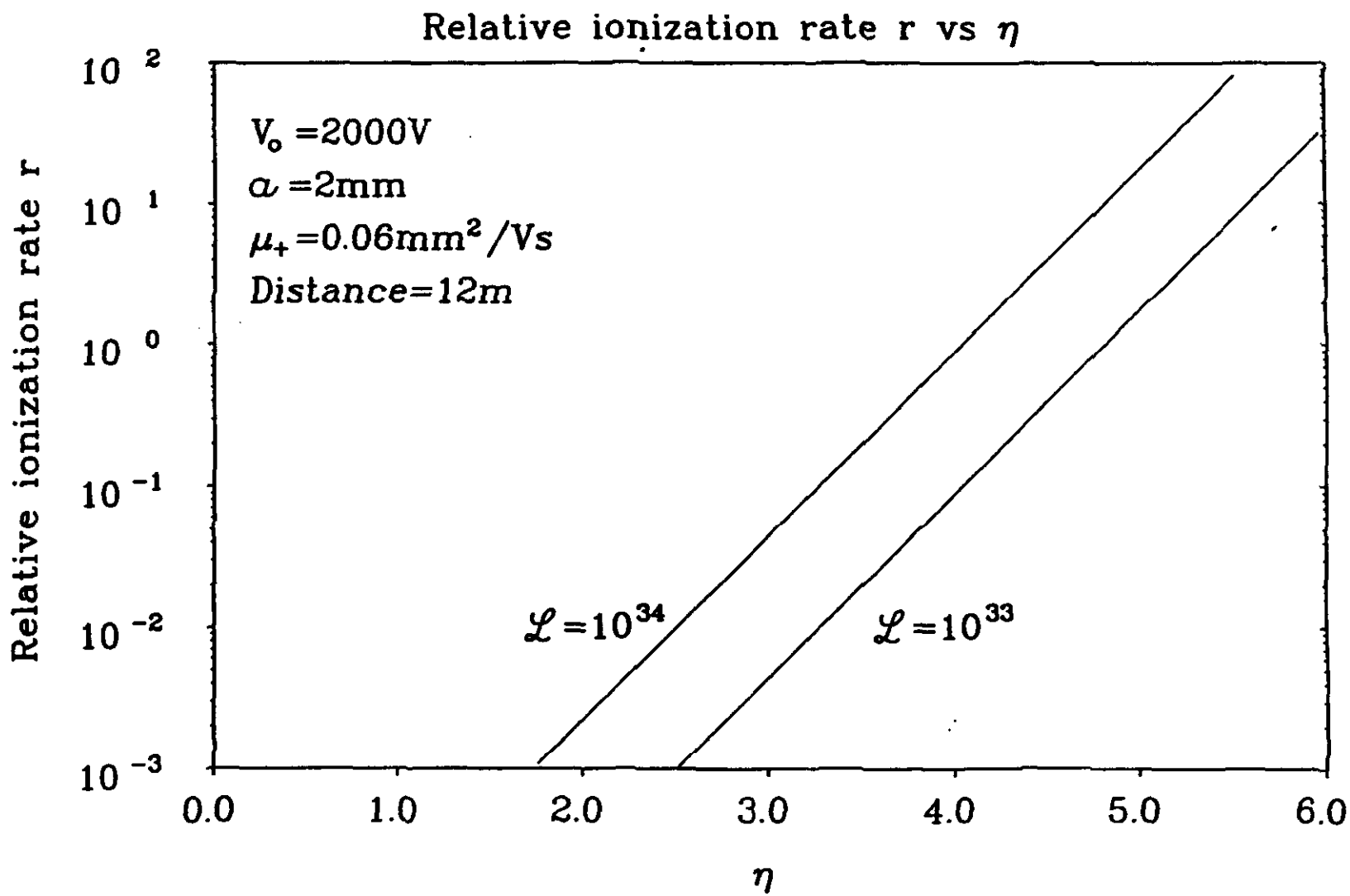

Ea/V_c
$2r^{1/4}$
$2r^{1/4}\sqrt{\mathcal{R}}\left(\frac{\mu_+}{\mu_-}\right)$
ρ_+/ρ_c
$r^{1/2}$
$\sqrt{\frac{r}{\mathcal{R}}}$
$\frac{a}{r^{1/4}}\left(\frac{\mu_+}{\mu_-}\right)$
J/J_c
$J_+ + J_-$
$r^{3/4}$
J_+
J_-
$r^{3/4}\left(\frac{\mu_+}{\mu_-}\right)$
$-\rho_-/\rho_c$
$2r^{3/4}\left(\frac{\upsilon_+}{\upsilon_-}\right)$
$\sqrt{\frac{r}{\mathcal{R}}}$
D/D_c
r
$\frac{a}{r^{1/4}}\left(\frac{\mu_+}{\mu_-}\right)$
$a/r^{1/4}$
0
z_r
a
z
ρ/ρ_c
$r^{1/2}$
z_r
a
$-\sqrt{\frac{r}{\mathcal{R}}}$

Fig. 1


Relative signal vs relative ionization rate r
1.0
0.0
Relative signal
0.
2.
4.
6.
8.
10.
12.
14.
16.
18.
20.
Relative ionization rate r


Relative ionization rate r vs η
Relative ionization rate r
10 2
10 1
10 0
10 -1
10 -2
10 -3
V₀ =2000V
a =2mm
μ₊ =0.06mm²/Vs
Distance=12m
ℒ=10³⁴
ℒ=10³³
0.0
1.0
2.0
3.0
4.0
5.0
6.0
η