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Superconducting Super Collider Laboratory

Options to Make PATRICIA Symplectic

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I Summary

Various options to "symplectify" the off-momentum linear transformations used in PATRICIA are presented. Strengths and weaknesses of each method are discussed. Numerical results from each method are outlined, and a recommendation for a 7-point Lagrange interpolation with single pass resymplectification is made.

II Options for "Symplectification"

Earlier SSC TM's have outlined various methods for computing off-momentum matrices for PATRICIA [1]. It was found that the existing Taylor's series based method was not symplectic and alternatives were suggested. We now review each of the alternatives.

A Leave Well Enough Alone

The current "third order in momentum deviation" scheme is not "well enough" as it concievably violates the symplectic condition at the present parameters. They could be misleading for the long (10^5 turn) runs currently envisioned.

B Improve the Exisiting Scheme

Perform the Taylor expansion in PATRICIA to $O(\delta^4)$ in an effort to make the violations of the symplectic condition smaller than machine roundoff. This has been done in a program described elsewhere [2]. Using some caution in obtaining the

coefficients of the expansion [3], it is possible to compute a nonsymplectic approximation to the desired off-momentum matrix, accurately through $O(\delta^4)$ with resulting $O(\delta^5)$ errors in symplecticity.

Although the resulting matrices are more accurate and more nearly symplectic than in the $O(\delta^3)$ approximation, numerical work with the aforementioned program show that the violations of the symplectic condition can occur within 64-bit arithmetic for sufficiently large momentum deviations. This technique will therefore not insure absolute symplecticity.

Moreover, the violations of the symplectic condition vary as δ^5 and thus the number of iterations required by the "resymplectification" algorithm [3] varies with δ. This implies that checks of symplecticity must be made when this method is employed; an undesireable situtation from the viewpoint of our c.p.u. - bound tracking calculation.

C Use a Symplectic Interpolation Scheme

Although desireable from a dynamical viewpoint, the fact that such schemes require matrix inversion and/or logical checks to determine appropriate generating functions indicates that they are undesireable in terms of computation time. This is especially true on the CRAY, where logic will impede compiler attempts to vectorise the calculation.

D Use of Alternative Interpolation Schemes

Using the aforementioned program, we have examined the use of 5-point and 7-point Lagrange interpolations [4] to establish a nonsymplectic approximate to an off-momentum matrix.

We have found that with the 7-point interpolation, the error lies consistently beyond the length of a 64-bit word. Moreover, it is of a consistent magnitude, in contrast to the error from the Taylor expansion currently in use, for which the error varies as $O(\delta^5)$. The Lagrange interpolation is therefore ideal for use in conjunction with resymplectification. The error expected in the result of such an interpolation is only weakly dependent on δ. Consequently, a set number of iterations of the resymplectification algorithm will "fix" any errors in symplecticity; no program logic is required.

Sample results from the δ^4 Taylors series expansion and 5 and 7 point Lagrange interpolations are given in Table 1. The relative "flatness" of the error from Lagrange interpolation is evident.

III Conclusion

A comparison of various methods of bringing PATRICIA to symplectic form indicates that use of a 7-point Lagrange interpolation, with resymplectification, appears to be the most desireable option from the viewpoint of accuracy and computing ease.

This method requires only slight changes in the PATRICIA coding, no additional program logic (unlike other methods considered) and is potentially very fast (requiring perhaps only a few more multiplies and adds than the $O(\delta^4)$ Taylor series method).

As a final check on the desireability of this method, we will make a timing comparison of the various methods to determine which is the most computatinually efficient.

IV References

1) Furman, M., SSC-TM-4001
Douglas, D., SSC-TM-4003

2) Douglas, D., op.cit.

3) Abramowitz and Stegun, AMS-55, p.914.

4) Furman, M., op.cit.

5) Abramowitz and Stegun, AMS-55, p.879.

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Table 1

Example Compairing Precision of Various Approximation Schemes for Case of 2 x 2 Quadrupole Matrix

($B' = 150T/m$, $L = 5m$, $B\rho = 66712.8$ T.m);

Errors Relative to Exact Matrix; Interpolation on the Range $\pm$ .008

	Error in M_{11}			Error in $(MJM)_{12}$		
$\delta = \delta P/Po$	δ^4 Taylor Series Expansion	5-Point Lagrange Interpolation	7-Point Lagrange Interpolation	δ^4 Taylor Series Expansion	5-Point Lagrange Interpolation	7-Point Lagrange Interpolation
.008	1.14×10^{-12}	0.	0.	1.92×10^{-13}	0.	0.
.004	3.15×10^{-14}	0.	5.73×10^{-19}	1.19×10^{-14}	0.	3.78×10^{-20}
.002	1.10×10^{-15}	3.93×10^{-14}	2.47×10^{-19}	6.93×10^{-16}	1.49×10^{-15}	1.42×10^{-20}
.001	4.13×10^{-17}	2.58×10^{-14}	2.86×10^{-19}	3.97×10^{-17}	9.81×10^{-16}	1.64×10^{-20}
.0005	1.73×10^{-18}	1.37×10^{-14}	1.67×10^{-19}	2.45×10^{-18}	5.21×10^{-16}	9.58×10^{-21}
.00025	8.12×10^{-20}	6.96×10^{-15}	9.65×10^{-20}	1.51×10^{-19}	2.62×10^{-16}	4.97×10^{-21}
.000125	4.12×10^{-21}	3.50×10^{-15}	4.36×10^{-20}	9.44×10^{-21}	1.32×10^{-16}	2.51×10^{-21}
.0000625	2.36×10^{-22}	1.75×10^{-15}	2.19×10^{-20}	5.76×10^{-22}	6.65×10^{-17}	1.26×10^{-21}
.00003125	1.37×10^{-23}	8.74×10^{-16}	1.09×10^{-20}	3.62×10^{-23}	3.32×10^{-17}	6.28×10^{-22}
.000015625	8.1×10^{-25}	4.37×10^{-16}	5.47×10^{-21}	2.21×10^{-24}	1.66×10^{-17}	3.14×10^{-22}
.0000078125	$5. \times 10^{-26}$	2.19×10^{-16}	2.74×10^{-21}	1.3×10^{-25}	8.31×10^{-18}	1.57×10^{-22}
.00000390625	0.	1.09×10^{-16}	1.37×10^{-21}	$1. \times 10^{-26}$	4.15×10^{-18}	7.86×10^{-23}

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