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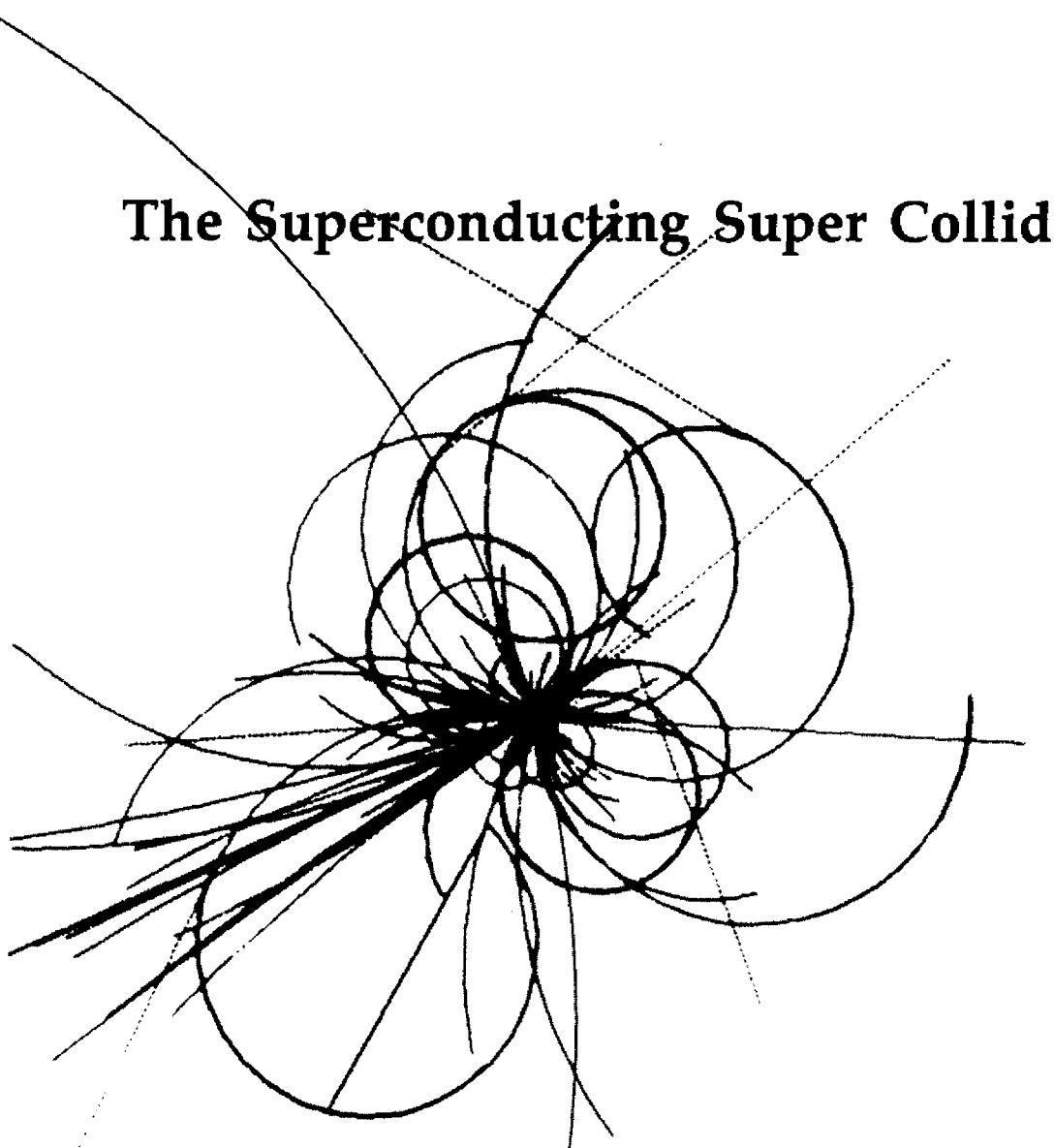
The Superconducting Super Collider



Long-Range Tune Shifts and Other Matters in High-β^* IRs

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1. Introduction

Two experimental configurations for the SSC have been discussed in which $\beta^\star$ is likely to be quite large:

1. Very low $p_\perp$ physics, especially for elastic scattering at less than 1 μrad. For the scattering angles to be large compared with the spread due to beam emittance, $\beta^\star \gtrsim 4$ km. There is no ancillary requirement for a long free space, but just for warm sections of modest size at phase advance $\approx 90°$ away from the interaction point (IP). For purposes of this Note, we take $\beta^\star = 4000$ m, and L (the distance between the IP and the place the beams are separated) as 85 m.
2. High-rapidity physics, or, more accurately, physics which covers as much of the available rapidity region as possible. Because of the need to measure particles at angles between 0.2 mrad and 400 mrad, the interaction region (IR) free space needs to be ± 1000 m or even ± 2000 m long. While $\beta^\star$ as small as 50 m would be desirable, it is likely to be very large compared with the bunch spacing. To focus ideas, we take $\beta^\star = 500$ m and $L = 1000$ m for the "bj IR." We also consider the case where the beams cross at an angle α and after a short distance (say 70 m) are brought parallel at a separation d for the remaining 930 m.

Both of these configurations present problems not discussed in the Conceptual Design Report (CDR), which in most cases used approximations in which $\beta^\star/S_B$ was a small number. (Here S_B is the bunch spacing, nominally 4.8 m.) One could of course solve all problems connected with high-$\beta^\star$ IRs if the machine were run in a dedicated mode with $S_B \approx 4$ km, but we assume for purposes of this Note that such an option would fail to win a DPF plebiscite.

The discussion by Diebold and Johnson in the Snowmass '84 Proceedings[1] is followed closely, the main departure being from their object of defining a low-$\beta^\star$ IR configuration.

For simplicity, "round" beams are assumed, *i.e.* the horizontal and vertical rms widths of the beams are assumed equal. This constraint is easily removed; it is just that the equations become longer and more opaque.

2. Geometry

Bunches of length $\sigma_\ell \approx 7$ cm are separated by $S_B \approx 4.8$ m. We will usually assume that straight beams in a (nearly) field free region of length $2L$ cross at an angle α, of order 100 μrad. The rms width σ of the beam is given by*

$$\sigma^2 = \beta\epsilon_N/\gamma \tag{1}$$

where β is the betatron amplitude, ϵ_N is the normalized emittance (nominally 10^{-6} m-rad everywhere), and γ is the usual Lorentz factor (2.13×10^4 at 20 TeV). At the interaction point (IP) β reaches its minimum value, $\beta^\star$, and it grows quadratically with distance ℓ from the IP:

$$\beta \approx \beta^\star + \ell^2/\beta^\star \tag{2}$$

From (1) and (2), we find

$$\sigma^2 = \sigma^{\star 2}(1 + (\ell/\beta^\star)^2) \quad . \tag{3}$$

In a low-$\beta^\star$ IR, we are used to thinking of σ as growing linearly with ℓ, since the first term is usually small compared with $(\ell/\beta^\star)^2$ at typical values of ℓ. The situation can be reversed, *e.g.* in the small-angle elastic scattering IR, where σ is nearly constant, as shown in Fig. 1. It is the persistence of a constant term (added in quadrature to a term which grows linearly) throughout the IR which makes the physics different when $\beta^\star$ is large.

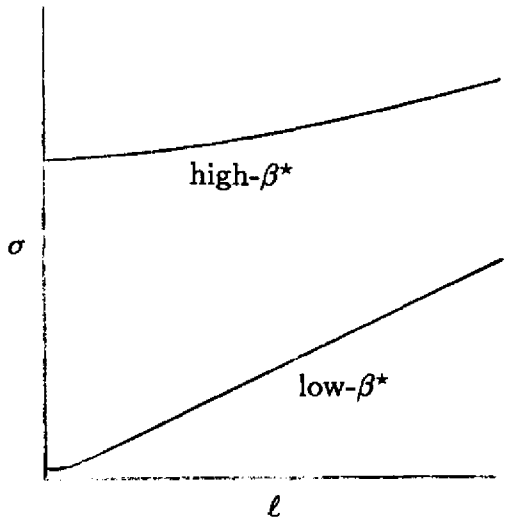

Fig. 1. $\sigma(\ell)$ in high-$\beta^\star$ and low-$\beta^\star$ IRs.

The picture is summarized in Fig. 2, where with the usual exaggerated vertical scale we indicate the bunch configuration at a time $S_B/2c$ after the crossing. Note that close encounters occur *every* time the bunches travel $S_B/2$.

* The denominator should actually be $(v/c)\gamma$, rather than γ. We ignore the distinction since (a) $v/c \approx 1$, and (b) β has already been used for something else.

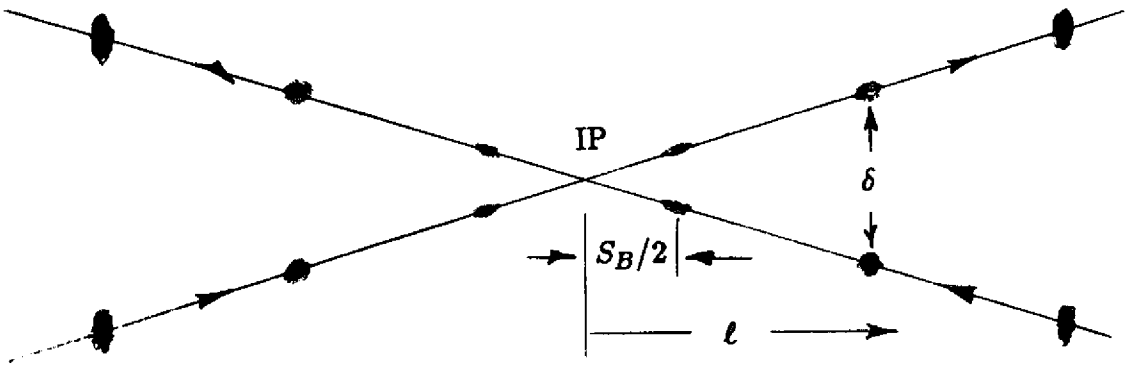

Fig. 2. IR geometry $S_B/2c$ after a crossing.

3. Bunch interaction at the first encounter

It should be clear that the relevant measure of transverse bunch separation is δ/σ, where δ is the actual separation of the centers. In general, it is smallest at the first encounter, where $\delta_1 = \alpha S_B/2$. Two considerations argue for a minimum value m of δ_1/σ:

1. If m is small, then there is substantial bunch interpenetration, and the transverse impulses received by protons at different transverse positions vary considerably—in other words, the beam-beam interaction is quite disruptive. *How* small is unclear, but the CDR uses $m \geq 7$.
2. There is also a high probability of p-p collisions if m is small, so the close encounter point becomes a "satellite luminosity" source. It is easily shown that the luminosity of such a source is $\exp(-\delta^2/4\sigma^2)$ times the central luminosity; only for $\delta > 5\sigma$ is it less than 10^{-3} of the central luminosity. Although one could probably live with satellite luminosity sources, it is better to avoid them.

The requirement $\delta/\sigma > m$ means that

$$\begin{aligned} \alpha &> \frac{2m}{S_B}\sigma^\star\sqrt{1+(S_B/2\beta^\star)^2} \\ &\gtrsim \frac{2m\sigma^\star}{S_B} = \frac{2m}{S_B}\sqrt{\beta^\star\epsilon_N/\gamma} \end{aligned} \tag{4}$$

For the nominal values of S_B, ϵ_N, and γ, we then have

$$\alpha \gtrsim m \times 90\ \mu\text{rad} \times \sqrt{\beta^\star/1000\ \text{m}} \tag{5}$$

The requirement that m should be greater than 5 or 7 will have especially serious

implications for the low-$p_\perp$ IR, where $\beta^\star \approx 4$ km, triplets start only ≈ 20 m from the IP, and satellite events will be hard to detect.

4. Long-range tune shifts

Diebold and Johnson give the long-range tune shift as

$$\Delta\nu_{LR} = 2\Delta\nu_{HO} \sum (\sigma_k/\delta_k)^2 \tag{6}$$

where $\Delta\nu_{HO}$ is the "head-on tune shift," produced by the interaction of the bunches at the IP. The summation is over close encounters in the IR, which in the case of the low-$\beta^\star$ and intermediate-$\beta^\star$ IRs essentially means those occurring between the ends of the first vertical bend dipoles. If the length of the common-beam region in the IR is $2L$, then the number of encounters n is $4L/S_B$. Except for a small reduction because of the finite crossing angle, the head-on tune shift is given by

$$\Delta\nu_{HO} = \frac{N_B r_p}{4\pi\epsilon_N} \quad , \tag{7}$$

where N_B is the number of protons in a bunch (nominally 7.3×10^9) and r_p is the proton classical radius, 1.535×10^{-18} m. For a normalized emittance of 10^{-6} m rad, $\Delta\nu_{HO} = 0.89 \times 10^{-3}$.

Although one can correct for tune shift, a *spread* in the tune shift leads to beam disruption. Our criterion that the long-range tune shift be small is really a requirement that the difference in the tune shifts experienced by different protons be small. There are as yet no good guidelines as to how large $\Delta\nu_{LR}$ can be before stability problems occur, but evidently $\Delta\nu_{LR}/\Delta\nu_{HO}$ should not be much larger than unity. (In the CDR, the ratio is about 3 for a high-$\beta^\star$ IR.)

Using (1) and (3), we may re-write (6) as

$$\Delta\nu_{LR}/\Delta\nu_{HO} = 2\sigma^{\star 2}\, 2\sum_{k=1}^{2L/S_B} \frac{[1 + (\ell_k/\beta^\star)^2]}{\delta_k^2} \tag{8}$$

$$= \frac{2\epsilon_N}{\gamma\beta^\star}\, 2\sum_{k=1}^{2L/S_B} \left[\left(\frac{\ell_k}{\delta_k}\right)^2 + \left(\frac{\beta^\star}{\delta_k}\right)^2\right] \tag{9}$$

where the factor of two in front of the summation accounts for encounters on both sides of the IP.

Let us first consider a "straight-beam" IR, with crossing angle α, so that $\delta_k = \alpha\ell_k$, and $\delta_k = k\alpha S_B/2$. From (9) we then obtain

$$\Delta\nu_{LR}/\Delta\nu_{HO} = \frac{2\epsilon_N}{\alpha^2\gamma\beta^\star} 2 \sum_{k=1}^{2L/S_B} \left[1 + \left(\frac{2\beta^\star}{S_B}\right)^2 \frac{1}{k^2}\right] \quad . \tag{10}$$

With $n = 4L/S_B$ (the number of encounters a bunch experiences on both sides of the IP), we thus obtain

$$\Delta\nu_{LR}/\Delta\nu_{HO} = \frac{2\epsilon_N}{\alpha^2\gamma\beta^\star} \left[n + 8\left(\frac{\beta^\star}{S_B}\right)^2 \sum_{k=1}^{2L/S_B} \frac{1}{k^2}\right] \quad . \tag{11}$$

Since $2L/S_B$ is a large number, we may replace the sum by an infinite sum, which we recognize as $\pi^2/6$. Finally,

$$\Delta\nu_{LR}/\Delta\nu_{HO} = \frac{2\epsilon_N}{\alpha^2\gamma\beta^\star} \left[n + \frac{4\pi^2}{3}\left(\frac{\beta^\star}{S_B}\right)^2\right] \quad . \tag{12}$$

The contribution of the second term is almost independent of the IR length, since most of the contribution to the sum comes from the first few close encounters. As was pointed out earlier, the width of the beam consists of a constant component and a piece which grows linearly with ℓ, added in quadrature. The first term in (12) comes from the growing part, and the second from the constant contribution. This result* is to be compared with Eq. 4.1-9 of the CDR, in which the second term is absent. The approximation is justified, for in both the low-$\beta^\star$ and intermediate-$\beta^\star$ IRs the first term easily dominates. However, for the low-$p_\perp$ IR, $\beta^\star/S_B \approx (4000\text{ m})/(4.8\text{ m})$, while n is still about 70. The second term is almost 10,000 times larger, although the problems this engenders are somewhat alleviated by the $1/\beta^\star$ out front. With the nominal values of ϵ_N and γ, we find

$$\begin{aligned}\Delta\nu_{LR}/\Delta\nu_{HO} &= (2.35\times10^{-14}/\alpha^2)(70 + 9.14\times10^6) \\ &= (463\ \mu\text{rad}/\alpha)^2 \quad .\end{aligned} \tag{13}$$

For the straight-beam case in the bj IR ($L = 1000$ m, $\beta^\star = 500$ m), we find

$$\begin{aligned}\Delta\nu_{LR}/\Delta\nu_{HO} &= (1.88\times10^{-13}/\alpha^2)(830 + 1.43\times10^5) \\ &= (164\ \mu\text{rad}/\alpha)^2 \quad .\end{aligned} \tag{14}$$

As was mentioned above, an alternate scheme for the bj IR is to cross the beams at a convenient angle α, then bend them parallel at some modest distance

* Curiously, Diebold and Johnson give this coefficient as 12, rather than $4\pi^2/3$. Close, but no cigar.

from the IR, which we take as $L_b = 70$ m. The separation d of the parallel beams is then αL_b. In this case it is easiest to start over with (9). We obtain the form (12) for the first 70 m (so $n = 58$), plus an additional term

$$\begin{aligned} &\frac{2\epsilon_N}{\gamma\beta^\star}\, 2 \sum_{k=1}^{2(L-L_b)/S_B} [1 + (\beta^\star/\alpha L_b)^2] \\ &= \frac{2\epsilon_N}{\gamma\beta^\star}[1 + (\beta^\star/\alpha L_b)^2][4(L - L_b)/S_B] \quad . \end{aligned} \tag{15}$$

for the effect of encounters in the remaining length. Combining the contributions of (12) and (14) for the assumed values, we find*

$$\begin{aligned} &\frac{2\epsilon_N}{\gamma\beta^\star}\left[\frac{1}{\alpha^2}(58 + 1.43 \times 10^5) + 775.0(1 + 51.0/\alpha^2)\right] \\ &= 1.88 \times 10^{-13}[775.0 + 1.83 \times 10^5/\alpha^2] \quad . \end{aligned} \tag{16}$$

The first term may be neglected since we expect $\alpha = O(10^{-4})$, and

$$\Delta\nu_{LR}/\Delta\nu_{HO} \approx (185\ \mu\text{rad}/\alpha)^2 \quad . \tag{17}$$

For $\alpha = 185\ \mu$rad, $d = 1.3$ cm. Comparing (14) and (17), we see that the penalty for fairly close parallel beams is small.

5. Piwinski effect

In a recent SSC Report,[2] Piwinski studies the effect of head-tail interactions of intersecting bunches with a finite crossing angle. He finds that the resulting coupling of synchrotron and betatron oscillations excites undesirable resonances if the crossing angle is much in excess of an angle α_0, where $\alpha_0 = 2\sigma^\star/\sigma_\ell$.† With the nominal SSC emittance and bunch length, we obtain

$$\alpha_0 = 6.19 \text{ mrad} \times \sqrt{\beta^\star/1000 \text{ m}} \quad . \tag{18}$$

Unlike the angles calculated in previous sections, α_0 is an *upper* limit. In terms

* At first sight, the dominance of $1/\alpha^2$ in the term from the parallel beam section is somewhat surprising. It is something of an artifact, arising from the $1/d^2$ dependence and the fact that $d = \alpha L_b$.

† We remember that the luminosity is reduced due to a finite crossing angle α by a factor $\sqrt{1 - (\alpha/\alpha_0)^2}$.

of the lower limits discussed in Section 3, we may write

$$\alpha_0 = m\frac{\sigma_\ell}{S_B} \quad . \tag{19}$$

For the nominal SSC parameters given in the CDR, $\sigma_\ell/S_B \approx 1/68$. The constraint does not seem to be very serious for either of the large-$\beta^\star$ IRs, unless one is attempting a wide, early beam separation in order to use separate quadrupoles.

6. Conclusions

A variety of effects having to do with beam crossing angle are examined for IRs in which $\beta^\star$ is large. Because the beam width does not grow substantially larger than $\sigma^\star$, results are qualitatively and quantitatively different than those for a low-$\beta^\star$ or intermediate-$\beta^\star$ IR. The results are shown graphically in Fig. 3, which may be compared with Fig. 5 in Ref. 1. Long-range tune shifts turn out never to be as important as the requirement for adequate bunch separation at the first encounter, which provides the limiting lower limit on crossing angle. Excitation of synchro-betatron oscillations provides an upper limit; IRs should be designed to work between these limits.

For the two examples studied, results may be summarized as follows:

1. *Low-$p_\perp$ IR*, in which $L \approx 85$ m but $\beta^\star \approx 4000$ m. To minimize long-range beam-beam effects, the crossing angle should exceed 460 μrad (or at worst, maybe $\sqrt{3}$ less). With nominal triplet spacing, as in the insertions presently under discussion, this would result in a 4 cm beam separation at the separation dipole. To avoid synchro-betatron effects, the angle must be less than 12.2 mrad, so separated triplets could be used if the triplet entrance were at $\ell \gtrsim 25$ m.

   There are also problems in obtaining sufficient beam separation at the first encounter. For $m = 7$, we see from (5) that $\alpha = 1.3$ mrad, with a resulting beam separation of 3.2 cm at 25 m. For satellite luminosity below the 10^{-3} level we require $\alpha > 900$ μrad.

   We are left with 12.2 mrad $> \alpha >$ 1.2 mrad, which is awkward from the point of view of beam separation at the triplet. While a solution might be to separate beams before the triplet and use a crossing angle of ~10 mrad, this would preclude sometimes operating at a smaller $\beta^\star$, which is considered essential.

2. *bj IR*, for which we have arbitrarily taken $L = 1000$ m and $\beta^\star = 500$ m. Beam-beam interaction constraints on the crossing angle are studied for two cases: In the first, no dipoles are used and the beams are assumed straight

for ± 1000 m. In this case the minimum crossing angle is (conservatively) 160 μrad, for a 16 cm spacing at the first triplet element. Secondly, we consider dipole bending to make the beams parallel after 70 m. As expected, the increased beam-beam interaction over a longer distance forces some increase in the crossing angle, but only to 185 μrad. The beam separation remains constant at 1.3 cm.

Our bunch separation criteria for the first encounter say $\alpha = 320$ μrad for $m = 5$, and $\alpha = 445$ μrad for $m = 7$. For the second and more conservative number, the dipole strength needed to produce parallel beams is 14.8 T-m. Without dipoles or close triplets, the beam separation would be 45 cm at the triplet. Many options exist for this IR (including obtaining a smaller β^*), and no insurmountable problems appear to be raised by the issues discussed in this report.

REFERENCES

1. R. Diebold and D. E. Johnson, *Proc. of the 1984 Summer Study on the Design and Utilization of the Superconducting Super Collider*, June 23-July13, 1984, Snowmass, Colorado, pp 399-408.
2. A. Piwinski, SSC Central Design Group Report SSC-N-57 (January 1986).

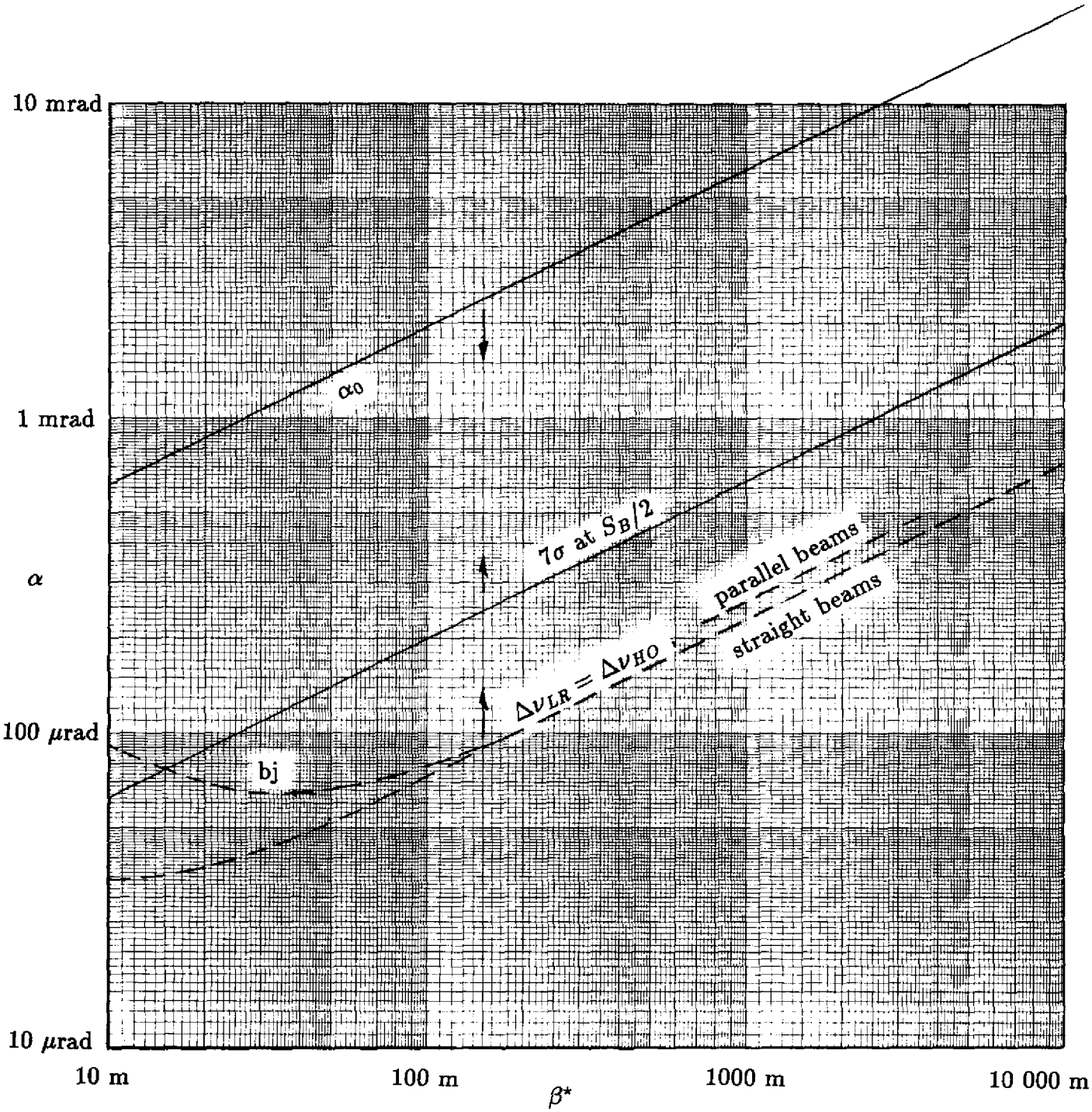

Fig. 3. Various limits on the crossing angle α as a function of β^*. Lower limits are marked by upward arrows; the upper limit by a downward arrow.